HAL
open science

The extension to the transverse momentum of the statistical parton distributions

Claude Bourrely, Jacques Soffer, Franco Buccella

To cite this version:

Claude Bourrely, Jacques Soffer, Franco Buccella. The extension to the transverse momentum of the statistical parton distributions. Modern Physics Letters A, 2006, 21, pp.143-150. hal-00007731v2

HAL Id: hal-00007731

https://hal.science/hal-00007731v2

Submitted on 14 Nov 2005

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

ccsd-00007731, version 2 - 14 Nov 2005

The extension to the transverse momentum of the statistical parton distributions

Claude Bourrely and Jacques Soffer

Centre de Physique Théorique, UMR 6207 [1],
CNRS-Luminy, Case 907
F-13288 Marseille Cedex 9 - France

Franco Buccella

Dipartimento di Scienze Fisiche, Università di Napoli,
Via Cintia, I-80126, Napoli and INFN, Sezione di Napoli, Italy

Abstract

By extending the statistical distributions to the transverse degree of freedhom, we account for a multiplicative factor in the Fermi-Dirac functions of the light quarks, we were led to introduce in a previous work to comply with experiment. We can also get light antiquark distributions, similar to those we proposed earlier.

keywords: polarized partons; transverse momentum; statistical models
PACS numbers: 13.88+e, 13.60.Hb, 12.40.Ee

CPT-2005/P.036
UNIV. NAPLES DSF-15/2005

[1] UMR 6207 is Unité Mixte de Recherche du CNRS and of Universités Aix-Marseille I and Aix-Marseille II and of Université du Sud Toulon-Var, laboratoire affilié à la FRUMAN.

Resulting from a research which begun many years ago [1], we proposed for the parton distributions [2], some expressions inspired by quantum statistics, with a relationship between the light q and $\bar{q}$ distributions following from the chiral properties of QCD [3]. We determined the few free parameters involved, from a selected set of precise deep inelastic scattering data [2]. Our approach has a strong predictive power, because, once the parameters, occuring in the better known unpolarized $u(x)$ and $d(x)$ distributions, are fixed, the polarized distributions, $\Delta u(x)$ and $\Delta d(x)$, as well as the antiquark distributions $\bar{u}(x)$, $\bar{d}(x)$, $\Delta\bar{u}(x)$ and $\Delta\bar{d}(x)$, are also fixed.
First, we made a prediction for a positive $A_1^n(x, Q^2)$ at high x [4], which was successfully verified by recent data at Jefferson Lab [5]. Next, we predicted for the high x behavior of the ratio $\bar{d}(x)/\bar{u}(x)$ a monotonously increasing function of x. An earlier result from E866 at FNAL [6], seemed to contradict our prediction, but the comparison with the most recent results on Drell-Yan processes by the NUSEA [7], shows that this is not the case [8]. Finally, the agreement with the data on $e^{\pm}p$ neutral and charge current reactions [9] [10], is particularly successful. In Ref. [8] we stressed the experimental evidence in several structure functions for a typical property of the statistical distributions: the change of slope for $x > X_{0u}^{+}$, where X_{0u}^{+} denotes the largest "thermodynamical potential", whose definition and value are given below.
We now recall some of the basic features of the statistical approach. The fermion distributions are given by the sum of two terms [2], a quasi Fermi-Dirac function and a helicity independent diffractive contribution equal for all light quarks:

$$xq^h(x, Q_0^2) = \frac{AX_{0q}^h x^b}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \ , \tag{1}$$

$$x\bar{q}^h(x, Q_0^2) = \frac{\bar{A}(X_{0q}^{-h})^{-1}x^{2b}}{\exp[(x + X_{0q}^{-h})/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \ , \tag{2}$$

at the input energy scale $Q_0^2 = 4\mathrm{GeV}^2$.
The parameter $\bar{x}$ plays the role of a *universal temperature* and $X_{0q}^{\pm}$ are the two *thermodynamical potentials* of the quark q, with helicity $h = \pm$. For the gluons we consider the black-body inspired expression

$$xG(x, Q_0^2) = \frac{A_G x^{b_G}}{\exp(x/\bar{x}) - 1} \ , \tag{3}$$

a quasi Bose-Einstein function with $b_G = \tilde{b} + 1$, since we believe that the diffractive contribution in Eqs. (1,2) is strongly related to the gluons.
We also assume that, at the input energy scale, the polarized gluon distribution vanishes, so

$$x\Delta G(x, Q_0^2) = 0 \ . \tag{4}$$

For strange quarks we assumed that $s = \bar{s}$, but we did not introduce any more parameters and we simply related them to $\bar{u} + \bar{d}$ to comply with experimental data [11]. Similarly we took $\Delta s = \Delta\bar{s}$ and we related them to $(\Delta\bar{d} - \Delta\bar{u})$, to agree with the second Bjorken sum rule.
The *eight* free parameters in Eqs. (1,2) (A, $\bar{A}$ and A_G are fixed by the normalization conditions $u - \bar{u} = 2$, $d - \bar{d} = 1$ and by the proton momentum sum rule) were determined at the input scale from the comparison with a selected set of very precise unpolarized and polarized DIS data from NMC, BCDMS, E665, ZEUS, H1 for $F_2^{p,d}(x, Q^2)$, from CCFR for $xF_3^{\nu N}(x, Q^2)$ and from SMC, E154 and E155 for $g_1^{p,d,n}(x, Q^2)$.
We obtained

$$\bar{x} = 0.09907, \ b = 0.40962, \ \tilde{b} = -0.25347 \ , \tag{5}$$

$$X_{0u}^{+} = 0.46128, \ X_{0u}^{-} = 0.29766, \ X_{0d}^{-} = 0.30174, \ X_{0d}^{+} = 0.22775 \ , \tag{6}$$

$$\tilde{A} = 0.08318, \ A = 1.74938, \ \bar{A} = 1.90801, \ A_G = 14.27535 \ . \tag{7}$$

In Eqs. (1,2) the factors X_{0q}^{h} and $(X_{0q}^{-h})^{-1}$ introduced in the numerators of the q's and $\bar{q}$'s distributions, imply a modification of the quantum statistical form, we were led to introduce to agree with experimental data. As we will see now, the justification of these factors can be understood by studying the role of the transverse degree of freedhom for the partons.
Let us denote $p_i(x, p_T^2)$ [2] a parton (quark or gluon) distribution with transverse momentum p_T, which is assumed to be much smaller than its longitudinal momentum xp_z, where p_z is the momentum of the proton with mass M. We first consider the momentum sum rule, which reads now

$$\sum_i \int_0^1 dx \int_0^{(p_T^2)_{max}} xp_i(x, p_T^2) dp_T^2 = 1 \ , \tag{8}$$

[2] Here i stands for the quark flavour and for the spin component of the partons; $p_i(x, p_T^2)$ has now the dimension of GeV^{-2}.

where the upper limit for the p_T^2 integral, $(p_T^2)_{max}$, will be determined below. Next, we consider, the condition for the energy sum rule, analogous to Eq. (8), where we assume that the proton mass is much smaller than its momentum, an approximation which holds in the deep inelastic regime. Consequently, the proton energy is $p_z + M^2/2p_z$, similarly the energy of a massless parton is $xp_z + p_T^2/2xp_z$, so this energy sum rule reads

$$\begin{aligned} p_z \sum_i \int_0^1 dx \int_0^{(p_T^2)_{max}} xp_i(x, p_T^2) dp_T^2 \quad + \\ \sum_i \int_0^1 dx \int_0^{(p_T^2)_{max}} p_i(x, p_T^2) \frac{p_T^2}{2xp_z} dp_T^2 \quad = \quad p_z + \frac{M^2}{2p_z} \ . \end{aligned} \tag{9}$$

By using Eq. (8) in Eq. (9), it is easy to derive the constraint

$$\sum_i \int_0^1 dx \int_0^{(p_T^2)_{max}} p_i(x, p_T^2) \frac{p_T^2}{x} dp_T^2 = M^2 \ . \tag{10}$$

By comparing Eq. (10) and Eq. (8), where it is clear that x is bounded by 1, we see that p_T^2/x is bounded by M^2, so $(p_T^2)_{max} = xM^2$.
The above sum rule will be used to determine the p_T dependence, as we will explain now. We recall the general method of statistical thermodynamics [12], to find the most probable occupation numbers n_i, for the energy levels ϵ_i, when the total energy of N distinguishable particles is E. Since we have

$$N = \sum_i n_i \quad \text{and} \quad E = \sum_i n_i \epsilon_i \ , \tag{11}$$

one should look for the maximum of

$$\ln[N!/\prod_i n_i!] + \alpha(N - \sum_i n_i) + \beta(E - \sum_i n_i \epsilon_i) \ , \tag{12}$$

with the Lagrange multipliers method and we find

$$n_i = \exp(-\alpha - \beta\epsilon_i) \ . \tag{13}$$

The Lagrange multipliers α and β are fixed by the two above constraints Eq. (11). Now by putting in correspondence n_i with $p_i(x, p_T^2)$, N with 1 and

α with $1/\bar{x}$, for the sum rule (8), and E with M^2, ϵ_i with p_T^2/x and β with $1/\mu^2$, for the sum rule (10), one finds, in correspondence with Eq. (13)

$$p_i(x, p_T^2) = \exp(\frac{-x}{\bar{x}} + \frac{-p_T^2}{x\mu^2}) \ . \tag{14}$$

μ is a parameter which has the dimension of a mass and it will be determined later. We recall that Eq. (14) describes well the p_T behavior of the particles produced in very high energy hadron-hadron scattering [13]. For fermions, the Boltzmann exponential form Eq. (14) is expected to be modified into the product of two Fermi-Dirac expressions.
For the sake of clarity, we will first ignore the p_T dependence of the diffractive contribution of quarks, antiquarks (second terms in Eqs. (1,2)) and of the gluons, which dominate the very low x region. It certainly corresponds to very small p_T values, since $(p_T^2)_{max} = xM^2$ as seen above. So let us consider now the non-diffractive terms and instead of the helicity, as in Eqs. (1,2), the spin component of the partons along the momentum of the proton, which coincides with helicity only when $p_T = 0$. The quantum statistics distributions for quarks and antiquarks read in this case

$$xq^{S_z}(x, p_T^2) = \frac{F(x)}{[\exp[(x - X_{0q}^{S_z})/\bar{x}] + 1]} \frac{1}{[\exp[(p_T^2/x\mu^2 - Y_{0q}^{S_z})/\bar{x}] + 1]} \ , \tag{15}$$

$$x\bar{q}^{S_z}(x, p_T^2) = \frac{\bar{F}(x)}{[\exp[(x + X_{0q}^{-S_z})/\bar{x}] + 1]} \frac{1}{[\exp[(p_T^2/x\mu^2 + Y_{0q}^{-S_z})/\bar{x}] + 1]}, \tag{16}$$

where $F(x)$ and $\bar{F}(x)$ do not depend on spin and flavour, X and Y are the thermodynamical potentials related to the sum rules (8) and (10), respectively [3]. At high p_T, one has a Gaussian behavior, with a width proportional to $\sqrt{x}$. It is important to note that this is at variance with the usual fatorization assumption of the dependences in x and p_T [14, 15]. We observe that the upper limit of the integral in x, means that one parton is taking all the proton momentum. Similarly $(p_T^2)_{max} = xM^2$ for the p_T^2 integral corresponds to the situation where one parton is taking all the transverse energy. Therefore to simplify the formulas, we shall take as upper limit infinity, since the

[3]Here for convenience, we have divided by $\bar{x}$ the argument of the exponential in the Fermi-Dirac expression of the p_T dependence, in agreement with the expression for the x dependence.

contribution of the tail is negligible. In fact when one now integrates over p_T^2, the right-hand side of Eq. (15), one finds

$$\int_0^\infty \frac{dp_T^2}{\exp[(p_T^2/x\mu^2 - Y_{0q}^{S_z})/\bar{x})] + 1} = -x\mu^2\bar{x}\mathrm{Li}_1(-\exp[Y_{0q}^{S_z}/\bar{x}]) \ . \tag{17}$$

Here Li_1 denotes the polylogarithm function of order 1, which is known to arise from the integral of Fermi-Dirac distributions and is such that

$$-\mathrm{Li}_1(-\mathrm{e}^y) = \int_0^\infty \frac{d\omega}{\mathrm{e}^{(\omega-y)} + 1} = \ln(1 + \mathrm{e}^y) \ . \tag{18}$$

It is reasonable to assume the proportionality relationship

$$Y_{0q}^{S_z} = kX_{0q}^{S_z} \ . \tag{19}$$

It implies that the partons with a larger contribution to their first moments from the non-diffractive part, not only have a broader x dependence, but also have a broader p_T^2 dependence, at every x. We recall that Li_1 is an increasing function of its argument and we see from Eq. (18) that for large values of y, it becomes approximately proportional to y and one has the high degeneracy of the Fermi gas. Therefore this fully justifies, the phenomenological assumption of the proportionality to X_{0q}^h made in Ref. [2], as seen in Eq. (1). By taking in Eq. (15) [4]

$$F(x) = -\frac{Ax^{b-1}X_{0u}^+}{\mathrm{Li}_1(-\exp[Y_{0u}^+/\bar{x}])\mu^2\bar{x}} \ , \tag{20}$$

we recover the first term in Eq. (1) for the u^+ quark, which is the dominating parton at large x. Eq. (20) implies for the other quarks, that instead of X_{0q}^h, one should use the factor $X_{0u}^+\mathrm{Li}_1(-\exp[Y_{0q}^h/\bar{x}])/\mathrm{Li}_1(-\exp[Y_{0u}^+/\bar{x}])$.
As long as the non-diffractive term for the antiquarks, by integrating on p_T^2, one finds the factor $\mathrm{Li}_1(-\exp[-Y_{0q}^h/\bar{x}])$. At large negative y, Li_1 becomes proportional to $\exp(-y)$, which corresponds to the Boltzman limit. Like for the quarks, we will fixe the normalization using the largest distribution, namely $\bar{d}^-$, so similarly to Eq. (20) we will take in Eq. (16)

$$\bar{F}(x) = -\frac{\bar{A}x^{2b-1}(X_{0d}^+)^{-1}}{\mathrm{Li}_1(-\exp[-Y_{0d}^+/\bar{x}])\mu^2\bar{x}} \ . \tag{21}$$

[4] We identify $X_{0q}^{S_z}$ with X_{0q}^h given in Eq. (6) and similarly for the potentials $Y_{0q}^{S_z}$.

We recover the first term in Eq. (2) for $\bar{d}^-$ and for the other antiquarks instead of $(X^h_{0q})^{-1}$, one should use $(X^+_{0d})^{-1}\mathrm{Li}_1(-\exp[-Y^h_{0q}/\bar{x}])/\mathrm{Li}_1(-\exp[-Y^+_{0d}/\bar{x}])$. Now, in order to determine k, we require no change for the quark distributions, once integrated over p_T^2. This can be achieved by choosing

$$k = 1.42 \ . \tag{22}$$

This value indicates that the potentials Y^h_{0q} are of the same order as X^h_{0q} and moreover it implies lower values for the non-diffractive contributions of $\bar{u}^+$, $\bar{d}^+$ and $\bar{u}^-$. Since the antiquark distributions are dominated by the diffractive contribution, these small modifications are still consistent with the data.
At this stage we can try to use the energy sum rule Eq. (10) to determine μ, the only unknown parameter, so far. We find to a very good approximation, the remarkable value,

$$\mu = 1\mathrm{GeV} \ . \tag{23}$$

Moreover the energy sum rule is found to be saturated by a quark contribution of 98%, whereas the antiquark contribution is only 2%. This value of μ was found by neglecting the gluon and the diffractive contributions and it is clear that if they turn out to be important, μ will be smaller.
For completeness we now turn to the universal diffractive contribution to quarks and antiquarks in Eqs. (1,2), namely $xq^D(x, Q_0^2) = \tilde{A}x^{\tilde{b}}/[\exp(x/\bar{x})+1]$. Since $\tilde{b} < 0$ (see Eq. (5)), the introduction of the p_T dependence cannot be done similarly to the non diffractive contributions, because in the energy sum rule Eq. (10), it generates a singular behavior when $x \to 0$. Therefore we propose to modify our prescription by taking at the input energy scale

$$xq^D(x, p_T^2) = \frac{\tilde{A}x^{\tilde{b}-2}}{\ln(2)\mu^2\bar{x}} \frac{1}{[\exp(x/\bar{x})+1]} \frac{1}{[\exp(p_T^2/\bar{x}x^2\mu^2)+1]} \ , \tag{24}$$

whose p_T fall off is stronger, because $x\mu^2$ is now replaced by $x^2\mu^2$. Note that this is properly normalized to recover $xq^D(x, Q_0^2)$ after integration over p_T^2. We have checked that $xq^D(x, p_T^2)$ gives a negligible contribution to Eq. (10), as expected, and we are led to a similar conclusion for the gluon, which has the same small x behavior. However since the gluon is parametrized by a quasi Bose-Einstein function, one has to introduce a non-zero potential Y_G, to avoid the singular behavior of $\mathrm{Li}_1(\exp[-Y_G/\bar{x}])$, when $Y_G = 0$ (see Eq. (18)). The value of Y_G is not constrained, but by taking a very small Y_G, it does not affect the energy sum rule. Of course, one may ask why the

gluon contributes so much to the momentum sum rule, Eq. (8), and so little to the energy sum rule, Eq. (10). Using a better regularization procedure, we might find a larger contribution and consequently a smaller μ value. For this reason, we give the p_T dependence as a function of the dimensionless variable p_T^2/μ^2.
For illustration, we display in Fig. 1, $x\mu^2 u(x, p_T^2/\mu^2, Q_0^2)$, the predicted dimensionless u-quark parton distribution at input energy scale, versus p_T^2/μ^2 for different x values. Note that the curves are limited to the value $(p_T^2)_{max} = xM^2$ and the Gaussian behavior in the high p_T region follows a rather flat behavior for $10^{-2} < p_T^2/\mu^2 < 10^{-1}$, or so. This feature is a natural consequence of the statistical approach and it coincides, for a mean x value, with the simplifying assumption made in recent papers (see e.g. [13]) without real justification. For $p_T^2/\mu^2 < 10^{-2}$, the very low x region is strongly enhanced by the diffractive contribution, due to the factor $x^{\tilde{b}-2}$ (see Eq. (24)). However, since the regularization procedure proposed above is not unique, it should not be considered as a definite prediction.

The theoretical fundation of the statistical parton distributions proposed earlier in Ref. [2] is strengthened by the present extension to the transverse degrees of freedhom. Indeed the energy sum rule Eq. (10) cuts naturally high p_T^2 values and this is very appealing, because an increasing phase-space for p_T^2 linear in Q^2 would support the absence of quantum statistical effects for diluted fermions and bosons. Our predicted light partons distributions $p_i(x, p_T^2)$, both unpolarized and polarized given by Eqs. (15,16), remain to be checked against appropriate experimental data, sensitive to some effects arising from the transverse momentum of partons inside the proton. Here it is important to emphasize that the p_T dependence is only valid in a limited kinematic region, say $0.1 < x < 0.9$, which is not dominated by the diffractive contribution. Clearly in this large x region, we predict the mean value of p_T^2, namely $< p_T^2 >$, to increase with x, a behaviour which is at variance with what one expects in the very small x region from the BFKL evolution, assuming the emission of a hudge number of gluons [16]. However, so far there is no experimental evidence for the BFKL evolution.

Finally, we note that our procedure restricts to rotational invariance, the transverse momentum dependence of the parton distributions. The next step should be to introduce some azimuthal dependence, which is needed to study the important topic of single-spin asymmetries [17].

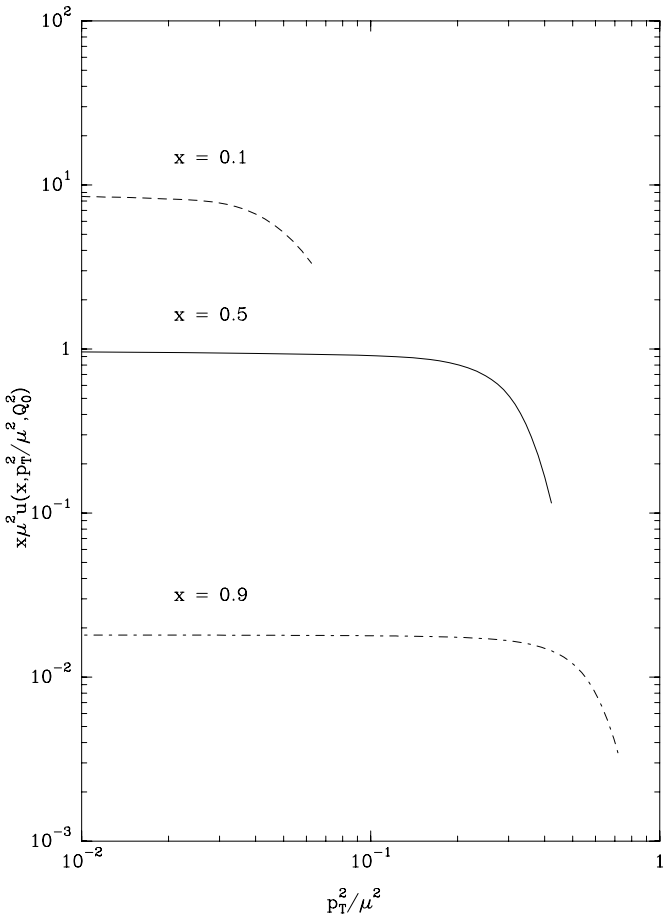

Figure 1: The predicted $x\mu^2 u(x, p_T^2/\mu^2, Q_0^2)$ statistical distribution, versus p_T^2/μ^2 for different values of x.

References

[1] C. Bourrely, F. Buccella, G. Miele, G. Migliore, J. Soffer and V. Tibullo, Z. Phys. C **62**, 431 (1994)

[2] C. Bourrely, F. Buccella and J. Soffer, Eur. Phys. J. C **23**, 487 (2002)

[3] R.S. Bhalerao, Phys. Rev. C **63**, 025208 (2001)

[4] C. Bourrely, F. Buccella and J. Soffer, Mod. Phys. Lett. **18**, 771 (2003)

[5] Jefferson Lab Hall A Collaboration, X. Zheng *et al.*, Phys. Rev. C **70**, 065207 (2004)

[6] FNAL Nusea Collaboration, E. A. Hawker *et al.*, Phys. Rev. Lett. **80**, 3715 (1998); J. C. Peng *et al.*, Phys. Rev. D **58**, 092004 (1998); R. S. Towell *et al.*, Phys. Rev. D **64**, 052002 (2001)

[7] FNAL E866/NuSea Collaboration, J.C. Webb *et al.*, submitted to Phys. Rev. Lett. [hep-ex/0302019]

[8] C. Bourrely, F. Buccella and J. Soffer, Eur. Phys. J. C **41**, 327 (2005)

[9] H1 Collaboration, C. Adloff *et al.*, Eur. Phys. J. C **30**, 1 (2003), ibid **19**, 269 (2001)

[10] ZEUS Collaboration, S. Chekanov *et al.*, Eur. Phys. J. C **21**, 443 (2001); Eur. Phys. J. C **28**, 175 (2003); Phys. Rev. D **70**, 052001 (2004); Eur. Phys. J. C **32**, 1 (2003); Phys. Lett. **B539**, 197 (2002)

[11] CCFR Collaboration, P. Z. Quintas *et al.*, Phys. Rev. Lett. **71**, 1307 (1993); W. C. Leung *et al.*, Phys. Lett. B **317**, 655 (1993); W. G. Seligman *et al.*, Phys. Rev. Lett. **79**, 1213 (1997); J. H. Kim *et al.*, Phys. Rev. Lett. **81**, 3595 (1998); U. K. Yang *et al.*, Phys. Rev. Lett. **86**, 2741 (2001)

[12] E. Schrödinger, *Statistical Thermodynamics*, Cambridge University Press (1964).

[13] F. Buccella and L. Popova, Mod. Phys. Lett. A **17**, 2627 (2002)

[14] M. Anselmino, U. D'Alesio and F. Murgia, Phys. Rev. D **67**, 074010 (2003)

[15] I. Schmidt, J. Soffer and J.J. Yang, Phys. Lett. B **612**, 258 (2005)

[16] The Small x Collaboration, J.R. Andersen *et al.*, Eur. Phys. J. C **35**, 67 (2004) and references therein

[17] D. Boer, P.J. Mulders and F. Pijlman, Nucl. Phys. B **667**, 201 (2003)