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On the perturbative expansion of a quantum field theory around a topological sector

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Abstract

The idea of treating general relativistic theories in a perturbative expansion around a topological theory has been recently put forward in the quantum gravity literature. Here we investigate the viability of this idea, by applying it to conventional Yang–Mills theory on flat spacetime. We find that the expansion around the topological theory coincides with the usual expansion around the abelian theory, though the equivalence is non–trivial. In this context, the technique appears therefore to be viable, but not to bring particularly new insights. Some implications for gravity are discussed.

1 Introduction

Conventional quantum field theory (QFT) treats interacting theories in a perturbative expansion around the quadratic term of the action. This procedure can be applied to general relativity (GR) by expanding the metric field around a fixed background, yielding the background–dependent perturbative quantum gravity of Feynman, DeWitt, Veltman and many others. The theory is non–renormalizable [1], and the approach makes sense only for computing low energy gravitons scattering. The question is open whether the non–renormalizability is an intristic feature of the theory –and GR must be modified to be a consistent theory–, or it has to do with the perturbative expansion chosen. In this case, GR could be non–perturbatively renormalizable, or even finite [2]. In this case however, a perturbative expansion, different from the standard one, could still be well defined.

An implementation of this idea has been recently proposed in [3], based on the fact that GR can be written as a modified BF theory. BF is a topological field theory that can be quantized exactly and is finite, as first showed by Witten in 3d [4]. The relevance of topological theories also for 4d quantum gravity is suspected since long [5], and has been exploited in a number of ways. The idea of [3], in particular, is to construct quantum GR via a perturbative expansion around the topological theory (see also [4]). This approach is radically new and raises a number of questions. In particular, the unperturbed theory has no local degrees of freedom. Does it make sense to expand a non–trivial QFT around a topological theory? In this paper, we address this question by studying such expansion in a simpler case: Yang–Mills (YM) theory on a flat background.

In fact, YM theory too can be written as a modified BF theory [6], sometimes called BFYM. The conventional perturbative expansion of BFYM, where the zero'th order is the kinetic term, has been investigated in detail in the literature (see for instance the last reference in [6]). Here we show, however, that the coupling constant g_0 can be moved in such a way that the zero'th order is BF theory. Therefore, the expansion in g_0 becomes an expansion around the topological theory, precisely as the one proposed for quantum gravity. We thus investigate the perturbative expansion of the theory in this formulation. In particular, we compute the propagator in a power series in g_0.

We find that the resulting expansion is equal to the standard one. In the non–abelian case, an interesting subtlety arises: the leading non–vanishing order is abelian, as expected, in spite of the non–abelian nature of the zero'th order BF term. We discuss the implications of this result for gravity in the final section 3.

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The possibility of expanding YM around BF opens the way to a homogeneous treatment of quantum gravity and YM theory. A proposal in this direction is under study [7], and the present work provides support for the viability of the approach.

2 Expansion around BF of Yang–Mills theory

Consider YM theory with gauge group $\mathcal{G}$, say SU(N). We call A^a_μ the YM connection. The index a is in the algebra $\mathfrak{g}$ of $\mathcal{G}$. The YM curvature is $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu$, where f^{abc} are the gauge group structure constants. We use greek letters $\mu = 0, \ldots, 3$ for spacetime indices and $[\mu\nu] = \mu\nu - \nu\mu$. For a compact notation, we use the formalism of p-forms, where $F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu = dA + \frac{1}{2}[A, A]$ and introduce the Hodge star $*$, defined by $*F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu}{}^{\rho\sigma} F_{\rho\sigma}$. We also introduce a $\mathfrak{g}$-valued 2-form B^a.

Consider the following two actions:

1. the standard second–order YM action

$$S_{\text{YM}}[A^a_\mu] = \frac{1}{4{g_0}^2} \int d^4x \; F^a_{\mu\nu} F^{\mu\nu}_a = -\frac{1}{2{g_0}^2} \int \; \text{Tr} \; F \wedge *F; \tag{1}$$

2. and the first order action

$$S_{\text{BFYM}}[B^a_{\mu\nu}, A^a_\nu] \quad = \quad i \int \; \text{Tr} \; B \wedge F - \frac{{g_0}^2}{2} \int \; \text{Tr} \; B \wedge *B. \tag{2}$$

The traces Tr are over the algebra indices. The equivalence of the two actions can be easily checked: the equation of motion

$$0 = \frac{\delta S_{\text{BFYM}}}{\delta B^a_{\mu\nu}} = \frac{i}{2} \epsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma} + {g_0}^2 B^{\mu\nu}_a$$

implies

$$B^a_{\mu\nu} = -\frac{i}{2{g_0}^2} \; \epsilon_{\mu\nu}{}^{\rho\sigma} F^a_{\rho\sigma}. \tag{3}$$

Plugging (3) into (2) and using $\epsilon^{\mu\nu\lambda\tau}\epsilon_{\mu\nu\rho\sigma} = -2\delta^{\lambda\tau}_{[\rho\sigma]}$, we obtain (1). Therefore, the two actions describe the same classical theory.

For $g_0 = 0$ the action (2) reduces to the action for BF theory. When using the action (1), one usually rescales the connection $A \mapsto g_0 A$. In this way the coupling constant is moved in front of the commutator term of the curvature, $F = dA + \frac{1}{2} g_0 [A, A]$. For $g_0 = 0$, thus, the scaled YM action reduces to the quadratic term.

To construct the quantum theory, we define the (Euclidean) n-point Green functions

$$\Gamma^{\text{YM}\, a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}[x_1 \ldots x_n; g_0] := \frac{1}{Z} \int \mathcal{D}A \; A^{a_1}_{\mu_1}(x_1) \ldots A^{a_n}_{\mu_n}(x_n) \; e^{-\frac{1}{4}\int \text{Tr}\,(dA + g_0[A,A])^2 - S_{\text{gf}}(A)}, \tag{4}$$

for the (rescaled) action (1), and

$$\Gamma^{\text{BFYM}\, a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}[x_1 \ldots x_n; g_0] := \frac{1}{Z} \int \mathcal{D}B \; \mathcal{D}A \; A^{a_1}_{\mu_1}(x_1) \ldots A^{a_n}_{\mu_n}(x_n) \; e^{-i\int \text{Tr}\, B\wedge F + \frac{{g_0}^2}{2}\int \text{Tr}\, B \wedge *B - S_{\text{gf}}(B,A)}, \tag{5}$$

for the action (2). Here Z is the partition function, namely the functional integral without the field insertions, and $S_{\text{gf}}(A)$ and $S_{\text{gf}}(B, A)$ are appropriate gauge–fixing terms. In the following we use the Lorentz gauge.

Formally, these Green functions are equivalent: integrating over the B field in (5), we indeed obtain

$$\Gamma^{\text{YM}\, a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}[x_1 \ldots x_n; g_0] = \frac{1}{{g_0}^n} \Gamma^{\text{BFYM}\, a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}[x_1 \ldots x_n; g_0]. \tag{6}$$

The factor ${g_0}^{-n}$ comes from the rescaling used in (4). Since n-point Green functions are the basic building blocks of QFT, the equivalence above would guarantee that the two actions give rise to the same quantum theory. However, the equivalence is only formal, until we can actually define and evaluate the integrals. In QFT, the conventional way of doing so is by expanding the functional integrals in g_0 around a gaussian

integral, that can be evaluated. If the equivalence holds, then the two expansions should coincide, order by order. That is, (getting rid of indices and arguments)

$$\Gamma^{\text{YM}(0)} + g_0\, \Gamma^{\text{YM}(1)} + {g_0}^2\, \Gamma^{\text{YM}(2)} + \ldots = \frac{1}{{g_0}^n} \left[\Gamma^{\text{BFYM}(0)} + {g_0}^2\, \Gamma^{\text{BFYM}(1)} + {g_0}^4\, \Gamma^{\text{BFYM}(2)} + \ldots\right]. \tag{7}$$

However, there are different reasons for doubting this equivalence, and it is far from obvious how the two expansions could coincide. In particular, the two zero'th orders are extremely different:

- In (4), the zero'th order is a free abelian theory, namely a gaussian integral, which depends on the metric and has lost track of the non–abelian structure,

$$\Gamma^{\text{M}a_1\ldots a_n}_{\mu_1\ldots\mu_n}[x_1\ldots x_n] := \frac{1}{Z} \int \mathcal{D}A\; A^{a_1}_{\mu_1}(x_1)\ldots A^{a_n}_{\mu_n}(x_n)\; e^{-\frac{1}{4}\int \text{Tr}\,(dA)^2 - S_{\text{gf}}(A)}.$$

- In (5), the other way around, the zero'th order is not gaussian, the theory depends on the non–abelian structure and has lost track of the metric of spacetime,

$$\Gamma^{\text{BF}a_1\ldots a_n}_{\mu_1\ldots\mu_n}[x_1\ldots x_n] := \frac{1}{Z} \int \mathcal{D}B\; \mathcal{D}A\; A^{a_1}_{\mu_1}(x_1)\ldots A^{a_n}_{\mu_n}(x_n)\; e^{-i\int \text{Tr}\, B\wedge F - S_{\text{gf}}(B,A)}.$$

How can starting points that are so different give rise to the same Green functions? Below we show that they do. The differences are harmless, and (7) holds, order by order. For simplicity, we focus on the 2-point Green function.

The perturbative expansion of (5) reads

$$\begin{aligned}
\Gamma^{\text{BFYM}ab}_{\mu\nu}[x,y;g_0] &= \frac{1}{{g_0}^2}\,\frac{1}{Z}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{{g_0}^2}{2}\right)^k \int \mathcal{D}B\; \mathcal{D}A\; A^a_\mu(x)A^b_\nu(y)\; \left(\int \text{Tr}\, B\wedge *B\right)^k\; e^{-i\int \text{Tr}\, B\wedge F - S_{\text{gf}}(B,A)} \\
&= \frac{1}{{g_0}^2}\,\Gamma^{\text{BFYM}(0)ab}_{\mu\nu}[x,y] + \Gamma^{\text{BFYM}(1)ab}_{\mu\nu}[x,y] + {g_0}^2\,\Gamma^{\text{BFYM}(2)ab}_{\mu\nu}[x,y] + \ldots
\end{aligned} \tag{8}$$

For $k=0$ we have

$$\Gamma^{\text{BFYM}(0)ab}_{\mu\nu}[x,y] = \frac{1}{Z}\int \mathcal{D}B\; \mathcal{D}A\; A^a_\mu(x)A^b_\nu(y)\; e^{-i\int \text{Tr}\, B\wedge F - S_{\text{gf}}(B,A)}; \tag{9}$$

the gauge–fixing term for the B field is needed to cancel the components of the B fields, which are integrated over, but do not enter the BF action because of the Bianchi identity on the curvature. We perform the $\mathcal{D}B$ integral and obtain

$$\Gamma^{\text{BFYM}(0)ab}_{\mu\nu}[x,y] = \frac{1}{Z}\int \mathcal{D}A\; A^a_\mu(x)A^b_\nu(y)\; \delta(F)\; e^{-S_{\text{gf}}(A)} \equiv 0, \tag{10}$$

because in the Lorentz gauge $F=0$ implies $A=0$. Therefore, the zero'th order does not contribute to the physics: its correlators vanish.

The leading order of (7) is then proved if we can show that, in the Lorentz gauge,

$$\Gamma^{\text{BFYM}(1)ab}_{\mu\nu}[x,y] = \Gamma^{\text{M}ab}_{\mu\nu}[x,y] = \int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip\;(x-y)}}{p^2}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\delta^{ab}. \tag{11}$$

To do so, we consider separately the abelian and the non–abelian cases.

2.1 Free propagator: abelian case

In the abelian case, the curvature is simply given by the exterior derivative of the connection, $F = dA$, and both the curvature and the B field are gauge–invariant. The gauge–fixing term only concernes the connection. To calculate the propagator, we introduce a source $j^\mu(x)$ for the connection, and write ($j\cdot A$ stands for $j^\mu A_\mu$)

$$\begin{aligned}
\Gamma^{\text{BFYM}(1)}{}_{\mu\nu}[x,y] &= \frac{1}{2}\int \mathcal{D}B\; \mathcal{D}A\; A_\mu(x)A_\nu(y)\int B\wedge *B\; e^{-i\int B\wedge dA - S_{\text{gf}}(A)} = \\
&= -\frac{1}{2}\frac{\delta^2}{\delta j^\mu(x)\delta j^\nu(y)}\int \mathcal{D}B\; \mathcal{D}A\int B\wedge *B\; e^{-i\int B\wedge dA - S_{\text{gf}}(A) + i\int j\cdot A}\Big|_{j=0}.
\end{aligned} \tag{12}$$

We perform first the $\mathcal{D}A$ integral. Because of the Lorentz gauge–fixing term, this integral selects only the transverse components of the source, $j^\mu_{\rm T} := (\delta^\mu_\nu - \frac{p^\mu p_\nu}{p^2})j^\nu$. Integrating by parts in the action, we thus get

$$\Gamma^{{\rm BFYM}(1)}{}_{\mu\nu}[x,y] = -\frac{1}{2}\frac{\delta^2}{\delta j^\mu(x)\delta j^\nu(y)} \int \mathcal{D}B \int B\wedge *B\ \delta(*dB + j_{\rm T}). \tag{13}$$

In order to evaluate the remaining integral, we have to find solutions of the equation $*dB = -j_{\rm T}$, which in Fourier–transformed components reads $\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}p_\nu B_{\rho\sigma}(p) = -j^\mu_{\rm T}(p)$. This can be easily solved in B, to give

$$\epsilon^{\alpha\beta\rho\sigma}B_{\rho\sigma}(p) = \frac{2}{p^2}\ p^{[\alpha}j^{\beta]}_{\rm T}(p).$$

It follows that

$$\int \mathcal{D}B \int B\wedge *B\ \delta(*dB + j_{\rm T}) = -2\int \frac{d^4p}{(2\pi)^4}\ \frac{1}{p^2}\ j^\alpha_{\rm T}(p)\ j^{\rm T}_\alpha(-p).$$

Plugging this result in (13) it is straightforward to obtain

$$\Gamma^{{\rm BFYM}(1)}{}_{\mu\nu}[x,y] = \int \frac{d^4p}{(2\pi)^4}\ \frac{e^{-ip\ (x-y)}}{p^2}\Big(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\Big).$$

It is easy to check that in this abelian case $\Gamma^{{\rm BFYM}(k)}$ for all $k>1$, so that we actually verify (11) exactly.

2.2 Free propagator: non–abelian case

In the non–abelian case, the situation is more subtle. As above, we introduce a source j^μ_a for the connection, but also a source $\eta^{\mu\nu}_a$ for the B field. We can then write (here $\eta\cdot B$ stands for $\eta^{\mu\nu}B_{\mu\nu}$)

$$\begin{aligned}\Gamma^{{\rm BFYM}(1)ab}_{\mu\nu}[x,y] &= \frac{1}{2}\int \mathcal{D}B\ \mathcal{D}A\ A^a_\mu(x)A^b_\nu(y)\int {\rm Tr}\, B\wedge *B\ e^{-i\int {\rm Tr}\, B\wedge F - S_{\rm gf}(B,A)} = \\ &= \frac{1}{8}\frac{\delta^2}{\delta j^\mu_a(x)\delta j^\nu_b(y)}\int dz\frac{\delta}{\delta\eta^c_{\rho\sigma}(z)}\frac{\delta}{\delta\eta^{\rho\sigma}_c(z)}\int \mathcal{D}B\ \mathcal{D}A\ e^{-i\int {\rm Tr}\, B\wedge F - S_{\rm gf}(B,A) + i\int {\rm Tr}\, j\cdot A + i\int {\rm Tr}\,\eta\cdot B}\Big|_{j=\eta=0}.\end{aligned}$$

Once again, the gauge–fixing term for B is needed to cancel the components which do not enter the BF action. It is convenient this time to perform the $\mathcal{D}B$ integral first, obtaining

$$\begin{aligned}\Gamma^{{\rm BFYM}(1)ab}_{\mu\nu}[x,y] &= \frac{1}{8}\frac{\delta^2}{\delta j^\mu_a(x)\delta j^\nu_b(y)}\int dz\frac{\delta}{\delta\eta^c_{\rho\sigma}(z)}\frac{\delta}{\delta\eta^{\rho\sigma}_c(z)}\int \mathcal{D}A\ \delta\Big(*F(A)-\eta\Big)\ e^{-S_{\rm gf}(A)+i\int {\rm Tr}\, j\cdot A}\Big|_{j=\eta=0} \\ &= \frac{1}{8}\frac{\delta^2}{\delta j^\mu_a(x)\delta j^\nu_b(y)}\int dz\frac{\delta}{\delta\eta^c_{\rho\sigma}(z)}\frac{\delta}{\delta\eta^{\rho\sigma}_c(z)}\ e^{i\int {\rm Tr}\, j\cdot A_\eta}\Big|_{j=\eta=0} = \\ &= -\frac{1}{8}\int dz\ \frac{\delta A_{\eta\mu}{}^a(x)}{\delta\eta^{\rho\sigma}_c(z)}\ \frac{\delta A_{\eta\nu}{}^b(y)}{\delta\eta^c_{\rho\sigma}(z)}\Big|_{\eta=0}.\end{aligned} \tag{14}$$

Here we have introduced the solution A_η of the equation

$$F^a_{\mu\nu}(A) = \epsilon_{\mu\nu\rho\sigma}\,\eta^{\rho\sigma}_a \tag{15}$$

for the connection. The equation is non–linear, and we do not know the analitic form of the solution. However, as we see from (14), only the linear dependence of A_η on η matters. Let us therefore expand A_η in a power series in η,

$$A_{\eta\mu}{}^a(x) = \int dy\ G_1{}^{ab}_{\mu\nu\rho}(x,y)\ \eta^{\nu\rho}_b(y) + \int dydz\ G_2{}^{abc}_{\mu\nu\rho\sigma\tau}(x,y,z)\ \eta^{\nu\rho}_b(y)\eta^{\sigma\tau}_c(z) + \dots \tag{16}$$

When we insert (16) in (15), the leading order, in Fourier components, is

$$p_{[\mu}\,G_1{}^{ab}_{\nu]\alpha\beta}(p)\,\eta^{\alpha\beta}_b(p) = \epsilon_{\mu\nu\rho\sigma}\,\eta^{\rho\sigma}_a(p).$$

Remarkably, G_1 is only sensible to the abelian structure of the theory. In the Lorentz gauge we can solve this equation, obtaining

$$G_1{}^{ab}_{\nu\alpha\beta}(p)\,\eta_b^{\alpha\beta}(p) = \frac{1}{p^2}\,\epsilon_{\mu\nu\rho\sigma}\,p^\mu\,\eta_a^{\rho\sigma}(p). \tag{17}$$

Using this in (14) we immediately get

$$\begin{aligned}\Gamma^{\mathrm{BFYM}(1)ab}_{\mu\nu}[x,y] &= -\frac{1}{2}\int dz\,G_1{}^{ac}_{\mu\rho\sigma}(x,z)\,G_1{}^{bc}_{\nu\rho\sigma}(y,z) = \\ &= \int\frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip\ (x-y)}}{p^2}\left(\eta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\delta^{ab}.\end{aligned} \tag{18}$$

The non–abelian structure, yet present at the beginning of the calculation, plays no role in the final result.

Notice how the non–abelian structure comes into play at the next order: when we evaluate $\Gamma^{\mathrm{BFYM}(2)}$, the term $\left(\int \mathrm{Tr}\,B\wedge *B\right)^2$ gives four functional derivatives $\delta/\delta\eta$, which make the quadratic dependence of A_η on η enter the final expression. But (15) at quadratic order in η depends on the non–abelian structure, and we expect to obtain the usual ${g_0}^2$ corrections to the free propagator. We do not perform here the explicit calculations.

From the discussion above, we conclude that the perturbative expansion around BF of YM theory on flat spacetime coincides with the conventional one.

3 Consequences for gravity

Two different formulations of GR as modified BF theories have been considered in the literature, as starting point for the quantisation:

1. The Plebanski action,

$$S[B^{IJ}_{\mu\nu},\omega^{IJ}_\mu,\mu_{IJKL}] = \frac{1}{16\pi G}\int \mathrm{Tr}\,B\wedge F(\omega) + \int \mu_{IJKL}\,B^{IJ}\wedge B^{KL}, \tag{19}$$

used in [8]. The latin indices are in the group $SO(3,1)$ (or $SO(4)$ in the Riemannian version). This has been used for the quantum gravity models described in [9].

2. The MacDowell–Mansouri action,

$$S[B^{IJ}_{\mu\nu},\omega^{IJ}_\mu] = \int \mathrm{Tr}\,B\wedge F(\omega) - 8\pi G\int \epsilon_{IJKLM}\,v^M\,B^{IJ}\wedge B^{KL}, \tag{20}$$

used in [10, 11]. The latin indices are in the group $SO(4,1)$ ($SO(5)$ in the Riemannian version). This is a more recent approach to the problem, and its quantisation has been considered in [3].

Both actions describe modified BF theories. However, there is a substantial difference between the two. The additional term in (19) is a constraint, whereas the additional term in (20) is a genuine interaction. The B field in (19) is a fundamental variable (and μ is a Lagrange multiplier), while the B field in (20) is only a Lagrange multiplier. Notice also the different position of the Newton's constant G.

The perturbative treatment of the additional term appears more natural with the action (20). In the rest of the discussion we focus on this approach. We assume $8\pi G/\hbar \ll 1$, and we discuss the perturbative expansion

$$Z_{\mathrm{GR}} = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{-i8\pi G}{\hbar}\right)^n\int \mathcal{D}B\,\mathcal{D}\omega\,\left(\int \epsilon_{IJKLM}\,v^M\,B^{IJ}\wedge B^{KL}\right)^n\,e^{\frac{i}{\hbar}\int \mathrm{Tr}\,B\wedge F(\omega)} \tag{21}$$

of the partition function of quantum GR.

It is important to distinguish the two cases, of compact or non–compact spacetime. If spacetime is non–compact, appropriate boundary conditions on the fields must be chosen, in order for the functional integrals to be well–defined. In the previous discussion of YM on flat spacetime, we implicitely chose vanishing boundary conditions for the fields at spacial infinity. Once the boundary conditions are chosen, the classical

equations of motion select only one solution (up to gauge), and the functional integral defining the Green functions is generically picked around this solution, as discussed above. In this situation, we showed in the previous section that the perturbative expansion around BF coincides with the conventional one. If we treat the non–compact case in gravity, we have to provide spacial boundary conditions for the gravitational field. The natural choice here is asymptotic flatness: $g_{\mu\nu} \mapsto \eta_{\mu\nu}$ at infinity. When using the action (20), this is a condition on the B field. Once this condition is fixed, we expect the quantum theory to be picked around a classical solution (see the discussion in [12]). In this setting, we expect our analogy with YM theory to hold through: namely, (21) should give rise to the conventional (non–renormalizable) perturbation theory in terms of gravitons. From this perspective, the "expansion around the topological sector" of the theory does not appear to be different from the conventional expansion.

The situation is far less clear in the compact case, where the background field that minimizes the gaussian action, or is selected by the delta function, is not determined by asymptotic conditions. In particular, it has been recently argued that the definition of the theory on a compact spacetime region with boundaries is more suitable to extract physical information from background independent theories [2, 13, 14, 15]. This is perhaps the situation where the proposal of [3] appears to be more interesting. We leave the discussion on this case open for further developments.

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