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Indoor Sonic Boom Reproduction Using ANC

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ABSTRACT

The European programs for development of supersonic air-flights involve new studies on the human perception of sonic boom. Because this noise includes high-level components at very low-frequency, the usual psycho-acoustic tests with headphones are not relevant; instead, the original sound-field can be reproduced with many loudspeakers in a small room, but the loudspeakers must be controlled for an accurate reproduction, both in time and space, in an area large enough to enclose a listener's head. In this paper, Active Noise Control is applied to sonic boom reproduction through Boundary Surface Control (as named by S.Ise) of the acoustic pressure around a listener. A small room was built at LMA with sixteen powerful low-frequency acoustic sources in the walls. Frequency and time-domain numerical simulations of sonic boom reproduction in this room are given, including a sensitivity study of the coupling between a listener's head and the incident sonic boom wave which combine into the effective sound-field to be reproduced.

1. INTRODUCTION

Most of the time, when the disturbance induced by some industrial or transport noises has to be evaluated, psycho-acoustic tests are conducted through sound reproduction using headphones. However, in some cases, tests with headphones are not relevant because of spectral or spatial specificities of the soundfield to be reproduced. Sonic boom is such a very special noise: it is very loud (120 dB is a common level for this type of sound), and most of its energy is localized at very low frequencies, down to 2 or 3 Hz. Headphones could reproduce such a pressure at the eardrums of a listener, but it is expected that the perception of such a very low frequency sound does not only depend on the pressure fluctuation at the listener's ear, but also at the whole listener's body, especially on his or her torso. Moreover, headphones do not replicate accurately the noise at the eardrums if the listener's own Head Related Transfer Functions (HRTF) are not included in the audio reproduction device. The sound image reproduced using headphones also moves with the listener's head, whereas slight rotations of the head in front of a fixed source are known to be an important factor in the source localization. Therefore, a soundfield reproduction technique is needed that works in an area large enough to enclose a person and to let him or her move slightly. The reproduction technique should also not depend on the physiognomy of the listener.

Because of the above drawbacks of sound reproduction using headphones, a Boundary Surface Control (BSC) technique [1], as named by S.Ise, has been chosen to perform indoor sonic boom reproduction at the LMA. This paper presents the preliminary work that has been conducted to that purpose. Firstly, the soundfield reproduction is formulated as an Active Noise Control (ANC) problem, and the theory underlying Boundary Surface Control is briefly introduced. Numerical simulations of soundfield reconstructions are then presented and discussed. Finally, the results of a study on the influence of the listener's presence upon the system performances are shown.

2. SOUND REPRODUCTION USING ANC

The simplest ANC set-up includes a single acoustic source cancelling the noise measured by a single microphone. If d denotes the disturbance signal and y the signal produced by the secondary source, the usual ANC problem is the minimization of the error signal $e = d + y$. The nearer y is to $-d$, the better is the control performance.

Therefore, cancelling a primary noise amounts to reproducing it and inverting its phase. The ANC problem can be transposed into a sound reproduction problem simply by denoting d the sound to be reproduced and y the reproduced sound, the error being $e = d - y$. This remains true whether the control filter is adaptive or not, and for the single-channel as well as for the multi-channel case where d , y and e are signals vectors. This means that any active noise

control device can perform sound reproduction. All the techniques, algorithms and hardware that have been developed for ANC can be used for sound reproduction.

In this paper, the sound reproduction is formulated as an ANC problem because of two features of ANC systems which are not present in usual reproduction techniques such as stereophony and 5.1 sound reproduction, nor in more advanced techniques such as Wave Field Synthesis [2]: ANC devices very often rely on the monitoring of error signals which directly measures the system performances, and they make use of adaptive filtering. These two features are of importance for an accurate sound reproduction: adaptive filtering can reduce the sound reproduction sensitivity to temperature changes or to a listener's presence; the inclusion of error signals in the reproduction process means that direct information about noise propagation is available. No a priori assumptions have to be made about the reproduction area, an accurate and adaptive reproduction can be achieved even in a room at low-frequency. Furthermore, the prediction of the primary noise, which requires a feedforward reference signal for control of broadband non-stationary noise, is not a problem in sound reproduction. The signal d , which has to be reproduced, must have been recorded or computed in advance so that it is available as a convenient reference signal. Moreover, it is possible to make use of this reference signal with a time advance as long as required in order to make causal the inversion of the secondary path matrix which is required for an accurate sound reproduction.

3. BOUNDARY SURFACE CONTROL

For psycho-acoustic tests, a soundfield reproduction must be performed over a 3D area which must be large enough to surround the listener's head. This means that, from one hand, many sensors and actuators are required. On the other hand loudspeakers and microphones cannot be placed too close to the listener, otherwise the tests would be uncomfortable for him or her. The sound reproduction quality could also decrease in this case, due to the stronger influence of his or her presence on the soundfield to be reproduced (see part 5).

Fortunately, sound reproduction, as well as noise cancellation, can be performed inside a 3D volume by controlling noise only at the boundary surface of the volume. To this purpose, Furuya et al. proposed in 1990 a method called Boundary Pressure Control, and, in the 90's, Ise suggested a substitute method called Boundary Surface Control [1]. Several other versions of the technique have then been presented [3]. As it is denoted by its name, BSC aims at controlling the pressure in a volume by monitoring noise at its boundary surface. It is justified by the Kirchhoff-Helmholtz integral expression of the acoustic pressure inside a bounded volume:

$$p(\mathbf{r}) = \iint_{\Sigma} \left[G(\mathbf{s}, \mathbf{r}) \frac{\partial p(\mathbf{s})}{\partial \mathbf{n}} - p(\mathbf{s}) \frac{\partial G(\mathbf{s}, \mathbf{r})}{\partial \mathbf{n}} \right] \mathbf{d}\mathbf{s} \quad (1)$$

where $p(\mathbf{r})$ is the sound pressure at a point \mathbf{r} of the volume Ω , \mathbf{s} is a point on the surface Σ of Ω , \mathbf{n} the unit vector which is normal to the surface and G the Green function of Helmholtz equation in free field. This equation shows that pressure at an interior point only depends on pressure and its normal derivative on the exterior surface. The idea of BSC is therefore to control these values in the same way as for multi-channel ANC, e.g. to impose some pressure values on a group of error microphones placed all over the boundary of the control region.

The Boundary Pressure Control method relies on the fact that the some redundant information can be found in the right hand side of the equation (1). Indeed, the limit of equation (1) for \mathbf{r} tending to a point \mathbf{s}_0 of Σ is:

$$\frac{1}{2}p(\mathbf{s}_0) = \iint_{\Sigma} \left[G(\mathbf{s}, \mathbf{s}_0) \frac{\partial p(\mathbf{s})}{\partial \mathbf{n}} - p(\mathbf{s}) \frac{\partial G(\mathbf{s}, \mathbf{s}_0)}{\partial \mathbf{n}} \right] \mathbf{d}\mathbf{s} \quad (2)$$

In addition to Eq. (1), Eq. (2) shows that surface pressure normal derivative can be seen as a function of the pressure on the surface. This means that ensuring the right pressure value on Σ gives the desired soundfield in any point inside the control region.

Several problems have been raised concerning the BPC and BSC methods. The first problem is that the integral in the Kirchhoff-Helmholtz equation is a continuous function of the space variable on the surface, which implies that the sound pressure at point \mathbf{r} depends on an infinite number of pressure values. Monitoring the pressure at a finite number of locations for control implicitly relies on the discretization of the integral in equation (1). In practice it is not possible to control the pressure at more than a few dozen different points, which imposes a limitation on the sound reproduction frequency range [4]. A second problem occurs for BPC at the eigenfrequencies of the internal Dirichlet problem in Ω , where the solution of equation (2) is not unique [3]. This means that, at the volume eigenfrequencies, both the acoustic pressure and its normal derivative are theoretically needed for the interior soundfield to be fully controlled.

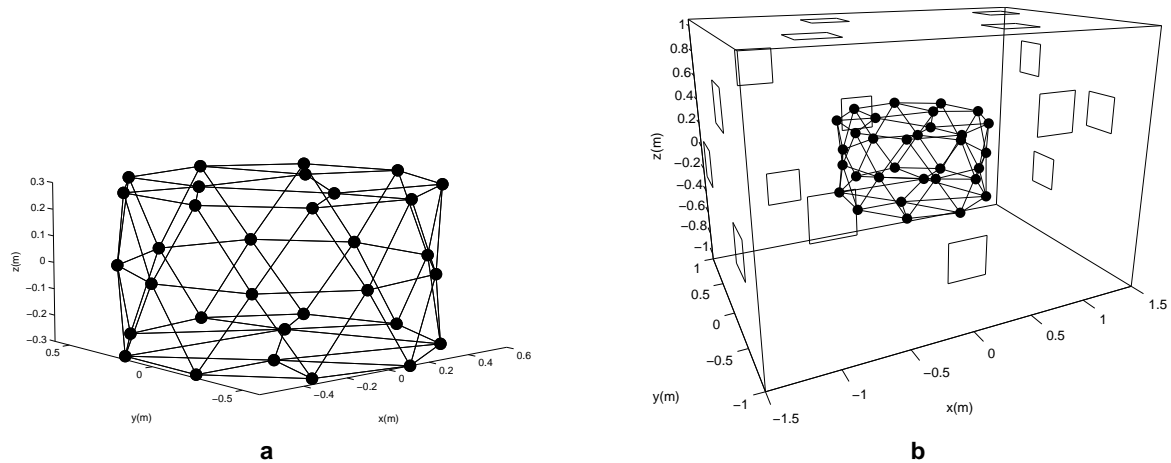


Figure 1: The virtual error microphones cylindrical net alone (a), and placed into the reproduction room (b). The sources are represented by the squares on the walls

Only BSC, which a priori requires twice as many sensors as BPC, is supposed to work at the eigenfrequencies of the inner volume. However numerical simulations showed that a slight dissimetry in a mesh of pressure sensors on the boundary surface could be sufficient for ensuring control of the noise inside the volume through control of only the pressure at the boundary (see Ref.[1] and [3]); the discretization of equation (2) using an irregular mesh could lead to a discretized problem with a unique solution. Finally, a few pressure gradient sensors or a few pressure sensors in the volume can be used for BPC in addition to the pressure sensors, which is the equivalent in the context of ANC of the CHIEF method implemented for computation of acoustic fields using a Boundary Element Method [5].

Because no experimental work has been reported on the failure of BPC at the inner volume eigenfrequencies, BPC has been chosen to perform sonic boom reproduction at LMA. One of the aims of the numerical simulations below is to determine if the theoretical control singularity at the eigenfrequencies is a real limitation to the use of control of pressure only.

4. NUMERICAL SIMULATIONS OF INDOOR SONIC BOOM REPRODUCTION

In order to reproduce the noise generated on the ground by a supersonic aircraft, a reproduction room has been built at the LMA, Marseilles, including sixteen powerful low-frequency acoustic sources in the walls. The sources were designed to reproduce the high low-frequency pressure levels that can be measured in real conditions. Each source includes two large loudspeakers driven with out-of-phase signals so that the first distortion harmonic is minimized. A simple model of the room and the noise sources has been elaborated for numerical simulations of the sound reproduction.

A. Modelling of the Reproduction Room

Several simplification hypothesis have been made for modelling the reproduction room. The room is assumed to be a perfect 3m x 2m x 2.1m parallelepiped so that the modal behaviour of the room can be easily written as a sum of cosine functions. The absorbing properties of the walls are included in the model through a constant real normal admittance, which leads to modal damping proportional to frequency [4]. Acoustic sources are considered as omnidirectional monopoles. Finally, it is assumed that the cavity is airtight, i.e. there is no acoustic leakage.

Using these assumptions, the pressure radiated by a monopole source at a measurement point can be written as a simple modal series. For the simulation this series was restricted to the first thousand eigenmodes. This number has been found sufficient to describe the acoustical paths in the room at low frequencies (below 500Hz), which is the frequency range for which it is intended to reproduce accurately the sonic boom soundfield.

The Boundary Pressure Control method requires the meshing of the surface enclosing the volume where sound reproduction is intended. For the simulations a cylindrical mesh was considered, which is adequate for enclosing a listener during psycho-acoustic tests. The cylinder includes 32 microphones/nodes, and has a 60 cm radius and a

60 cm height, as displayed on figures 1a and 1b. The microphones are supposed to be perfectly omnidirectional for the simulations.

B. Frequency-Domain Simulations

In the frequency domain, pressure values and acoustic paths can be described by single complex coefficients. The acoustic pressure field which has to be reproduced at the error microphones can be written as a complex vector \mathbf{p}_M^0 . Let \mathbf{p}_P^0 denote the noise that has to be reproduced inside the control volume at some observation points. At a given frequency, if \mathbf{H}_{SM} and \mathbf{H}_{SP} respectively denote the transfer matrix from the secondary sources to the control microphones, and from the sources to the observation point, the optimal vector of source command signal is:

$$\mathbf{q} = \mathbf{H}_{SM}^{-1} \mathbf{p}_M^0 \quad (3)$$

The reproduced soundfield pressure at the observation points is then given by:

$$\mathbf{p}_P = \mathbf{H}_{SP} \mathbf{q} = \mathbf{H}_{SP} \mathbf{H}_{SM}^{-1} \mathbf{p}_M^0 \quad (4)$$

Note that these computations are similar to the derivation of optimal noise cancellation: the reconstruction error at the microphones only depends on the source-to-microphone transfer matrix inversion.

The first soundfield whose reproduction is evaluated through numerical simulations is an harmonic plane wave. In order to provide easy-to-interpret figures, an additional horizontal mesh of 32 by 64 observation points is used to

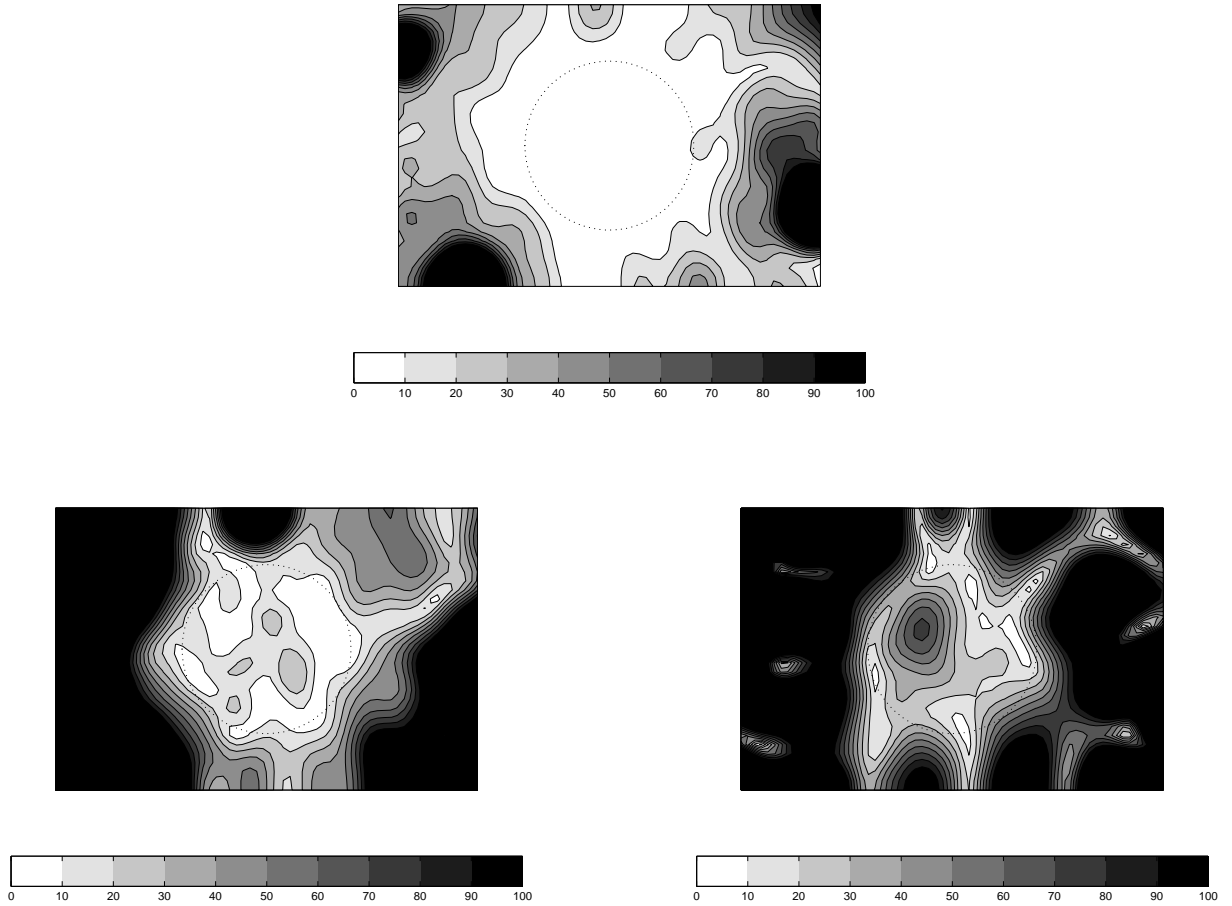


Figure 2: Contours of equal sound pressure error level (in percents) for plane waves of 100 Hz (top), 200 Hz (bottom-left) and 300 Hz (bottom-right)

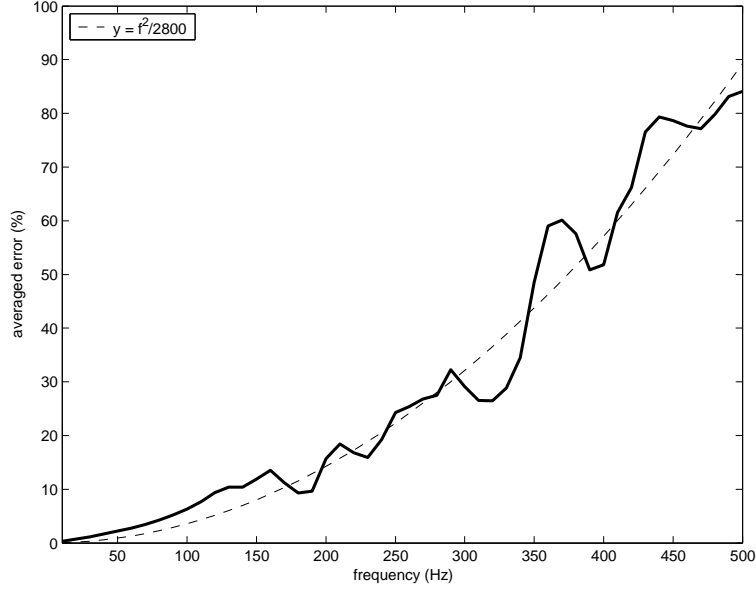


Figure 3: Averaged sound pressure error in the control region as a function of the frequency

observe the soundfield over the whole reproduction room. The height of the observation points corresponds to the middle of the cylinder on the surface of which the acoustic pressure is controlled. After computing the input to the control source, the pressure error vector can be computed by the following formula:

$$e(x,y) = 100 \left| \frac{p_r(x,y) - p_o(x,y)}{p_o(x,y)} \right| \quad (5)$$

where p_r and p_o are respectively the reproduced and original pressure values at the (x,y) point. The obtained value e is then in percents. Equal sound pressure error contours are then displayed for harmonic plane waves at 100, 200 and 300 Hz on figure 2. It can be seen that, as expected, the performance of the sound reproduction system decreases as the frequency of the original sound wave increases. This can easily be explained by the fact that the density of microphones per wavelength on the control surface decreases as the frequency increases. The microphones surface density D is given by:

$$D = \frac{n}{S} \quad (6)$$

where n is the number of error microphones and S the surface of the cylinder in squared meters. If S is a number of λ by λ squares, where λ is the wavelength of the original sound, then we have

$$D_\lambda = \frac{n\lambda^2}{S} = \frac{nc^2}{Sf^2} \quad (7)$$

where c is the sound speed and f the frequency. The number of error microphones for each λ by λ square decreases as a f^{-2} function, and so does the sound reproduction accuracy. This can be seen as a generalization of a frequently observed result in ANC, which is that three sensors by wavelength are necessary to ensure an efficient control along a one-dimensional microphone antenna (see Ref. [4],[6]).

In order to observe more accurately the influence of the primary wave frequency on the system performance, figure 3 displays the relative pressure error, averaged on a few dozens of regularly spaced points inside the cylinder where control is intended. It can be seen that the reproduction error behaves as a f^2 function, it is inversely proportionnal to the error microphones surface density D_λ of the error microphones, which confirms the significance of D as an indicator of the reproduction accuracy.

In the case of a cylindrical volume with radius 60 cm and height 60 cm, the first eigenfrequency of the Dirichlet problem for the internal volume takes place at about 283 Hz, which is within the range of the intended soundfield reproduction. It can be seen in figure 3 that no error peak can be observed around this frequency, which means that the soundfield reproduction through Boundary Pressure Control does not suffer from deficiencies at this cylinder eigenfrequency. The eigenfrequencies of higher order are out of the frequency range of the system.

Although the reproduction error increases quickly with the frequency, figures 4 shows that the phase of the secondary wave is quite well reproduced even for large values of f . This suggests that, even if for one frequency value the amplitude of the pressure is not perfectly reproduced inside the control region, the crossing of a transient sound, for instance from the left to the right, can be reproduced so that a listener perceives the direction where the sound is coming from, because an accurate phase reconstruction involves for the listener a good reproduction of the Interaural Time Difference, which is known to be the main cue for the localization of noise sources in an horizontal plane at low frequency.

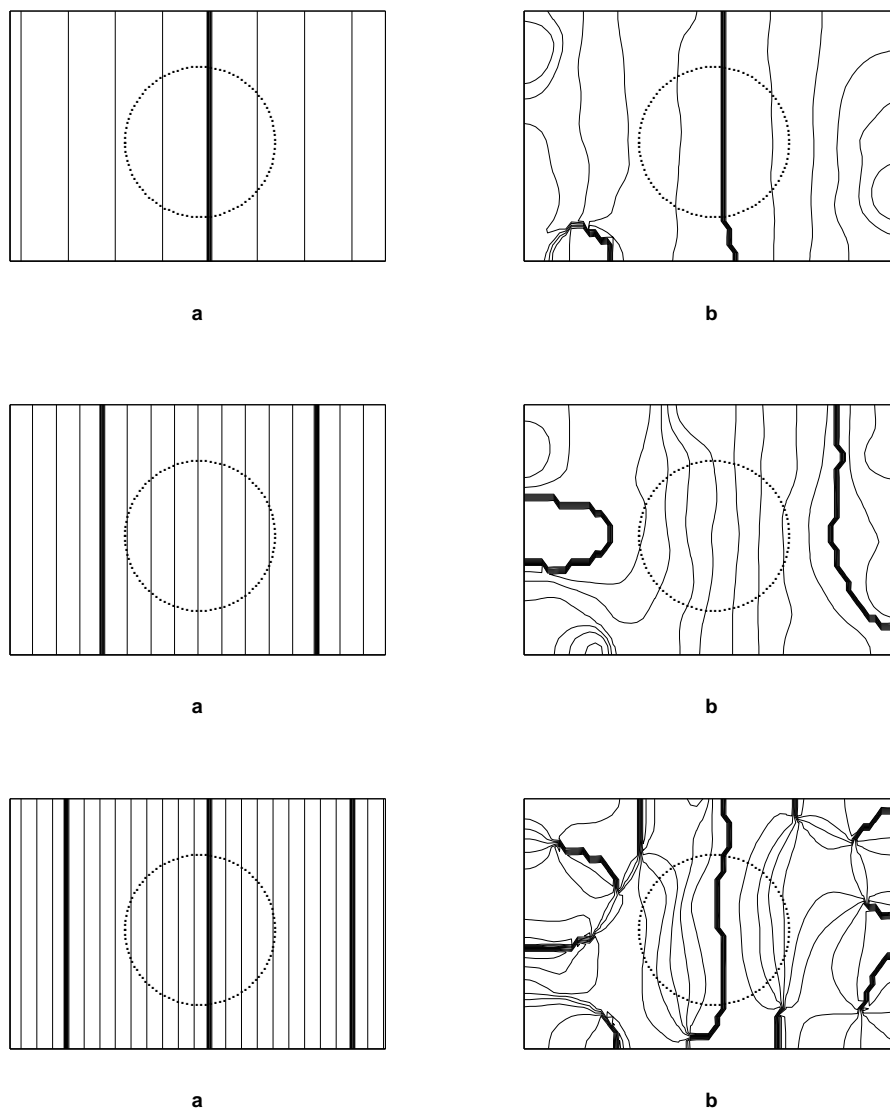


Figure 4: Contours of equal sound pressure phase for a 100 Hz (top), 200 Hz (middle) and 300 Hz plane wave (bottom); a: original soundfield, b: reproduced soundfield)

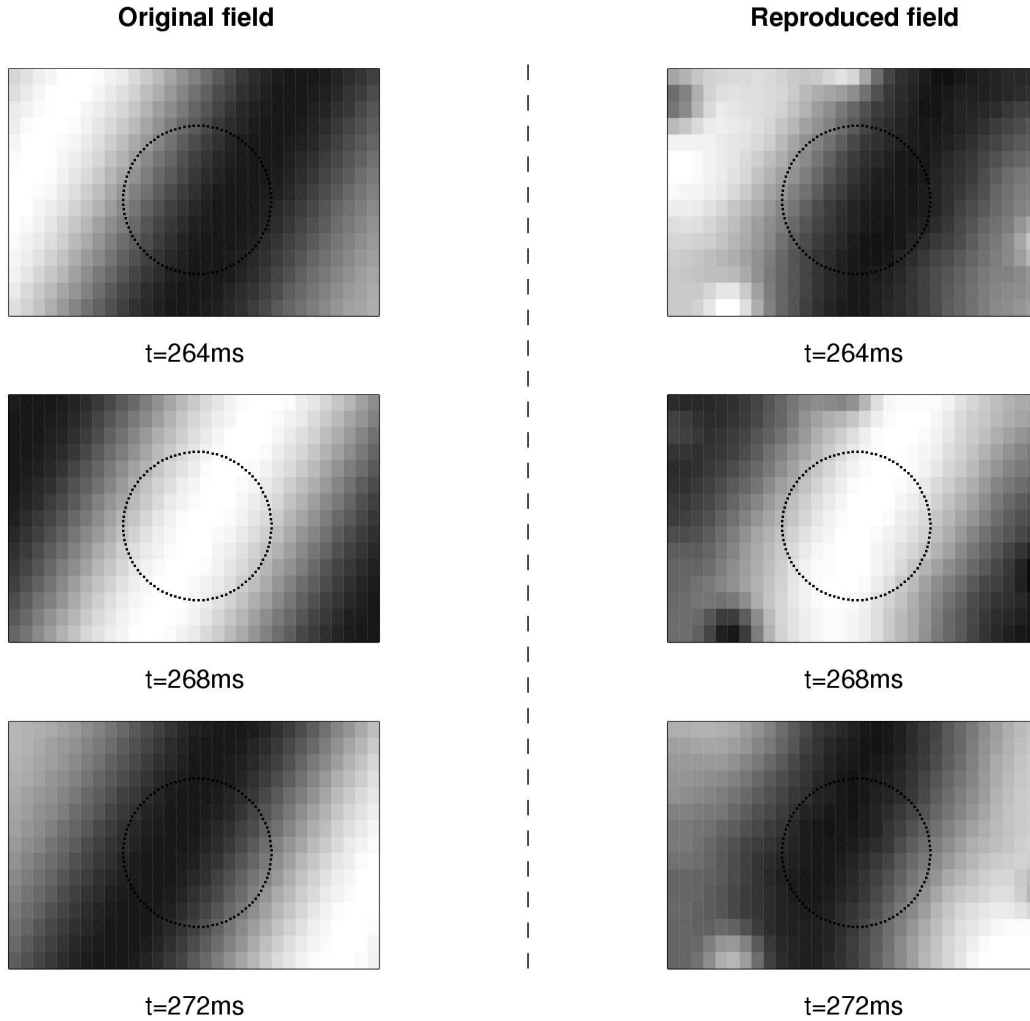


Figure 5: Original (left) and reproduced (right) sound pressure for several time values. The original wave is a plane gaussian pulse with center frequency 100 Hz.

C. Time-Domain Simulations

The sonic boom is a very unstationnary noise and, for psycho-acoustic tests, the reproduction of the transient noise components must also be accurate. Therefore time-domain simulations are required in addition to frequency-domain simulations, all the more since effective audio reproduction systems work in the time-domain.

Firstly, the matrix of secondary paths from noise sources to error microphones, which has been computed for a dense grid of discrete frequencies, gives the corresponding impulse response matrix by using Inverse Fast Fourier Transform. The impulse response for the inverse of the matrix of secondary paths is also computed through IFFT. Once these direct and inverse responses have been obtained, the computation of the residual error is made as in the frequency-domain, with the difference that the products become convolution products. The secondary field is the sum of the soundfield from each source, each one being calculated by filtering the original signals with the appropriate filters.

In a first step, simulations were performed for gaussian pulse plane waves, which are the signals with maximum

localization in both time and frequency. The visualization plane is the same as in the previous simulations. This gives, for each time sample, a map of the original and reproduced sound pressures in the observation plane, which is this time divided in 32 by 16 points. Excerpts of the results are presented on figures 5 and 6 for plane gaussian pulse waves with center frequency 100 and 200 Hz. The time-domain reconstruction of the original wave is very accurate for a 100 Hz pulse, and still quite accurate for the 200 Hz one in almost the whole controlled region, although the frequency-domain simulations suggested a 20 to 30 % error at this frequency. The results observed in the frequency domain for the phase reproduction are confirmed, the secondary wavelet travels through the room very like the original one.

In a second step we simulated the reconstruction of a recorded real sonic boom. Because of the frequency limitations of the system, the recorded sound was low-pass filtered in order to remove the signal components at frequencies higher than 300 Hz. The results are presented on figure 7 for an observation point placed at the center of the control region. Once again, the original soundwave is very well reproduced, even if a slight variation of the secondary pressure around the original value can be observed. When listening the two signals with headphones the difference is almost inaudible.

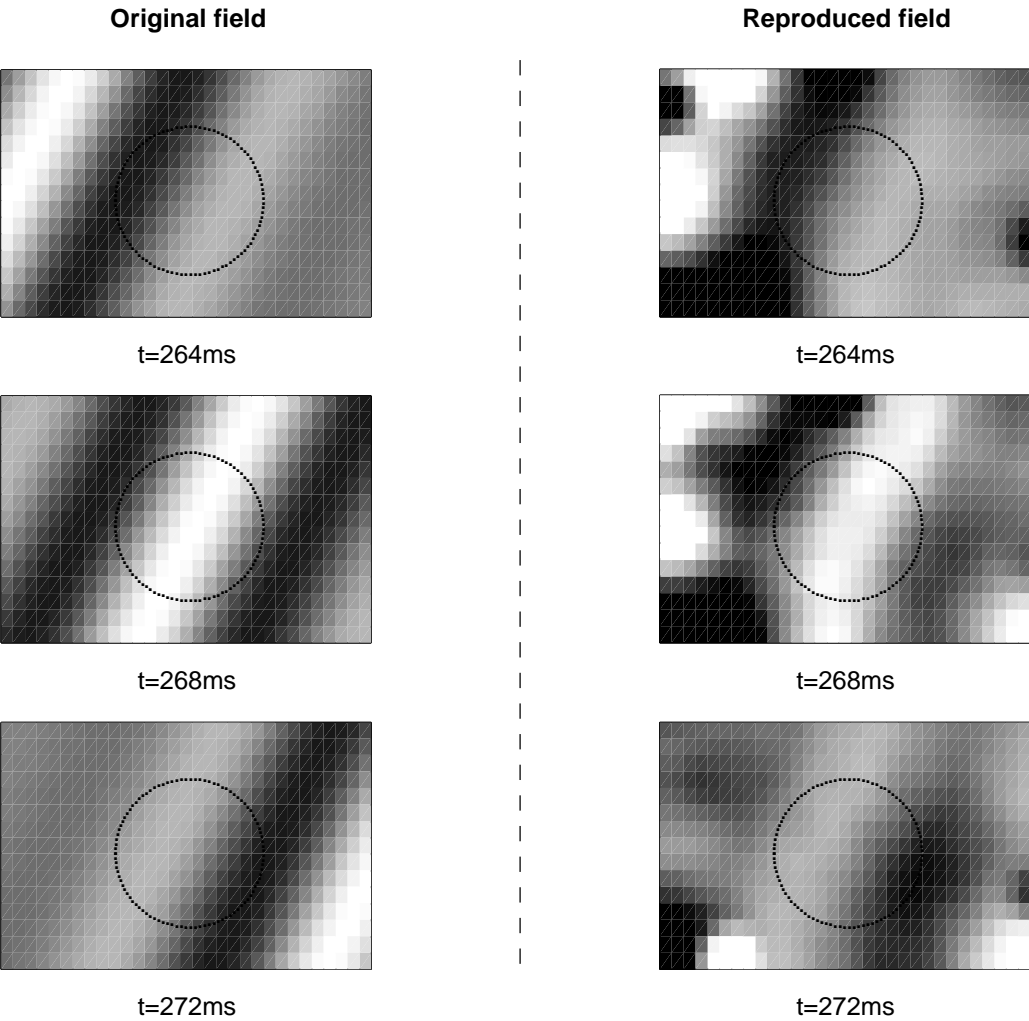


Figure 6: Original (left) and reproduced (right) sound pressure for several time values. The original wave is a plane gaussian pulse with center frequency 200 Hz.

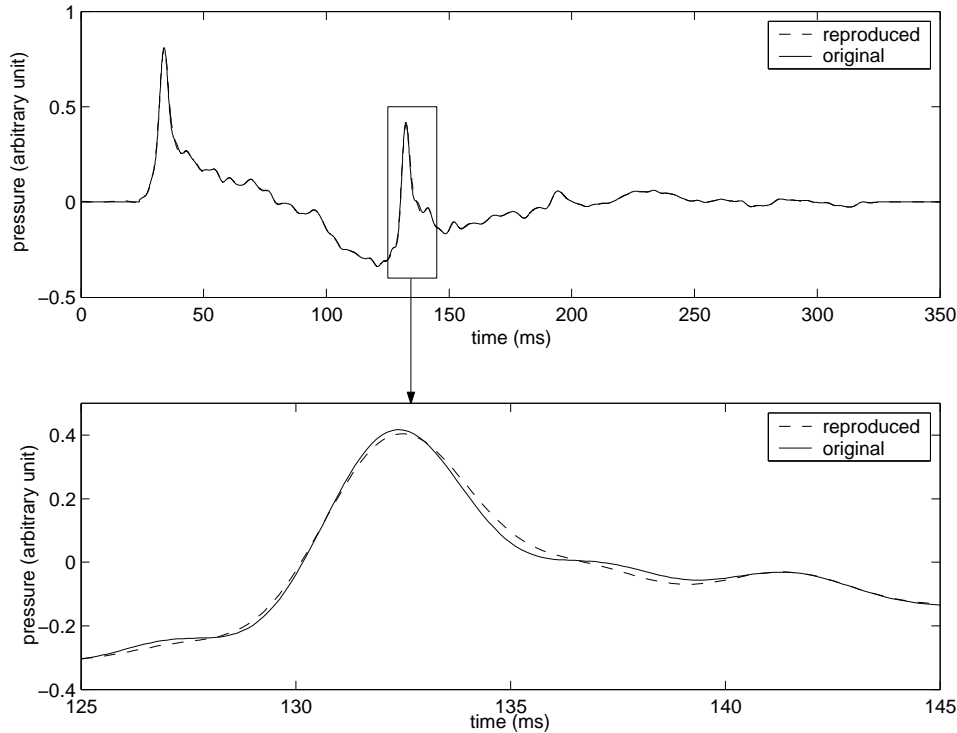


Figure 7: Original and reproduced sound pressure for a plane low-pass filtered sonic boom wave at the center of the control region.

D. Conclusions

Frequency and time-domain simulations of indoor soundfield reconstruction in the LMA reproduction room have been performed. These simulations correspond to an ideal case, where the inverse filters are the optimal ones. Thus, the obtained results cannot be considered as the real performances of the system in practice but as the maximal performances that can be expected. In practice the control filters will be Finite Impulse Response filters and the reproduced soundfield may not be as close to the original sonic boom field as the simulations suggest.

However, the simulation results are encouraging enough in implementing sonic boom reproduction through Boundary Pressure Control. In particular no singular behaviour of the reproduction process has been met at the resonance frequencies of the Dirichlet eigenproblem for the reproduction area.

5. INFLUENCE OF LISTENER'S PRESENCE ON THE SOUNDFIELD

When a soundwave, such as a sonic boom one, impings a listener, it is diffracted by his or her body, depending on what is the wave propagation direction and what are the shape and reflective properties of the body. In particular, it is known that the structures of the head and ear pinnae influence the properties of the sound measured at the ear drum. Therefore, each listener hears a different sound, and the soundfield to reproduce on control microphones around a listener by a BPC system is not only the single incident sound wave, but the sum of the diffracted and direct waves. Thus, an effective sound reproduction requires the presence of the listener during both the recording and reproduction stages, just as in the case of the use of binaural techniques involving HRTF.

However, since the recording microphones are more distant to the listener, his or her influence on the recorded soundfield is expected to be lower than in the binaural case. There is therefore a compromise to find between a lower listener influence on the soundfield and a lower maximum reproduction frequency, because the more distant are the microphones from the person, the lower are the surface density of sensors and the reproduction accuracy. Moreover,

since the amplitude of the diffracted pressure depends on the frequency of the incident wave (more precisely on the ratio between the wavelength and the characteristic dimensions of the diffractive object), it is probable that the difference between the direct and undirect soundwaves is very tight for low frequencies. For example, for lower frequencies under 340 Hz, the wavelength is more than 1 m, whereas a common diameter for human head is 17 cm.

Therefore a few questions need to be answered before implementing BPC for reproducing sonic boom around a listener. Firstly, are the soundfields recorded around two different persons very different? How does this difference depend on the control microphones distance and on the incident wave frequency? How accurate is the reproduction when the pressure field recorded around a listener A is reproduced around a listener B? Finally, is it possible to reconstruct the sonic boom around a person by reproducing only the single incident soundwave at the control microphones?

A. Simulations

In order to answer these different questions, a few numerical simulations have been made. Just as before, optimal reproduction was computed in the frequency domain for the case of plane waves travelling in free-field. Firstly, the total field around a rigid sphere (figure 8a) was computed. Figure 9a shows that the difference between the incident field and the total field (including the scattering by the sphere) decreases fastly as the distance to the sphere increases. Furthermore the decreasing error curve can be divided in two parts: a fast decrease occurs between 0 and 50 cm, whereas the decrease is slower at more than 50 cm from the sphere. For noise reproduction this means that the influence of the scattered noise can be efficiently reduced by moving the control microphones away from the sphere surface in the 0-50 cm zone, but not that much for larger distances.

In a second step the soundfield was computed for a wave impinging a finite element head model of a sphere with two “ears” and a “nose” (figure 8b). The difference between this soundfield and the soundfield around the mere sphere is shown in figure 9b. As for the previous figure, zones of fast and slow decrease appear. Again, moving away the microphones from the head of two different listeners minimizes the difference in the soundfield that must be reproduced. However, this is not so relevant at a distance of more than 50 cm when the decrease in the difference is very slow when compared to the increase in reproduction error resulting from enlarging the controlled area with a constant number of microphones.

Finally, optimal reproduction of a low-frequency plane wave soundfield in free-field was tested in three configurations: the soundfields computed firstly with the sphere, secondly with the head model, and thirdly without any scattering object was reproduced by BPC around the sphere. It appeared firstly that no large error was made when reproducing around the sphere the field computed with the head model, at least at low frequencies. This was expected because of the dimensions of the “ears” and “nose” details compared to the wavelength. Secondly, the error resulting from reproducing, around the sphere, the noise computed without an object is much larger.

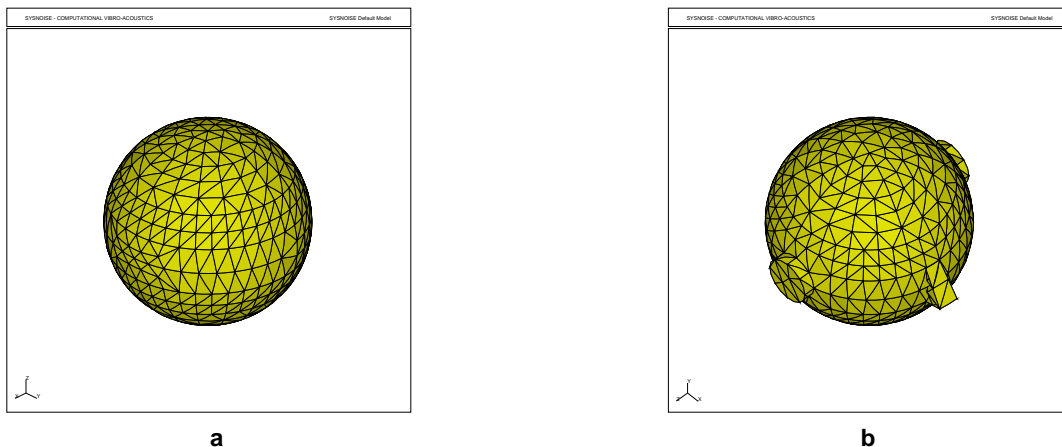


Figure 8: The two different finite elements head models used in the simulations (a: single rigid sphere; b: sphere with ears and nose).

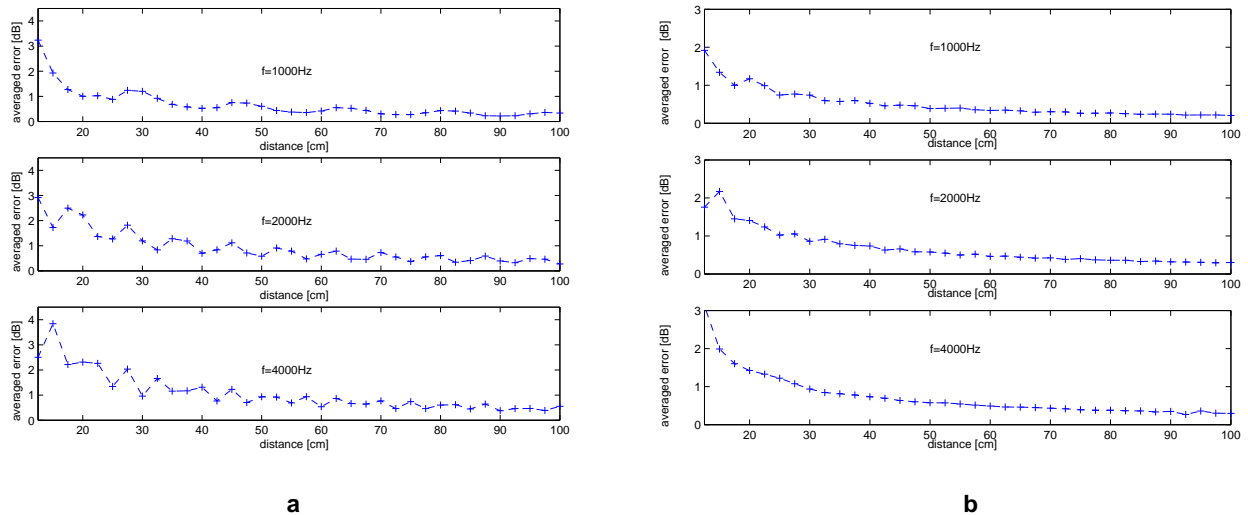


Figure 9: Averaged error between: (a) the single incident sound pressure and the total pressure around the single sphere; (b) the total pressure values around the two head models, as a function of the distance from the diffractive object.

B. Conclusions

In conclusion to these different simulations:

1. The influence of the diffractive object is reduced at low frequencies, i.e. when its dimensions are small when compared to the wavelength of the incident sound. It can be noted that this will always be true for a human head at frequencies inferior to 300 Hz.
2. At these frequencies, it is possible to use the soundfield recorded around a listener A as a reference for the reproduction of the soundfield around B.
3. It is useless to put the recording/control microphones at a distance larger than 50 cm from the listener since the resulting benefit in terms of error between the sound pressure values is then small when compared to the reproduction accuracy loss which results from the enlargement of the control surface.
4. Even at low frequency, the reproduction is inaccurate around a listener when the field which is reproduced is only made of the incident wave (and does not include the scattered wave). This shows that it is important to record or compute the original field with a person (or perhaps with a dummy head) inside the volume defined by the microphones.

6. GENERAL CONCLUSIONS AND PERSPECTIVES

Simulations of sonic boom reproduction in a room and, in free field, around a human head model, have been performed. Although these simulations involve theoretical optimal control, they provide an evaluation of the best achievable performances which encourages in using BPC for sonic boom reproduction. The LMA room is now ready for experiments, sound reproduction and psycho-acoustic tests will be conducted soon.

It has also been observed numerically that the eigenfrequencies of the Dirichlet problem pose no problem for sonic boom reproduction with BPC. A free-field experiment of ANC in a volume using the BPC method will also be conducted soon at the LMA in order to confirm this numerical result.

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