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# Time-controlled charge injection in a quantum Hall fluid

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We consider the injection of a controlled charge from a normal metal into an edge state of the fractional quantum Hall effect, with a time-dependent voltage $V(t)$. Using perturbative calculations in the tunneling limit, and a chiral Luttinger liquid model for the edge state, we show that the electronic correlations prevent the charge fluctuations to be divergent for a generic voltage pulse $V(t)$. This is in strong contrast with the case of charge injection in a normal metal, where this divergence is present. We show that explicit formulæ for the mean injected charge and its fluctuations can be obtained using an adiabatic approximation, and that non perturbative results can be obtained for injection in an edge state of the FQHE with filling factor $\nu = 1/3$. Generalization to other correlated systems which can be described with the Luttinger liquid model, like metallic Carbon nanotube, is given.

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## I. INTRODUCTION

It is now well known that the fluctuations of electric current contain valuable information both on the discreetness of the charge and on the quantum properties of transport [1, 2, 3, 4]. Many studies of these fluctuations, both experimental and theroretical, have been done on systems in a stationary regime, with constant or time-periodic voltage biases. We consider here a non-stationnary problem, where a voltage pulse is used to inject a given charge in a conductor, and call this process "time-controlled charge injection". Such a real-time transfer of charge might prove of great interest for applications, for example as a tool for the transfert of information. The current fluctuations also play an important role in the problem of time-controlled charge injection, as these fluctuations should be made as low as possible to transfer the charge as precisely as possible.

An interesting area to study time-controlled charge injection consists of conductors with strongly correlated electrons. Indeed, in these systems, the elementary excitations are collective electronic excitations, and may have a charge $e^*$ which is only a fraction of the "elementary" charge $e$. We will be particularly interested in edge states of the fractional quantum Hall effect (FQHE) for Laughlin fractions, where elementary excitations have a charge $e^* = e/(2n+1)$, with $n$ an integer (the most accessible charge being $e^* = e/3$) [5, 6, 7]. This problem is interesting on its own from a theoretical perspective, as it allows to see how a system with strongly correlated electrons behaves in a non-stationary setup. It is also interesting for potential experimental applications, as the injection of a well controlled charge, e.g. a *unique* electron, in an edge state of the FQHE is an important experimental challenge, which could prove an useful tool in the quantum information domain for example.

In the case where electrons are uncorrelated (normal metal conductors), this problem has been studied by Levitov and coworkers [8]. Defining the Faraday flux $\phi = e/\hbar \int_{-\infty}^{\infty} dt V(t)$, they have shown that the mean transmitted charge $\langle Q \rangle$ is simply proportional to the flux (Ohm's law), $\langle Q \rangle \sim \phi$, but that the charge fluctuations $\langle Q^2 \rangle$ are in general logarithmically divergent, except for "integer" values of the flux $\phi = 2\pi n$ (with $n$ an integer) where these fluctuations are finite. They have related this behavior to the Anderson orthogonality catastrophe [9].The goal of this report is to study this problem in the case where the charge is transfered into an edge state of the FQHE, or more generally in a chiral Luttinger liquid.

The setup of this paper is as follows. The system is described in section II. In section III, we consider the case of the pertrubative regime, where the tunneling between the normal metal and the edge state is low. The convergence of the mean charge and of its fluctuations is studied, and explicit formulæ are given within the adiabatic approximation, whose validity is confirmed by numerical calculations. In section IV, we give non-perturbative results for the special case of electron tunneling in an edge state of the FQHE with filling factor $\nu = 1/3$. Finally, in section V, some perspectives are discussed, like the relevance of our results for other systems, and conclusions are given.

## II. DESCRIPTION OF THE SYSTEM

The system we consider is composed of a normal metal (non-interacting electrons, with Fermi liquid behavior) close to an edge state of a 2D electron gas in the FQHE regime. A voltage pulse $V(t)$ is applied between the two conductors, which leads to the tunneling of electrons. The FQHE regime is characterized by the filling factor $\nu = 1/m$, where $m$ is an odd integer. Note that the case $m = 1$ correspond to the integer quantum Hall effect, where there are no correlations between electrons and the edge state describe a Fermi liquid. We expect thus to recover, when we take $\nu = 1$, the results of Levitov et

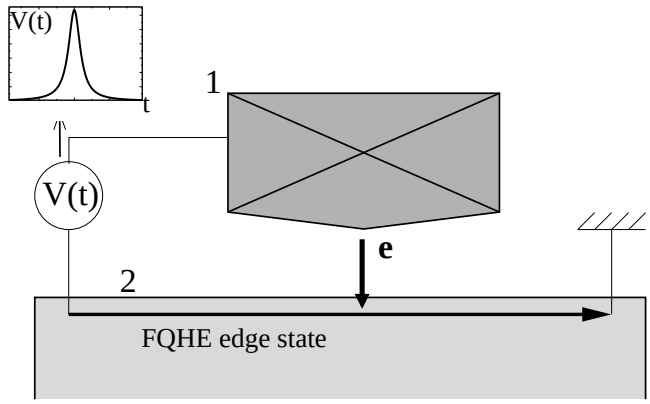


FIG. 1: The setup: electron tunneling between a usual conductor and an edge state of a 2D electron gas in the fractional quantum Hall effect (FQHE) regime, induced by a time dependent voltage $V(t)$.

al.

We call $\psi_1$ ($\psi_2$) the electron annihilation operator at the tunneling point in conductor 1 (conductor 2, the edge state)- see Fig.1. Using Peierls' substitution to have the voltage pulse as a vector potential only, we have the following tunneling Hamiltonian between the two conductors:

$$H_T(t) = \hbar v_F\, \Gamma e^{i\phi(t)} \psi_1^\dagger(t)\psi_2(t) + \text{h.c.} \tag{1}$$

where $\Gamma$ is the tunneling amplitude and $\phi(t) = e/\hbar \int_{-\infty}^{t} dt' V(t')$ is the time-dependent Faraday flux. Similarly, the tunneling current is given by:

$$I_T(t) = i\, e\, v_F \left(\Gamma e^{i\phi(t)} \psi_1^\dagger(t)\psi_2(t) - \text{h.c.}\right) \tag{2}$$

The quantities we need to calculate are the mean transmitted charge $\langle Q\rangle$:

$$\langle Q\rangle = \int_{-\infty}^{\infty} d\tau \, \langle I_T(\tau)\rangle \tag{3}$$

and its fluctuations $\langle \Delta Q^2\rangle = \langle Q^2 - \langle Q\rangle^2\rangle$:

$$\begin{aligned}\left\langle \Delta Q^2\right\rangle &= \left\langle \int_{-\infty}^{+\infty} dt\; I(t) \int_{-\infty}^{+\infty} dt'\; I(t')\right\rangle - \langle Q\rangle^2 \\ &= \int_{-\infty}^{+\infty} dt\, d\tau \left( \langle I(t) I(t+\tau)\rangle - \langle I(t)\rangle \langle I(t+\tau)\rangle \right) \\ &= \int_{-\infty}^{+\infty} dt\, d\tau S(t, t+\tau) \end{aligned} \tag{4}$$

The system being out of equilibrium, we use the Keldysh formalism, introducing a time contour going first from $-\infty$ to $+\infty$ (upper branch, $\eta = +1$) and then going back from $+\infty$ to $-\infty$ (lower branch, $\eta = -1$), and using the time ordering operator $T_K$ along this contour [10].

## III. PERTURBATIVE RESULTS

### A. Formalism

In this section, we will calculate the mean transmitted charge and its fluctuations in the tunneling regime, where $\Gamma \to 0$. This allows us to get the results by calculating only the lowest order in the tunneling amplitude $\Gamma$. Note that $\Gamma \to 0$ ensures the system is in the tunneling regime for any finite value of $V(t)$. Indeed, for electron tunneling from a normal metal to an edge state of the FQHE with filling factor $\nu = 1/(2p+1)$, one has in the tunneling limit the tunneling current $I \sim \Gamma^2 |\omega_0|^{2/\nu - 1}$, where $\omega_0$ is proportionnal to the applied voltage $V$. As $2/\nu - 1 > 0$, it is clear that when $V(t) \to 0$ the tunneling current goes to 0 and can thus be calculatd perturbatively. Note that the situation would be different for tunneling of fractionaly charged excitations between two edge states of the same FQHE fluid, where $V \to 0$ brings the system out of the tunneling regime

As we are considering the system in the tunneling regime, we can restrict ourselves to the lowest order contribution in the tunneling amplitude $\Gamma$. At order $\Gamma^2$, we have for the mean transmitted charge:

$$\langle Q\rangle = \frac{-i}{2\hbar} \int_{-\infty}^{+\infty} d\tau \sum_{\eta\eta_1 = \pm 1} \eta_1 \int_{-\infty}^{+\infty} dt_1 \, \langle T_K I_T(\tau^\eta) H_T(t_1^{\eta_1})\rangle \tag{5}$$

Using Eqs. (1) and (2), with standard properties of the Keldysh Green functions, and parity properties, we can write this expression as:

$$\begin{aligned}\langle Q\rangle = -2e\, v_F^2 \Gamma^2 \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} d\tau \; \text{Im}\left(G_1(t) G_2(t)\right) \\ \sin\left(\phi(\tau + t/2) - \phi(\tau - t/2)\right)\end{aligned} \tag{6}$$

where $G_i(t)$ is the standard Green function for conductor $i$ ($i = 1, 2$): $G_i(t) = \left\langle T \psi_i^\dagger(0) \psi_i(t)\right\rangle$. Similarly, we get for the charge fluctuations (Eq. (4)):

$$\begin{aligned}\left\langle \Delta Q^2\right\rangle = 2e^2 v_F^2\, \Gamma^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} d\tau \; \text{Re}\left(G_1(t) G_2(t)\right) \\ \left[\cos\left(\phi(\tau + t/2) - \phi(\tau - t/2)\right) - 1\right]\end{aligned} \tag{7}$$

Note that the term "-1" next to the cosine has been introduced to regularize the expression. This regularizing term is needed because we have permuted the order of the $t$ and $\tau$ integrals, and is not needed if the $t$ integral is performed before the $\tau$ integral, as $\int dt \text{Re}(G_1(t)G_2(t)) = 0$.

### B. Convergence of the integrals

Eqs. (6) and (7) will allow us to study the properties of the charge injection. On these formulæ, we see

that both the injected charge and its fluctuations are obtained with two elements: the Green functions product $G_1(t)G_2(t)$, which contains all the information about the two conductors, and a kernel obtained by integrating over $\tau$ a function of the Faraday flux $\phi(t)$, which contains all the information about the voltage pulse $V(t)$. To study the convergence/divergence of the time integrals for the charge and its fluctuations, we need the time behavior of these two elements. Note that as we consider tunneling through a single point contact, the normal metal can be mapped [12] to a chiral Luttinger liquid with parameter $\nu = 1$. The Green functions at zero temperature are simply [10]:

$$G_1(t) = \frac{1}{2\pi a}(1+iv_F t/a)^{-1} \qquad G_2(t) = \frac{1}{2\pi a}(1+iv_F t/a)^{-1/\nu} \tag{8}$$

where $a$ is a short length cutoff, and $\nu = 1/m$ the filling factor of the FQHE conductor. Introducing $K = (1/2)(1 + 1/\nu)$, with $K$ an integer, we see that the large time behavior of the real and imaginary part of $G_1(t)G_2(t)$ is:

$$\mathrm{Im}\,(G_1(t)G_2(t)) \sim t^{-(2K+1)} \qquad \mathrm{Re}\,(G_1(t)G_2(t)) \sim t^{-2K} \tag{9}$$

(the imaginary part does not contain a $t^{-2K}$ term as this term includes a $\sin(\pi K)$ factor which is zero). We now turn to the large time behavior of the kernels involving the flux $\phi(t)$. For the mean charge, it is given by $B_1(t) = \int_{-\infty}^{\infty} d\tau \sin\left(\phi(\tau + t/2) - \phi(\tau - t/2)\right)$. By hypothesis, the voltage pulse $V(t)$ is important only in a finite time domain, of width $\Delta t$. This means that for $t \gg \Delta t$, and for a $\tau$ interval of the order of $t$, we have:

$$\phi(\tau + t/2) - \phi(\tau - t/2) = \frac{e}{\hbar}\int_{\tau-t/2}^{\tau+t/2} V(t) \simeq \frac{e}{\hbar}\int_{-\infty}^{\infty} V(t) = \phi. \tag{10}$$

We have thus $B_1(t) \simeq \sin(\phi)t + C_1$ for $t \to \infty$, where $C_1$ is a constant. Similarly, we have for the kernel of the charge fluctuations, $B_2(t) = \int_{-\infty}^{\infty} d\tau[\cos\left(\phi(\tau + t/2) - \phi(\tau - t/2)\right) - 1]$, $B_2(t) \simeq (\cos(\phi) - 1)t + C_2$. Both the kernels $B_1(t)$ and $B_2(t)$ have thus a linear dependence in $t$ for large $t$, except for the special values of the flux $\phi = 2\pi n$ (with $n \in \mathbb{N}$) where they are constant for large $t$.

Combining the large time behavior of the Green functions and of the kernels, we see that the integrand for the mean charge (Eq. (6)) behaves for large $t$ as:

$$B_1(t)\,\mathrm{Im}\,(G_1(t)G_2(t)) \simeq \sin(\phi)t^{-2K} + C_1\, t^{-2K+1} \tag{11}$$

while for the charge fluctuations (Eq. (7)) we have:

$$B_2(t)\,\mathrm{Re}\,(G_1(t)G_2(t)) \simeq (\cos(\phi) - 1)t^{-2K+1} + C_2 t^{-2K} \tag{12}$$

Let us first check that this is compatible with the known results for non-interacting electrons. In this case, one has $\nu = 1$ and thus $K = 1$. We see then that the mean charge integral is always converging, while the charge fluctuations integral has a logarithmic divergence, except for $\phi = 2\pi n$, and we recover thus the results of Levitov et al. [8]. Turning now to interacting electrons, one has $\nu = 1/m$ with $m > 1$ an odd integer, and thus $K$ is an integer with $K > 1$. In this case, we see that the mean charge integral is as before always converging, and that the charge fluctuations integral is also always converging, independently of the value of the flux $\phi$ ! This means that, because of the electronic correlations, the divergence of the charge fluctuations is removed.

### C. Explicit formulæ and the adiabatic approximation

It is possible to go further and to get explicit formulæ for the integrals of the mean injected charge and its fluctuations. For simplicity, we will restrict ourselves to the case $\nu = 1/3$, but the results shown here can be extended to any value of $\nu = 1/(2n + 1)$. For $\nu = 1/3$, the Green function product is:

$$\begin{aligned}\mathrm{Re}\,(G_1(t)\,G_2(t)) &= \frac{1}{4\pi^2 a^2}\frac{1 - 6(v_F t/a)^2 + (v_F t/a)^4}{(1 + (v_F t/a)^2)^4} \\ \mathrm{Im}\,(G_1(t)\,G_2(t)) &= \frac{1}{4\pi^2 a^2}\frac{4(v_F t/a)(1 - (v_F t/a)^2)}{(1 + (v_F t/a)^2)^4}\end{aligned} \tag{13}$$

Let us first consider the integral for the mean transmitted charge (Eq. (6)), which involves the imaginary part of the Green function product. On Eq. (13), we see that this part is important in a time domain of the order of $a/v_F$, and then quickly decreases to 0 for $t \gg a/v_F$ as $t^{-5}$. For the $t$ integral giving the mean transmitted charge, we can thus consider that times up to $t \sim a/v_F$ only contribute importantly to the integral. In the kernel $B_1(t)$, time $t$ appears in the bounds $\tau \pm t/2$ of the $V(t')$ integral in the sinus. As the short time cutoff $a/v_F$ is much smaller that the typical time of variation of $V(t')$, the function $V(t')$ can be considered as constant in this integral, giving $\sin(e/\hbar\, V(\tau)\, t)$. The mean charge is then:

$$\langle Q\rangle = (-1)2e\, v_F^2\Gamma^2 \int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} dt\, \sin\left(\frac{e}{\hbar}\, V(\tau)\, t\right) \mathrm{Im}\,(G_1(t)G_2(t)) \tag{14}$$

The $t$ integral in (14) is simply the formula for the mean tunneling current when a constant voltage $V(\tau)$ (with a given $\tau$) is applied. We have thus performed an *adiabatic* approximation, valid because of the rapid decrease of the Green function product. Performing the time integral, using Eq. (13), we get the result:

$$\langle Q\rangle = \frac{e\Gamma^2 a^2}{2\pi\, 3! v_F^2}\int_{-\infty}^{+\infty} d\tau\, \left(\frac{e}{\hbar}\, V(\tau)\right)^3 \tag{15}$$

We see that the mean transmitted charge is not proportional to the total flux $\phi$, but rather to the integral of

the cube of the voltage pulse $V(t)$. This means that, for a voltage pulse of given shape whose flux is varied by an overall scale factor only, the mean transmitted charge varies as the cube of the flux $\phi$. This behavior is a consequence of the non-linear relation between voltage and current for tunneling in a Luttinger liquid. The presence of the cutoff $a$ in the formula is typical of electron tunneling in a Luttinger liquid. Note that for another filling factor $\nu = 1/m$, the result would be proportional to the integral of $(V(\tau))^m$.

Turning now to the charge fluctuations, one could expect to obtain similar results: from Eq. (13), we see that the real part of the product of the Green functions is important for $t \sim a/v_F$ only, and decreases rapidly to 0 as $t^{-4}$. However, when $V(\tau)$ is not large enough the adiabatic approximation breaks down, because $\int_{-\infty}^{+\infty} dt\, \mathrm{Re}(G_1(t)G_2(t)) = 0$, which means that the contribution of the time interval up to $t \sim a/v_F$ may vanish. We have thus to distinguish between two regimes, small flux $\phi \ll 1$ and large flux $\phi$.

For the small $\phi$ regime let us consider a voltage pulse $V(t) = \phi V_1(t)$, where $V_1(t)$ is of fixed shape and unit flux ($\int dt\, V_1(t) = 1$), and $\phi \ll 1$. The cosine in the kernel of the charge fluctuations $B_2(t)$ can then be developped at second order, giving $\frac{(-1)}{2}\phi^2 \left(\int_{\tau-t/2}^{\tau+t/2} dt' V_1(t')\right)^2$. When we vary the flux $\phi$ corresponding to the voltage pulse $V(t)$, we see that the kernel $B_2(t)$, and thus the charge fluctuations, varies as $\phi^2$. The charge fluctuations are thus proportional to $\phi^2$ for small $\phi$, contrarily to the mean transmitted charge which varies as $\phi^3$.

In the other regime, with a larger flux $\phi$, we can use the same adiabatic approximation as for the mean charge. We get:

$$\left\langle \Delta Q^2 \right\rangle = 2e^2 v_F^2 \Gamma^2 \int_{-\infty}^{+\infty} dt\, \mathrm{Re}\left(G_1(t)G_2(t)\right) \left(\int_{-\infty}^{+\infty} d\tau \left[\cos\left(\frac{e}{\hbar}V(\tau)\, t\right) - 1\right]\right) \quad (16)$$

Permuting the $t$ and $\tau$ integrals (the "-1" term does not contribute then), we see that for $V(t)$ large enough the variations of the cosine are rapid enough to get a non-zero integral on the domain where the Green functions are important ($t \sim a$). Performing the $t$ integral, we get for the charge fluctuations, for large $\phi$:

$$\left\langle \Delta Q^2 \right\rangle = \frac{e^2 \Gamma^2 a^2}{2\pi\, 3! v_F^2} \int_{-\infty}^{+\infty} d\tau \left(\frac{e}{\hbar}V(t)\right)^3 \quad (17)$$

In this regime, we recover for the charge fluctuations a behavior similar to the one of the mean charge, with a dependence as $\phi^3$ for a voltage pulse of given shape. Comparing Eqs. (15) and (17), we see that the charge fluctuations are simply $e$ times the mean charge. This means that for larger flux, although the system is non-stationary, the charge fluctuations have a poissonian character, as is common in time-independent problems.

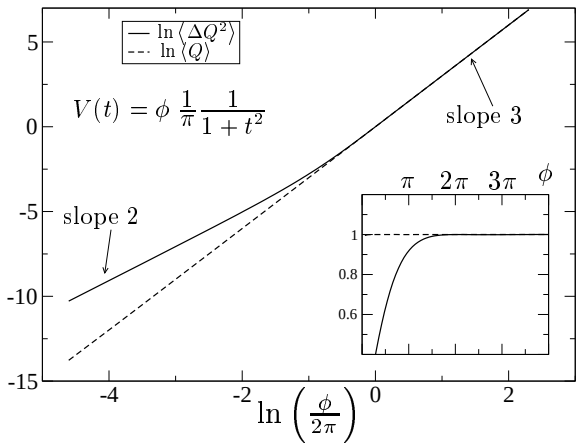


FIG. 2: Results of the numerical integration of Eqs. (6) and (7), for a pulse of Lorentzian shape ($V(t) = \phi\ (1/\pi)(1+t^2)^{-1}$). Mean transmitted charge (dashed curve) and its fluctuations (full curve) as a function of $\phi$, on a log-log plot. The mean charge has a slope 3, while the charge fluctuations has a slope 2 for small $\phi$ and a slope 3 for large $\phi$. Inset: ratio between the results of the adiabatic approximation (Eqs. (15) and (17)) and numerical integration of Eqs. (6) and (7), for the mean charge (dashed line) and its fluctuations (full line). The adiabatic approximation is clearly valid for the mean charge, while it is valid for the charge fluctations when $\phi \gtrsim 2\pi$.

To confirm the results obtained with the adiabatic approximation, we have performed a numerical integration of Eqs (6) and (7). The results for the case of a Lorentzian voltage pulse, $V(t) = \phi\ (1/\pi)(1+t^2)^{-1}$ are shown on Fig. 2. On this figure, it is clear that the mean transmitted charge behaves as $\phi^3$ for all $\phi$, while the charge fluctuations behaves as $\phi^3$ for large $\phi$, but as $\phi^2$ for small $\phi$. The inset of Fig. 2 shows the ratio between the numerical integrations and the results of the adiabatic approximation, Eqs. (15) and (17); we see that the adiabatic approximation gives excellent results for $\langle Q\rangle$ for all fluxes $\phi$, while it gives excellent results for $\langle \Delta Q^2\rangle$ when $\phi \gtrsim 2\pi$. Numerical integration with other shapes of voltage pulses (not shown) gives similar results.

## IV. NON PERTURBATIVE RESULTS

In the previous section, we have shown that, except for the charge fluctuations at small $\phi$, we obtain a very good approximation of the exact results by using an adiabatic approximation, where the transmitted charge due to the voltage pulse $V(t)$ is computed by integrating over $t$ the stationnary current $I_{st}$ due to $V = V(t)$. As the adiabatic approximation is related to the rapid decrease of the Green function product $G_1(t)G_2(t)$, and as higher orders of the tunneling current imply higher powers of this product, we expect the adiabatic approximation to be valid in the non-perturbative regime. We will thus compute non-pertubative results for the mean charge and

its fluctuations starting from non-pertrubative results for the stationnary tunneling current and noise. Because this method implies the calculation of stationnary current only, it is much simpler than the full calculations.

Non-perturbative results for the stationnary current are known in the case of tunneling of electrons from a normal metal to an edge state of the FQHE with filling factor $\nu = 1/3$. Indeed, for tunneling properties, this system is equivalent to the tunneling of electrons between two edges states of the same quantum Hall fluid with $\nu = 1/2$, as shown in [11]. This equivalence can be guessed from Eq. (8): the Green function product is $G_1(t)G_2(t) = (2\pi a)^{-2}(1 + iv_F t/a)^{-(1+1/v)}$, which for $\nu = 1/3$ is the same as $(2\pi a)^{-2}(1 + iv_F t/a)^{-1/v'}(1 + iv_F t/a)^{-1/v'}$ with $v' = 1/2$. In [11], non-perturbative results for the tunneling current and noise are obtained for this system, as it is linked by the duality symmetry to the tunneling of fractionnaly charged excitations (with $e^* = e/2$) between two $\nu = 1/2$ edge states, which can be treated non-perturbatively by refermionization.

Using results of [11], we have for the stationnary tunneling current, $I_{st}$, corresponding to voltage $V$:

$$I_{st} = \frac{e^2}{4\pi\hbar}V - \frac{e\,v_F}{4\pi\,\Gamma a}\mathrm{Arctan}\left(\frac{eV}{\hbar v_F}\,\Gamma a\right) \qquad (18)$$

This expression shows there are two extreme regimes for the current. For $V$ or $\Gamma \to \infty$, the tunneling is made through a barrier of large transparency, and the current goes to the maximal value $e^2V/(2h)$. On the opposite, for $V$ or $\Gamma \to 0$, the barrier transparency goes to 0, and we recover the perturbative results with $I \sim \Gamma^2 V^3$. To get the mean charge transmitted with a voltage pulse pulse $V(t)$, we now simply integrate over $t$ the current $I_{st}$ corresponding to $V(t)$. This gives:

$$\langle Q\rangle = \frac{e}{4\pi}\phi - \int_{-\infty}^{\infty} dt\ \frac{e\,v_F}{4\pi\,\Gamma a}\mathrm{Arctan}\left(\frac{eV(t)}{\hbar v_F}\Gamma a\right) \qquad (19)$$

In the tunneling regime, $\Gamma \to 0$, we recover the expression (15) obtained perturbatively, where the mean charge varies as $\phi^3$. Note that we obtain the same expression for the limit of very small voltage $V(t)$ for any given $\Gamma$ if $\Gamma V(t) \ll 1$ for all $t$. On the opposite, in the limit of a large $\Gamma$ or large $V(t)$, the charge becomes simply proportionnal to the flux $\phi$. The systems shows thus a crossover between $\langle Q\rangle \sim \phi^3$ at low $\phi$ and $\langle Q\rangle \sim \phi$ at large $\phi$, the position of the crossover being a monotonic decreasing function of $\Gamma$.

We now turn to the charge fluctuations. Using again the results of [11], we get the following expression for the non-perturbative stationnary current noise $S_{st}$ due to voltage $V$, with $x = (eV/\hbar v_F)\,\Gamma a$:

$$S_{st} = \frac{e^2 v_F}{4\pi\,\Gamma a}\left[\mathrm{Arctan}(x) - \frac{x}{1+x^2}\right] \qquad (20)$$

For $V$ or $\Gamma \to 0$, we recover the tunneling regime, with $S = e\,I \sim \Gamma^2 V^3$. As for the current, we obtain the charge fluctuations $\langle \Delta Q^2\ \rangle$ due to a voltage pulse $V(t)$ by integrating over $t$ the noise $S_{st}$ corresponding to $V = V(t)$. Defining $x(t) = (eV(t)/\hbar v_F)\,\Gamma a$, we get

$$\left\langle \Delta Q^2\right\rangle = \frac{e^2\,v_F}{4\pi\,\Gamma a}\int_{-\infty}^{\infty} dt\ \left[\mathrm{Arctan}(x(t)) - \frac{x(t)}{1+x(t)^2}\right] \qquad (21)$$

There is also a crossover in the behavior of $\langle \Delta Q^2\rangle$ as a function of $\phi$: when $\phi$ is small , we recover the results of Eq. (17) with $\langle Q^2\rangle \sim \phi^3$. As we know from the previous section, the adiabatic approximation does not reproduce in this case the correct $\phi^2$ behavior for $\phi \to 0$. When $\phi$ is large it is more difficult to get analytically the behavior of $\langle \Delta Q^2\rangle(\phi)$; numerical integration for different types of $V(t)$ shows that $\langle \Delta Q^2\rangle \sim \phi^{1/n}$ when $V(t) \sim t^{-n}$ for $t \to \infty$. This dependance on the asymptotic properties of $V(t)$ comes from the fact that for large $\phi$, noise comes mainly from the $t$ regions where $\Gamma V(t)$ is small (otherwise the barrier has a high transparency and noise is small), which are simply the tails of the voltage pulse.

## V. PERSPECTIVES AND CONCLUSIONS

The results we have obtained so far are valid for electron injection in any chiral Luttinger liquid, and can be applied to other systems than edge states of the fractional quantum Hall effect. In particular, the conduction electrons in a single-wall metallic Carbon nanotube can be described in terms of different modes of chiral Luttinger liquids. The electronic Green function in such a nanotube can be shown to be [13, 14]:

$$G_2(t) = (2\pi a)^{-1}(1+iv_F t/a)^{-\xi} \text{ with } \xi = \frac{3}{4}+\frac{1}{8}\left(g+g^{-1}\right) \qquad (22)$$

where $g$ is the parameter characterizing the interactions in the nanotube (typical experimental values are in the range $[0.2, 0.3]$). Note that $\xi > 1$, but $\xi$ is not an integer. We can then repeat, *mutatis mutandis*, the same reasoning as in section III to study the mean charge and its fluctuations. We define $K = (1/2)(1 + \xi)$, with $K > 1$. As $K$ is not an integer, the large time behaviors of the real and imaginary part of $G_1(t)G_2(t)$ are here the same:

$$\mathrm{Im}\left(G_1(t)G_2(t)\right) \sim t^{-2K} \qquad \mathrm{Re}\left(G_1(t)G_2(t)\right) \sim t^{-2K} \qquad (23)$$

Repeating the reasoning that leads to Eqs. (11) and (12), we see that because $K > 1$, both the mean charge and its fluctuations are finite, independently of the value of the flux $\phi$ (and thus of $V(t)$). As for an edge state of the FQHE, the electrons correlations in a nanotube prevent the divergence of the charge fluctuations for non-stationary injection.

Our results can also be of interest for the very general case of non-stationnary electron tunneling between two normal metals, when an Ohmic impedance is present in the system. Indeed, it is known that a mapping exists between a coherent one-channel conductor coupled to

an Ohmic environment (the dynamical Coulomb blockade problem [15]) and a Luttinger liquid with an impurity [16]. Our results suggest then that the divergence of charge fluctuations as predicted by Levitov and coworkers [8] are suppressed by the coupling to an Ohmic environment, as this coupling can be mapped to electron interaction leading to Luttinger liquid behavior.

To conclude, we have shown that for the non-stationnary charge injection from a normal metal into a chiral Luttinger liquid, using a voltage pulse $V(t)$, the electronic correlations in the Luttinger liquid prevent the charge fluctuations from being divergent for a generic voltage pulse. In the perturbative regime with respect to the tunneling amplitude, we have shown that explicit formulæ for the mean injected charge and its fluctuations can be obtained using an adiabatic approximation. We have identified when this adiabatic approximation breaks down, and shown that when it is valid, it leads to a Schottky-like relation bewteen the charge and its fluctuations. Finally, we have obtained non-perturbative results for the special case of charge injection in an edge state of the FQHE with filling factor $\nu = 1/3$. Centre de Physique Théorique is UMR 6207 du CNRS, associated to Université de la Méditérannée, Université de Provence and Université de Toulon.


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