# Diagnosis in Linear and Nonlinear Mixed-Signal Systems: a Parameter Identification Based Technique

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**Abstract**

*In this paper, we consider the nonlinear system modelling problem for on-chip testing and diagnosis of embedded mixed-signal systems. A Situation-Dependent AutoRegressive model with eXogenous variable (SDARX) is introduced to approximate the conventional Nonlinear-ARX (NARX). The parameter search space is divided into a linear weight subspace and the nonlinear parameter subspace. A nonlinear parameter estimation strategy combines the Levenberg-Marquardt method (LMM) for nonlinear parameter optimization and the least-square method (LSM) for linear parameter estimation. The diagnosis procedure requires a recursive estimation of the model parameters corresponding to the nominal behaviour, using input-output data recorded on the system under test. Emphasis is given to the characterisation of a particular failure mode by choosing the best model structure and identification of model parameters for diagnostic purposes. For fault identification, the parameter estimation technique is associated with the fault dictionary approach.*

## 1. Introduction

Due to the growing demands for reliability, safety, quality and cost efficiency, the call for fault tolerance in modern systems is gaining more and more importance. The detection and diagnosis of faults are necessary in order to achieve fault tolerance. The basic tasks of fault diagnosis are to detect and isolate occurring faults and to provide information about their size and source.

Analogue and mixed-signal devices are required in many applications such as communications, multimedia, and signal processing. These applications are often subject to severe area constraints. The complexity of analogue and mixed-signal cores, together with shrinking device dimensions limit accessibility to the internal nodes of the circuits. This makes circuit testing very difficult [14]. Ensuring high test quality at low cost for these mixed-signal designs has become an important test challenge [15]. In this context, we present a digital-compatible test and diagnosis scheme for analogue circuits. Analogue and mixed-signal cores are generally incorporated in a chip including intelligent digital components as microprocessors. Our purpose is to propose a test and diagnosis scheme that can be implemented using embedded smart resources.

Previous fault detection techniques have been restricted to check directly measurable variables for upward or downward transgression of fixed limits or trends. These techniques are automated by using a simple limit-value monitor. Various faults in the systems are recognised only when the controlled signal exceeds some predefined thresholds. For so called voting techniques, a fault occurrence was determined in a device when some redundant equipment allowing a comparison between two or more measures pointed out a possible mismatch. This technique, referred as hardware redundancy, is a powerful method but it has the disadvantage of requiring a lot of expensive fault-detection devoted equipment. In the last twenty years, the use of computers has allowed the development of new methods based on advanced signal processing techniques. Two main approaches may be identified: Model-Free Methods (MFM) and Model-Based Methods (MBM). Generally, MFM are simpler to implement, but they often suffer limitations coming from their inability to isolate the single faulted components or to detect even a small failure which has no evident effects, but causing just a slight and progressive degradation of performance. The MBM are based on the concept of analytical redundancy that is the process of identifying a failed component through

the comparison to an estimate which is derived by measurement of other functionally related components.

A fault is defined as any non-permitted deviation of a characteristic property, which leads to the inability to fulfil the intended objective. So, a fault is understood as any kind of malfunction in the actual system. The problems that are the main concerns in system testing include: fault detection, fault location, identification, and finally, fault prediction. For this last point, the response of the system is continuously monitored to check whether any component in the system is about to fail. In safety critical applications, the aim is to replace the concerned elements or systems before the imminent failure occurs with a minimum loss in the lifetime of the replaced elements or systems. This area is commonly referred to as system supervision. An essential prerequisite for development of automatic supervision techniques is an early fault detection tool. An attempt is made to detect the fault earlier and to locate them better by the use of a measured signal. This paper mainly addresses the on chip off-line fault detection and identification (FDI) problem for dynamic linear and nonlinear systems. Since the fault location and identification techniques assume that the system has already been identified as faulty, the fault detection is obviously a minimum requirement for fault location and identification.

Over the two last decades, the field of model-based fault detection and diagnosis has progressed significantly. At the present time, a significant body of results exists on off-line and on-line fault detection in linear systems (see for example [10], [12] and [13]). But the research results remain limited in the area of fault detection and diagnosis for linear systems. Even for this case, the few results existing in the area of fault location are limited to the particular case of faults that do not affect the linear structure of the faulty system. Unfortunately, it is well known that many real faults in a linear system can change its initial linear behaviour to a nonlinear one. If the classical detection scheme is generally sufficient to ensure the coverage of this type of faults, their efficient isolation is more problematic and requires the use of nonlinear modelling and identification procedure. A new type of models known as NARX (Nonlinear AutoRegressive with eXogenous input) has been discussed in [7] and [8] for discrete-time modelling of nonlinear systems. In the case of linear systems, parameter estimation based methods were proposed [5] for fault detection and isolation.

The main contribution of this paper is the development of a generic identification strategy applicable for both linear and nonlinear systems. Instead of conventional NARX modelling, the scheme proposed in this work uses a situation-dependent ARX (SDARX). This scheme represents not only the nonlinear behaviour of a nonlinear fault free system, but also to model the nonlinear faulty behaviour even if the nominal fault free system is linear. The parameter optimization algorithm separates the set of the model parameters into a linear part and a nonlinear part and applies different approaches for the estimation of both parts. Special attention has been given to numerical algorithms for the modelling and identification of nonlinear systems. The system under test will inherit the structure and the nonlinear parameters from the nominal model. Our claim is that a fault in the system under test will affect the estimated model in both its linear and nonlinear parameters. Hence, in our testing scheme, the nonlinear parameters of the system under test are assumed to be identical to those of the nominal model previously estimated.

The paper is organised as follows. Section 2 deals with linear and nonlinear system modelling using a regression model and introduces the SDARX modelling scheme. A parameter optimization algorithm for the generation of a nominal model by means of parameter identification is presented in Section 3. For an actual system under test, Section 4 considers parameter identification for fault detection and isolation on the basis of observed input/output data. Section 5 presents experimental results obtained using fault simulation on a microaccelerometer model. Finally, we will present some conclusions and directions for future work.

## 2. Regressive models for linear and nonlinear systems

### 2.1 The linear regressive model (ARX)

Although most processes in the real world display nonlinear dynamics, the linear modelling methods are often used in common practice. When the non-linearity of the process dynamics are not too severe, a linear approximation of this model in a neighbourhood of an operating point is generally satisfactory. It is well known that any single input single output (SISO) linear discrete-time dynamic system can be described by a regressive equation of the following form:

$$y(t) = \gamma^T(t)\theta + \nu(t) \tag{1}$$

where :

- $y(t)$ is the output of the system,
- $\gamma(t) = [y(t-1), y(t-2), \cdots, y(t-n_y), u(t-1), u(t-2), \cdots, u(t-n_u)]^T$ is the regression vector and its components include the previous input/output signal data (available at time t), required to calculate $y(t)$, where $n_y$ and $n_u$ are the orders of the system defined respectively by the maximal output and input delays required in the expression of $y(t)$.
- $\theta = [\theta_1, \theta_2, \cdots, \theta_{n\theta}]^T$ is the vector of constant parameters that characterises the system. Where $n_\theta = n_y + n_u$ is the total number of parameter to be estimated.
- The additive term $v(t)$ accounts for the fact that the next output $y(t)$ will not be an exact function of past data. However, a goal must be that $v(t)$ is small, so that we may think of $\gamma^T(t)\theta$ as a valid prediction of $y(t)$ given past data. $v(t)$ is a white noise assumed independent of input, output and system parameters.

Equation (1) represents the well-known AutoRegressive with eXogenous variables (ARX) models for linear systems, often called input/output equation. The autoregressive property is due to the use of previous output values as components in the regression vector and the exogenous (external) variables are the input signals of the system that are also included as components in the regression vector $\gamma(t)$.

### 2.2 The nonlinear regressive model (NARX)

When large deviations from operating point are possible, or when the dynamic non-linearity (eventually induced by faults) is severe, a linear modelling scheme is not efficient. In this case, the design of a nonlinear MBFDI scheme may be allowed by an accurate knowledge of a nonlinear model. A nonlinear model is then required to achieve efficient prediction of the system behaviour. For nonlinear systems, one difficult problem in MBFDI is the system modelling from a set of input-output observed data. Often, this is the only approach to modelling since just external data is available, given the difficulty to access internal data and the need to avoid any slight perturbation in the system. Wiener [1] was the first to use Volterra series to analyse the response of nonlinear systems to random excitation. Since then, intensive research effort has been directed toward the use of Volterra series and some other typical functions (e.g Laguerre expansions) for input-output modelling of nonlinear systems. Different approaches have been proposed in the specialised literature and interesting theoretical developments are available ([3]) but the practical aspects remain very limited. This is mostly due to the difficulty of obtaining the Volterra kernels for practical systems. In order to avoid this problem, a general realisation theory for nonlinear systems was proposed by Sontag in [4].

An extension of the ARX model of Equation (1) to nonlinear models, which predict the present output as a nonlinear function of the past inputs and outputs (as shown in Figure 1) is proposed and investigated by Billings et al. [8], [9]. It has been proved in [7] that a very large class of nonlinear discrete-time systems can be represented by the following nonlinear difference equation of form:

$$\begin{aligned} y(t) &= f(y(t-1), y(t-2), \cdots, y(t-n), u(t-1), u(t-2), \cdots, u(t-n_u)) + v(t) \\ &= f(\gamma(t)) + v(t) \end{aligned} \quad (2)$$

where $f(.)$ is a nonlinear function.

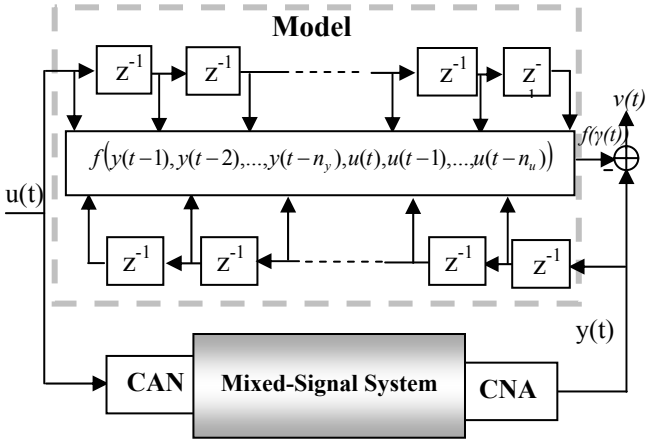


**Figure 1.** Example of NARX modelling of a mixed-signal system.

Because of its resemblance to ARX model (1), this model has been called NARX (as Nonlinear-ARX). Obviously, function $f(.)$ may have a complex (and often unknown a priori) structure.

### 2.3 The Situation dependent regressive model (SDARX)

The main problem in the NARX approach is the choice of a suitable expression for function $f(.)$. Various kinds of functions have been applied to approximate the unknown nonlinear function that maximizes the likelihood of the model. To simplify the fault detection procedure, our approach consists to use a situation-dependent ARX (SDARX) model described as follows:

$$y(t)=\phi_0(\gamma(t))+\sum_{i=1}^{n_y}\phi_i^y(\gamma(t))y(t-i)+\sum_{i=1}^{n_u}\phi_i^u(\gamma(t))u(t-i)+v(t) \quad (3)$$

where $\phi_0(\gamma(t))$, $\phi_i^y(\gamma(t))\big|_{1\le i\le n_y}$ and $\phi_i^u(\gamma(t))\big|_{1\le i\le n_u}$ are the data-dependent coefficients of the model.

In many practical cases, the data vector $\gamma(t)$ at time $t$ contains only a few number of past outputs, inputs or any measurable signal. The basic idea of the SDARX model (3) is to achieve the local linearization of the general NARX model (2) by introducing a locally linear ARX model with situation-dependent coefficients. Although the SDARX model (3) provides a useful framework for general nonlinear system modelling, the problem lies in specifying the functional form of its coefficients. In model (3), the signal $\gamma$(t) on which the time varying model coefficients depend may be the output signal, the input signal, or any other measured signal that is part of the system to be considered.

In the particular case where $\phi_0(.)=0$ and where $\phi_i^y(.)\big|_{1\le i\le n_y}$ and $\phi_i^u(.)\big|_{1\le i\le n_u}$ are all constant functions, the ARX model (1) is obtained with $\theta=[\theta_1,\theta_2,\cdots,\theta_{n\theta}]=[\phi_1^y(.),...,\phi_{n_y}^y(.),\phi_1^u(.),...,\phi_{n_u}^u(.)]^T$. This proves that the classical linear ARX model is a particular case of situation-dependent ARX model that is more general.

We have used a set of radial basis functions (RBF) to approximate the SDARX coefficients of the model (3), because an RBF network may approximate any functions by using members of a family of basis functions. Moreover, the locality of the basis functions makes the RBF much more suitable for modelling local variations [11]. Gaussian RBFs are selected as approximations of the coefficients of the model (3). The model coefficients derived are then given by:

$$\begin{aligned}
\phi_0(\gamma(t)) &= c_0^0+\sum_{k=1}^{m}c_k^0\exp\left\{-\rho_k^y\left\|\gamma(t)-Z_k^y\right\|^2\right\}\\
\phi_i^y(\gamma(t)) &= c_{i,0}^y+\sum_{k=1}^{m}c_{i,k}^y\exp\left\{-\rho_k^y\left\|\gamma(t)-Z_k^y\right\|^2\right\}\\
\phi_i^u(\gamma(t)) &= c_{i,0}^u+\sum_{k=1}^{m}c_{i,k}^u\exp\left\{-\rho_k^u\left\|\gamma(t)-Z_k^u\right\|^2\right\}
\end{aligned} \quad (4)$$

where $Z_k^\alpha\big|_{\substack{k=1,2,...,m\\ \alpha=y,u}}$ are the centers of the RBF, $\rho_k^\alpha\big|_{\substack{k=1,2,...,m\\ \alpha=y,u}}$ are the scaling parameters, $c_{i,k}^\alpha\big|_{\substack{k=1,2,...,m\\ i=1,2,...,n_\alpha\\ \alpha=0,y,u}}$ are scalar constant parameters and ||.|| denotes the vector Euclidean norm. $n_y$, $n_u$ and $m$ are the model orders, with $m$ the number of centers of each RBF function.

The complete system is given by the combination of the SDARX model of Equation (3) and the set of the coefficients defined in Equation (4). For embedded test and diagnosis purposes, our parameter estimation algorithm requires the separation of the set of parameters into a linear part and a nonlinear part and applies different approaches for the estimation of both parts. For the SDARX model defined by Equations (3) and (4), the linear parameters are given by:

$$\theta_L=\left\{c_{i,k}^\alpha\Big|_{\substack{k=1,2,...,m\\ i=1,2,...,n_\alpha\\ \alpha=0,y,u}}\right\} \quad (5)$$

and the nonlinear parameters are given by:

$$\theta_N=\left\{\rho_k^\alpha\Big|_{\substack{k=1,2,..,m\\ \alpha=y,u}},\ Z_k^\alpha\Big|_{\substack{k=1,2,...,m\\ \alpha=y,u}}\right\} \quad (6)$$

For estimation purposes, we can rewrite model (3) as:

$$y(t)=f(\theta_N,\theta_L,\gamma(t))+v(t) \quad (7)$$

where $\theta_N$ is the vector including all nonlinear parameters, $\theta_L$ is the vector including all linear weights, and the regression form of model (7), which is linear with respect to $\theta_L$, is given by :

$$y(t)=\varphi(\theta_N,\gamma(t))^T\theta_L+v(t) \quad (8)$$

## 3. Optimization algorithm for generation of the CUT nominal regressive model

The main idea in our identification approach is to divide the parameter search space into two subspaces (i.e., the linear weight subspace and the nonlinear parameter subspace). During the system design process, the search for finding the nominal model will concern the whole space including optimization in both linear and nonlinear parameter subspaces. To reduce the computational cost necessary during the testing process, optimization is applied in the linear parameter search subspace only.

The off-line identification procedure for the RBF model includes both order selection and estimation of all the parameters. Using a classical nonlinear parameter optimization algorithm, such as the Levenberg–Marquardt method (LMM) [2], to estimate all the parameters would involve extensive computation because the number of parameters to be estimated is usually quite large. In this paper, we propose a parameter estimation algorithm including both LMM

and the linear least square method (LSM). The first method optimizes the nonlinear parameters by using a nonlinear optimization iterative algorithm, which is based on an exhaustive search in the solution space and therefore requires extensive computation. The linear weights are estimated at each iteration, by the standard linear LSM, using the actual estimation of the nonlinear parameters.

First, the initial values $\theta_N^0$ for $\theta_N$ are chosen from prior knowledge of the system under test. At the k[th] iteration, fixing $\theta_N^k$ and estimating $\theta_L^k$ by LSM, yields:

$$\hat{\theta}_L^k = \left[X\left(\theta_N^k\right)^T X\left(\theta_N^k\right)\right]^{-1} X\left(\theta_N^k\right)^T Y \tag{9}$$

where:

$$Y = \begin{bmatrix} y(M) \\ y(M-1) \\ \vdots \\ y(\tau+1) \\ y(\tau) \end{bmatrix}, \quad X\left(\theta_N^k\right) = \begin{bmatrix} \varphi\left(\theta_N^k, \gamma(M-1)\right) \\ \varphi\left(\theta_N^k, \gamma(M-2)\right) \\ \vdots \\ \varphi\left(\theta_N^k, \gamma(\tau+1)\right) \\ \varphi\left(\theta_N^k, \gamma(\tau)\right) \end{bmatrix}, \tag{10}$$

$\{y(i), \gamma(i-1)\}\big|_{i=\tau+1,\ldots,M}$ is the measured dataset, $\tau$ is the largest time lag of any estimable variable in the model (7) or (8) and $M$ is the number of measured data observations.

To optimize the nonlinear parameters, efficient search routines are based on iterative local search in a "downhill" direction from the current point. We have chosen an iterative scheme of the following kind:

$$\hat{\theta}_N^{k+1} = \hat{\theta}_N^k + \eta_k R_k \nabla \hat{J}_k \tag{11}$$

Here $\theta_N^k$ is the nonlinear parameter estimate after iteration number *k*. The search scheme is thus made up from the three entities:

- $\eta_k$ is a scalar step length;
- $\nabla \hat{J}_K$ is an estimate of the gradient of the objective function $J(\theta_N, \theta_L)$
- $R_k$ *is a* matrix that defines the search direction.

The objective function is taken to be:

$$\begin{aligned} J(\theta_N, \theta_L) &= \sum_{k=\tau}^{M-1} \left(y(k) - \hat{y}(k+1/k)\right)^2 \\ &= \sum_{k=\tau}^{M-1} \left(y(k) - f(\hat{\theta}_N, \hat{\theta}_L, \gamma(k)\right)^2 \end{aligned} \tag{12}$$

where $\hat{y}(t+1/t) = f\left(\hat{\theta}_N, \hat{\theta}_L, \gamma(t)\right)$ is the one-step-ahead prediction of the output based on model (7). Hence:

$$\nabla \hat{J}_k = \frac{\partial J(\theta_N, \theta_L)}{\partial \theta_N} = -2 \sum_{k=\tau}^{M-1} \left(y(k) - f(\theta_N, \theta_L, \gamma(k))\right) \times h(\theta_N, \theta_L, \gamma(k) \tag{13}$$

where:

$$h(\theta_N, \theta_L, \gamma(k)) = \frac{\partial f(\theta_N, \theta_L, \gamma(k))}{\partial \theta_N} \tag{14}$$

It is well known that gradient search for the minimum is inefficient, especially for ill-conditioned problems close to the minimum. The true Newton direction will thus require that the second derivative be computed. In order to increase the robustness and the quickness of the search process, alternative search directions are more common in practice. In the LMM algorithm, the search direction $R_k$ of Equation (11) is given by:

$$R_k = H_k + \delta I \ . \tag{15}$$

where:

$$H_k = \sum_{k=\tau}^{M} h\left(\hat{\theta}_N^k, \hat{\theta}_L^k, \gamma(k)\right)^T h\left(\hat{\theta}_N^k, \hat{\theta}_L^k, \gamma(k)\right) \tag{16}$$

$\delta$ may be used instead of a step size. A large $\delta$ gives a small step in the gradient direction and a small (zero) $\delta$ gives a Gauss-Newton step.

## 4. Recursive LSM for fault detection and diagnosis

In our approach, we will suppose that embedded smart resources (such as microprocessors and memories) can be used to implement the test and diagnosis of the mixed signal cores or systems. Using the previous algorithm (9) and (11) to estimate all the parameters of the system under test during the test application would involve a large amount of computation that could not be implemented on the embedded resources. The computation time would soon be incompatible with the practical constrains of test application. In order to reduce the amount of computation necessary during the testing process, the search centers on the optimization of the linear parameter subspace only. The system under test will inherit the structure and the nonlinear parameters from the nominal model. Our claim is that a fault in the system under test will affect the estimate model in both its linear and nonlinear parameters.

Hence, in our testing scheme, the nonlinear parameters $\theta_N$ of the system under test are assumed to be identical to those of the nominal model previously estimated.

### 4.1. On chip estimation of CUT linear parameters

A simple LSM can be used to estimate the linear parameters of the system under test. An application of

conventional LSM requires large size matrix inversion. To avoid the intensive computation required by this operation, we have implemented a recursive least square (RLS) algorithm for $\theta_L$ estimation:

$$\hat{\theta}_L(t) = \hat{\theta}_L(t-1) + K(t)\left[y(t) - \varphi(\theta_N, \gamma(t))^T \hat{\theta}_L(t-1)\right] \quad (17)$$

where:

$$\begin{aligned} K(t) &= P(t)\varphi(\theta_N, \gamma(t)) \\ &= \frac{P(t-1)\varphi(\theta_N, \gamma(t))}{1 + \varphi(\theta_N, \gamma(t))^T P(t-1)\varphi(\theta_N, \gamma(t))} \\ P(t) &= P(t-1) - \frac{P(t-1)\varphi(\theta_N, \gamma(t))\varphi(\theta_N, \gamma(t))^T P(t-1)}{1 + \varphi(\theta_N, \gamma(t))^T P(t-1)\varphi(\theta_N, \gamma(t))} \\ &= \left[I - K(t)\varphi(\theta_N, \gamma(t))^T\right] P(t-1) \end{aligned} \quad (18)$$

The initial value of matrix *P(0)* and $\hat{\theta}_L(0)$ are chosen identical to those of the nominal model when the same sequence of input data is applied. Hence $\hat{\theta}_L(0) = \tilde{\theta}_L$ and $P(0) = \left(\sum_{k=\tau}^{M} \varphi(\theta_N, \gamma(t))\varphi(\theta_N, \gamma(t))^T\right)^{-1}$ where $\tilde{\theta}_L$ is the estimation of the linear parameters for the nominal system.

### 4.2. Fault dictionary composition

The fault dictionary technique for isolation is based upon the comparison of the failed system parameters with a set of fault signatures previously stored in a dictionary [6]. The first step in the fault dictionary building procedure is the fault definition. This step is a very critical aspect of the entire approach since only the most likely faults anticipated in this step could be identified. Large number of potential faults must be included; obviously, this will have an impact on the dictionary seize. For each faulty hypothesised case ($f_i$), the circuit under test is simulated and the corresponding input/output data are recorded. The corresponding faulty parameter vector $\hat{\theta}_i = \left(\hat{\theta}_{N,i}, \hat{\theta}_{L,i}\right)$ is determined with the LSM and LMM defined by (9) and (11) respectively, using the model structure of the fault free system. $\theta_i$ is stored in the fault dictionary as signature of fault $f_i$ for use in the identification of this fault.

### 4.3. Fault detection

The linear parameter vector $\hat{\theta}_L$ of the actual system is determined by the least square method described above (17), using the model structure defined in the previous system identification phase. $\hat{\theta}_L$ may take the value $\tilde{\theta}_L$ in the nominal case and $\hat{\theta}_L \neq \tilde{\theta}_L$ when the fault $f_i$ has occurred. The detection problem consists of the decision between the following two hypotheses:

$$H_0 : \hat{\theta}_L = \tilde{\theta}_L \text{ (no fault)} ; \quad H_1 : \hat{\theta}_L \neq \tilde{\theta}_L \text{ (fault)}.$$

The test can then be performed using a residual measure of the form:

$$r = (\hat{\theta}_L - \tilde{\theta}_L)^T (\hat{\theta}_L - \tilde{\theta}_L) \quad (19)$$

Finally, in any case, the residual is compared with a fixed threshold $\lambda_{max}$ and hypothesis $H_1$ is accepted if $\lambda < \lambda_{max}$.

### 4.4. Fault isolation

Since the physical parameters which indicate the faults of the system under test are generally not directly available for measurement, their change may be determined via the changes in the process model parameters. Under hypothesis $H_1$, the fault isolation is based on the idea of comparing the observed value of $\left(\tilde{\theta}_L - \hat{\theta}_L\right)^T \left(\tilde{\theta}_L - \hat{\theta}_L\right)$ with his fault free expected value for each class of fault. Fault isolation is performed using the following measures:

$$r_i = \left(\hat{\theta}_{L,i} - \hat{\theta}_L\right)^T \sigma_i^{-1} \left(\hat{\theta}_{L,i} - \hat{\theta}_L\right)\Big|_{i=1,2,\cdots,k} \quad (20)$$

where $\sigma_i$ is the fault free expected covariance matrix relative to fault class $f_i$, calculated as:

$$\sigma_i = Cov\left(\hat{\theta}_{L,i} - \tilde{\theta}_L\right) = \left(\hat{\theta}_{L,i} - \tilde{\theta}_L\right)\left(\hat{\theta}_{L,i} - \tilde{\theta}_L\right)^T \quad (21)$$

In either case the residual corresponding to each class of fault is compared with a particular fixed threshold $\lambda_i$. An occurring fault belongs to fault class $f_i$ if and only if $r_i < \lambda_i$.

## 5. Application to microsystems testing

As a case study, we consider a simple accelerometer based on a seismic mass suspended by a microbeam structure as shown in Figure 2.

Thanks to the high piezoresistivity of mono-crystalline silicon, an external acceleration is transformed into a proportional electrical signal through a piezoresistive bridge. A dynamic model of the accelerometer is of the form:

$$m\frac{d^2x(\tau)}{dt^2} = -b\frac{dx(\tau)}{dt} - kx(\tau) + m.u(\tau) \quad (22)$$

where $m$ is the seismic mass, $u(\tau)$ is the external acceleration, $x(\tau)$ is the seismic mass displacement, and $b$ and $k$ are the air damping and the beam spring constants, respectively.

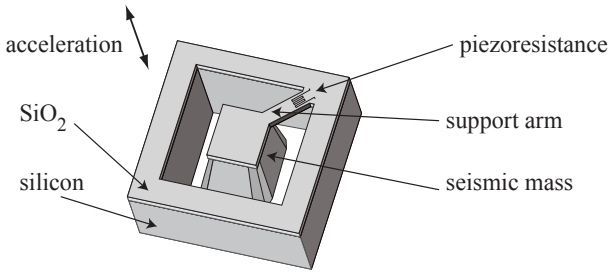


**Figure 2.** Piezoresistive accelerometer based on a suspended seismic mass.

The resistance variation $\Delta R(x)$ corresponding to a seismic mass position $x(\tau)$ is defined by:

$$\frac{\Delta R(x)}{R0} = \Gamma . x(\tau) \quad \text{with} \quad \Gamma = \frac{\pi_l e_p (L - 0.5 l_r).k}{2.I} \tag{23}$$

where $\pi_l$ is the longitudinal piezoresistive coefficient, $I$ and $e_p$ are the inertia moment and the thickness of the beam, $L$ and $l_r$ are the lengths on seismic mass and resistor, respectively. When a reference voltage $V_{DD}$ is applied to the bridge circuit, the measure of the output voltage $\Delta V$ is given by:

$$y(\tau) = \frac{\Delta R(\tau)}{2.R0 + \Delta R(\tau)} V_{DD} = \frac{\Gamma . x(\tau)}{2 + \Gamma . x(\tau)} V_{DD} \approx \left[ \frac{\Gamma . x(\tau)}{2} - \frac{\Gamma^2 . x^2(\tau)}{4} \right] V_{DD} \tag{24}$$

So the input/output behaviour of the microaccelerometer is described by a nonlinear dynamic continuous-time model defined by Equations (22) and (24). This model was used for both nominal and faulty system simulation. In the discrete time context, since the system is nonlinear, a NARX model is normally required. But in practice satisfactory modelling results were obtained with a simple second order three parameter ARX model of form:

$$y(t) = \theta_1 y(t-1) + \theta_2 y(t-2) + \theta_3 u(t-2) + v(t) \tag{25}$$

Parameter vector $\theta=(\theta_1, \theta_2, \theta_3)$ was estimated for each fault belonging to the fault dictionary. The difference with the nominal parameter vector and the corresponding distance were evaluated and stored as a fault signature. For a sample period of $10^{-4}$s, the estimated nominal parameter vector is $\theta_{nom} = (1.251, -0.983, 0.952)$. Faults affecting the damping $b$, the beam spring $k$, the beam inertia $I$ and the seismic mass $m$ have been considered in the dictionary. In Table 1, the results obtained for a seismic mass fault simulation are presented.

All the parametric failures on the seismic mass value greater than 10% are detected with a threshold of $\lambda_{max} = 0.01$. We can also notice that the first component of the parameter vector is more sensitive to seismic mass fluctuations than the two last components.

| Faulty value $\Delta$m/m (%) | Signature $\Delta\theta^T = (\theta_{nom}-\theta_{fault})^T$ | $\Delta\theta^T\Delta\theta$ |
|---|---|---|
| + 50 | [0.239, -0.006, 0.020] | 0.058 |
| + 40 | [0.203, -0.005, 0.027] | 0.042 |
| + 30 | [0.164, -0.004, 0.020] | 0.027 |
| + 20 | [0.1194, -0.003, -0.001] | 0.0143 |
| + 10 | [0.064, -0.002, 0.008] | 0.0042 |
| 0 | [0.0, 0.0, 0.0] | 0.0 |
| -10 | [-0.075, 0.002, -0.005] | 0.006 |
| -20 | [-0.167, 0.005, -0.017] | 0.0283 |
| -30 | [-0.286, 0.008, -0.030] | 0.083 |
| -40 | [-0.432, 0.015, -0.060] | 0.190 |
| -50 | [-0.634, 0.022, -0.077] | 0.410 |

**Table 1.** Seismic mass related fault dictionary.

The tolerance on parameter values is not yet taken into account in the actual version of our fault dictionary. This problem will be considered in our future works.

## 6. Conclusion

Fault detection and isolation for dynamic analogue systems is a very complex task. Even in the linear system case, this problem can not be solved by means of a unique method, but requires several different complementary techniques. When the systems to be monitored can be described by a linear model, a parameter identification procedure is proved to be very suitable to afford some diagnostic tasks particularly when there is little or none a priori knowledge about the system analytical characteristics. The purpose in this paper has been to show that existing parameter estimation techniques for linear systems can be extended to the general nonlinear dynamical case by the use of a suitable modelling structure. The SDARX model structure introduced leads to an implementation that is less costly in terms of computation time and resources than a conventional NARX model. From a computational point of view, a noticeable advantage offered by SDARX modelling is that it requires reasonable computational and memory resources, in such away that their implementation on PC, or even on a simple intelligent module using a standard microcontroller is realistic. An example of parameter estimation for modelling and testing of a simple microsystem has been presented. It is our intention to study the use of theses techniques for the testing of

more complex devices. When using the RLS algorithm in Section 4.1 to estimate the parameters of a system, some conditions are needed on the input signal *u(k)* to guarantee that the parameters estimated will converge to their unknown actual values. Moreover, this convergence must be *robust* to small disturbances such as measurement or calculation noises. Typically, such conditions are called "persistence of excitation" conditions. This paper did not deal with this issue that will be addressed in our future works.

## References


[1] N. WIENER, *"Nonlinear Problems in Random Theory",* MIT Press, Cambridge, MA 1958.

[2] D. MARQUARDT, *"An algorithm for least-squares estimation of nonlinear parameters"*, SIAM J. Appl. Math., Vol. 11, pp. 431–441, 1963.

[3] R. BROCKETT, *"Volterra Series and Geometric Control Theory ",* Automatica, No. 12, pp. 167-176, 1976.

[4] E. SONTAG, *"Polynomial Response Maps",* Lecture Notes in Control and Information Sciences, Vol. 13, Springer-Verlag, Berlin 1979.

[5] R. ISERMANN, *"Process Fault Detection Based Modelling and Estimation Methods--A Survey"*, Automatica, Vol. 20, No. 4, pp. 387-404, 1984.

[6] V. VISVANATHAN, A. SANGIOVANNI-VINCENTELLI, R. SAEK, *"Diagnosability of Nonlinear Circuits and Systems __ Part I & II"*, IEEE Trans. on Circuit and Systems, Vol. Cas-28, No. 11, pp. 1093-1108, 1981.

[7] I.J LEONTARITIS, S. A. BILLINGS, *"Input-output Parametric Models for Nonlinear Systems Part1 & Part2",* International Journal of Control, Vol. 41, pp. 303-344, 1985.

[8] S. CHEN, S. A. BILLINGS, *"Representation of Nonlinear systems: the NARX Model",* International Journal of Control, Vol. 49, pp. 1013-1032, 1989.

[9] S. CHEN, S. A. BILLINGS, C. F. N. COWAN, and P. M. GRANT, *"Nonlinear systems identification using radial basis functions*," Int. J. Syst. Sci., Vol. 21, pp. 2513–2539, 1990.

[10] A. CHATTERJEE, "*Concurrent Error Detection and Fault Tolerance in Linear Analog Integrated Circuits Using Continuous Checksums*", IEEE Transactions on VLSI Systems, Vol. 1, No. 2, pp. 138-150, 1993.

[11] J. SJÖBERG, Q. ZHANG, L. LJUNG, A. BENVENISTE, B. DELYON, P.Y. GLORENNEC, H. HJALMARSSON and A. JUDITSKYS, *"Nonlinear Black-box Modeling in System Identification: a Unified Overview"*, Automatica, Vol. 31, No. 12, pp. 1691-1724, 1995.

[12] E. SIMEU, A. W. PETERS and I. RAYANE *"Automatic Design of Optimal Concurrent Fault Detector for Linear Analog Systems",* 29th International Fault-Tolerant Computing Symposium (FTCS), Madison, Wisconsin, USA, June 15-18, 1999.

[13] M. W. TIAN and C.-J. R. SHI, *"Worst case tolerance analysis of linear analog circuits using sensitivity bands"*, IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications, Vol. 47, No. 8, pp. 1138–1145, 2000.

[14] P. VARIYAM and A. CHATTERJEE, *"Digital-compatible BIST for analog circuits using transient response sampling"*, IEEE Design & Test of Computers, Vol. 17, pp. 106–115, July–Sept., 2000.

[15] S.S. AKBAY, A. HALDER, A.CHATTERJEE, and D. KEEZER, *"Low-Cost Test of Embedded RF/Analog/Mixed-Signal Circuits in SOPs*", IEEE Transactions on advanced packaging, Vol. 27, No. 2, 2004