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On the determination of the deceleration parameter from Supernovae data

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Abstract

Supernovae searches have shown that a simple matter-dominated and decelerating universe should be ruled out. However a determination of the present deceleration parameter q_0 through a simple kinematical description is not exempt of possible drawbacks. We show that, with a time dependent equation of state for the dark energy, a bias is present for q_0 : models which are very far from the so-called Concordance Model can be accommodated by the data and a simple kinematical analysis can lead to wrong conclusions. We present a quantitative treatment of this bias and we present our conclusions when a possible dynamical dark energy is taken into account.

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Observations [1, 2] of Type Ia Supernovae (SNe) allow to probe the expansion history of the universe. The measurements of the apparent magnitudes $m(z)$ of SNe determine the luminosity distance versus redshift $d_L(z)$ which reads for a flat ($\Omega_T = 1$) universe :

$$\begin{aligned} d_L(z) &= c(1+z)\int_0^z \frac{du}{H(u)} \\ &= \frac{c(1+z)}{H_0}\int_0^z du\, e^{\left[-\int_0^u [1+q(x)]d\ln(1+x)\right]} \end{aligned} \quad (1)$$

where $d_L(z)$ is written here in terms of the epoch-dependent deceleration parameter $q(z) \equiv (-\ddot{a}/a)/H^2(z)$ with $H(z) = \dot{a}/a$. $d_L(z)$ is related to the measured magnitude by $m(z) = M_S + 5log_{10}(H_0/c\, d_L(z))$, M_S being a normalisation parameter which combines the absolute SNe magnitude and the Hubble constant.

The present behavior of the expansion is attributed to a new "Dark Energy" (DE) component with negative pressure : $p_X = w\,\rho_X$, with a possibly time dependent equation of state (EoS) $w(z)$ (for a recent review see Ref. [3]). Ω_X (Ω_M) will denote the ratio of the present DE (Matter) density to the critical density. In this framework, with $\Omega_T = 1$ and neglecting the radiation component :

$$\begin{aligned} d_L(z) &= \frac{c(1+z)}{H_0}\int_0^z du \\ &\left[(1+u)^3\Omega_M + \Omega_X e^{3\int_0^u [1+w(x)]d\ln(1+x)}\right]^{-1/2} \end{aligned} \quad (2)$$

The connection with Eq.(1) is given by :

$$q(z) = \frac{1}{2} + \frac{3}{2}w(z)\Omega_X(z)\ ;\ \Omega_X(z) = \Omega_X\frac{\rho_X(z)H_0^2}{\rho_X(0)H^2(z)} \quad (3)$$

In Ref.[2] Riess et al. have recently presented an analysis of 156 SNe including a few at $z > 1.3$ from the Hubble Space Telescope (HST) GOODS ACS Treasury survey. They use a kinematical description (no dynamical hypothesis) with a simple parametrization of $q(z)$ in Eq.(1), $q(z) = q_0 + q_1\, z$, and conclude to the evidence for present acceleration $q_0 < 0$ at 99% C.L. ($q_0 \approx -0.7$) and for past deceleration $q(z) > 0$ beyond $z_t = 0.46 \pm 0.13$. Concerning the dynamics in a flat universe, they conclude on the validity of the Cosmic Concordance version of the ΛCDM Model that is $\Omega_M \approx 0.3$, $w(z=0) \approx -1$ and no rapid evolution of the EoS.

Performing the same kind of analysis, we confirm the numbers and errors obtained with the Gold Sample (see Ref. [2]) of SNe for the "kinematical fit" : $q_0 = -0.74 \pm 0.18$ and $q_1 = 1.59 \pm 0.63$. In spite of the goodness of the fit and of the relatively small errors it yields, this simple strategy leads to some bias which we quantify in the following.

As noticed several times, [4, 5, 6, 7, 8] the physical parameters Ω's and $w(z)$ are related to the measured quantity $d_L(z)$ through a multiple integral relation which results in a somewhat uncertain determination of these parameters, even at their present values Ω_M and $w(0)$. In Ref.[8] we have analysed quantitatively the bias which occurs when one tries to extract $w(0)$ neglecting a possible redshift dependence of $w(z)$. In the same time, other fitted quantities (essentially Ω_M) are affected, due to strong correlations between Ω_M, $w(0)$ and $dw(z)/dz$. Another pitfall originates from the assumed value (and uncertainty) of the prior on Ω_M which is used for the fit [9] : an artificial convergence towards the Concordance Model seems to occur whereas the simulated fiducial model is very different.

Concerning the determination of the deceleration parameter, one encounters a similar problem : the extraction of q_0 is not independent of the assumed form of the function $q(z)$, a form which is related (dynamical approach) or not (kinematical approach of Riess et al.) to the evolution of the DE EoS and to the value of Ω_M. Indeed, when choosing the kinematical approach, Riess et al. are in some sense model independent but they must use a particular *description* of the kinematics : e.g. the linear form $q(z) = q_0 + q_1\, z$. It is valuable to question this description, for instance in the light of some dynamical models which include an evolution of the DE EoS.

To modelize the evolution of the EoS, in the absence of deep physical insight, the choice of a parametrization is arbitrary. Simplicity imposes a two-parameter parametrization for $w(z)$, then consistency with the data from the Cosmic Microwave background (CMB) imposes $w(z) \leq 0$ at high redshift. Therefore, the simple linear parametrization :

$$w(z) = w_0 + w_1\, z \quad (4)$$

we have used previously [8, 9] (with many other authors) is too badly behaved at high z. We prefer to switch to the parametrization advocated in [10, 11] and which is now widely used in the literature [12, 13, 14, 15, 16].

$$w(z) = w_0 + w_a z/(1+z) \quad (5)$$

Then from Eq.(5), one gets :

$$\Omega_X(z) = \Omega_X\,\frac{H_0^2}{H^2(z)}\,(1+z)^{3(1+w_0+w_a)}e^{-3w_a z/(1+z)} \quad (6)$$

and $q(z)$ reads :

$$\begin{aligned} q(z) &= \frac{1}{2} + \frac{3}{2}(w_0 + w_a\frac{z}{1+z}) \\ &\times\left[1 + \frac{\Omega_M}{\Omega_X}(1+z)^{-3(w_0+w_a)}e^{3w_a z/(1+z)}\right]^{-1} \end{aligned} \quad (7)$$

The behavior of $q(z)$ is plotted in Fig.1 for various models listed in Table I : it is obvious that $q(z)$ is in general far from a linear shape. Even in the ΛCDM model, $q(z)$ is not exaclty linear in z as can be seen from Eq.(7) and Fig.1.

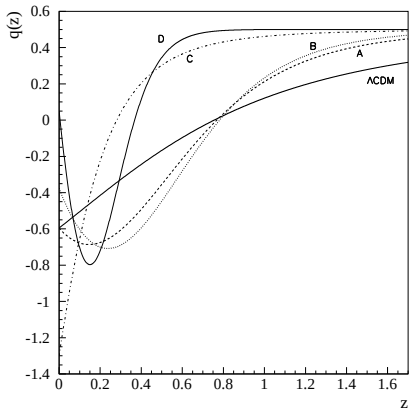

FIG. 1: $q(z)$ from Eq.(7) with $\Omega_T = 1$ and some particular values of the parameters (Ω_M, w_0, w_a) given in Table I.

TABLE I: Model examples with fiducial values (Ω_M, w_0, w_a). The fiducial $q_0(\Omega_M, w_0)$ is calculated from Eq.(7). The fitted q_0 are obtained with a linear 3-fit (M_S, q_0, q_1) of the simulated data. These values are well in agreement with the one extracted from the Gold Sample ($q_0^d = -0.74 \pm 0.18$). For consistency, Models A,B,C,D have been chosen to give a good dynamical 4-fit $(M_S, \Omega_M, w_0, w_a)$ of the Gold Sample.

model	Ω_M	w_0	w_a	$q_0(\Omega_M, w_0)$	fitted q_0
ΛCDM	0.27	-1	0	-0.6	-0.57 ± 0.17
A	0.27	-1	-2	-0.6	-0.79 ± 0.17
B	0.27	-0.8	-3.2	-0.38	-0.74 ± 0.17
C	0.50	-2.4	1.4	-1.3	-0.75 ± 0.18
D	0.50	-0.6	-15	0.05	-0.75 ± 0.18

To illustrate the consequence of the non-linear form of $q(z)$ on the kinematical fit, we simulate SNe data samples corresponding to the same statistical power as the true data sample by fixing the fiducial(^{F}) values for the parameters M_S^F, Ω_M^F, w_0^F and w_a^F. In the following, we consider a flat cosmology and we fix $M_S^F = -3.6$. Then we perform a three-parameter (M_S, q_0, q_1) kinematical fit of the simulated data. The results for the models we use are presented in Table I.

First, the ΛCDM Concordance model ($w_0^F = -1$, $w_a^F = 0$) with the Ω_M value (0.27) of [2], yields a fitted value: $q_0 = -0.57 \pm 0.17$ which reproduces well the fiducial value in the limit of the errors. Note however that the value $q_0^{data} = -0.74 \pm 0.18$ obtained from the real data is 1σ away from the actual ΛCDM value.

With Model A, where a time variation of the EoS is allowed ($w_a^F \neq 0$), we illustrate the fact that even if q_0^F from Eq. (7) is independent of w_a^F, its fitted value is not.

With model B (this model could correspond to one of the k-essence models [17]), the fitted q_0 value is now 2σ away from the fiducial ($q_0^F = -0.38$) .

In model C, we have changed the fiducial Ω_M^F value to 0.5. We have $w_0^F < -1$ (as in phantom models [18], see e.g. [19] for a list of references) and a positive w_a^F. The bias is then very large since the fitted q_0 is more than 3σ away from the fiducial value.

One can even get a truly slowly decelerating model

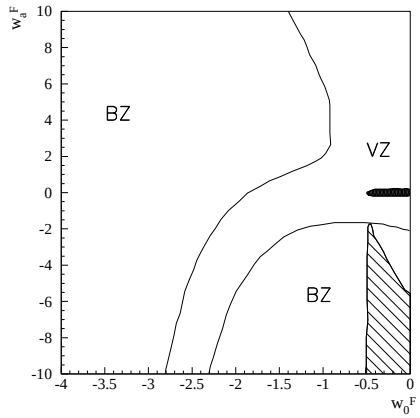

FIG. 2: Biaised Zone (BZ) and Validity Zone (VZ) for the deceleration parameter q_0, for the kinematical fit (M_S, q_0, q_1) of the simulated data in the fiducial plane (w_0^F,w_a^F) with $\Omega_M^F = 0.27, M_S^F = -3.6$. Small dark zone : where the linear approximation $q(z) = q_0 + q_1 z$ is acceptable ; hatched zone : where the conclusion on the sign of q_0 could be misleading.

with model D : $q_0^F = 0.05$, still with $\Omega_M^F = 0.5$, $w_0^F = -0.6$ and a very rapid and recent evolution of $w(z)$: $w_a^F = -15$. Of course the behavior of Model D seems somewhat artificial and the result of the fit can be seen from the non-monotonicity of $q(z)$ in this model (see Fig.1), which leads to compensations in the integral of Eq.(1). However, let us stress again that Model D as well as Models A,B,C fit perfectly the present SNe data, yielding in particular $q_0 \approx q_0^{data}$.

More quantitatively, it is interesting to pin down some regions in the parameter space where the kinematical fit gets in trouble. Following Ref.[8], we fix Ω_M^F and M_S^F and we consider the plane of the fiducial (w_0^F, w_a^F). In this plane, we define the Biased Zone (BZ) for q_0 as the zone where the kinematical fit converges perfectly but where the difference between the fitted value and the fiducial value q_0^F is larger than the statistical error $\sigma(q_0)$. The validity zone (VZ) is the complementary region. It must be stressed that the BZ is undetectable with real data. We give the two zones in Fig. 2 for $\Omega_M^F = 0.27$. The ΛCDM falls in the VZ as expected, and the models A,B,C,D belong to the BZ. Within the VZ, the zone where the linear approximation $q(z) = q_0 + q_1 z$ of Eq.(7) is acceptable is quite small. In fact the rest of the VZ is due to accidental cancellations when Eq.(7) is injected in Eq.(1). The lower-right corner of the BZ corresponds to models such that $w_a \lesssim -3w_0^2\Omega_M$; they display a non-monotonic behavior for $q(z)$ which explains the bias. In addition, even with $\Omega_M^F = 0.27$, there exists a region in this zone where the conclusion on the sign of q_0 could be misleading, i.e. a zone where $q_0 + \sigma(q_0) \leq 0$ whereas $q_0^F \geq 0$. This zone corresponds to $-0.5 < w_0^F < 0$ with large and negative w_a^F.

Therefore, it appears that the linear expression of $q(z)$ at first order : $q(z) = q_0 + q_1 z$, cannot be satisfactorily used over a large range of EoS parameters. Going to a

TABLE II: Results of a "dynamical" 4-fit with $w(z) = w_0 + w_a z/(1+z)$ using the Gold data from [2].

Ω_M prior	Ω_M	w_0	w_a	q_0
0.27 ± 0.04	$0.27^{+0.04}_{-0.04}$	$-1.49^{+0.31}_{-0.34}$	$3.20^{+1.60}_{-1.60}$	$-1.12^{+0.33}_{-0.36}$
0.27 ± 0.20	$0.33^{+0.15}_{-0.21}$	$-1.70^{+0.60}_{-0.60}$	$3.50^{+2.20}_{-4.30}$	$-1.21^{+0.49}_{-0.45}$

second order parametrization will not improve the issue, as it will only enlarge the number of parameters, will degrade the errors, and will not solve the bias problem.

Alternatively, one can deduce a q_0 value from real data using Eq.(7) (with $z = 0$) where we consider the dynamical description Eqs.(2)and (5), by performing a 4-parameter fit $(M_S, \Omega_M, w_0, w_a)$ of the Gold Sample of [2]. We have calculated asymetric standard deviations (deviation $\delta\chi^2 = 1$ for a given parameter), marginalizing over the other parameters by minimisation. For q_0, which is a derived parameter, we use a Monte-Carlo technique : generating 1000 simulated experiments around the data best fit, we estimate the width of the q_0 distribution.

The extracted values are displayed in Table II. In this Table, we have used a strong prior on Ω_M (0.27 ± 0.04) and also a more reasonable weaker uncertainty of ± 0.2, since adopting strong priors is recognized to be dangerous (see [9, 16]). Comparing with our previous fits, we see that changing the $w(z)$ parametrization from Eq.(4) to Eq.(5) has not changed the behaviors discussed in [9]. In particular choosing a strong prior around $\Omega_M = 0.3$ pushes w_0 towards -1 whereas, with a weaker prior the central value of w_0 is clearly lower than -1, a fact which has been noticed by many authors (see e.g. [14, 16, 20, 21, 22]).

The estimated q_0 values confirm that the Universe is presently in an acceleration phase, at 95% $C.L.$ We note that the extracted value for a weak prior is 2σ away from the ΛCDM value $q_0 = -0.6$. This is a direct consequence of the extracted value of the point (w_0, w_a), which is 2σ away from $(-1, 0)$. With the strong prior, the errors on q_0, as on other parameters, are systematically smaller when Ω_M is close to the Concordant Model value ($\approx$ 0.3). This conclusion would be different if we use a larger central value for Ω_M : in this case, the q_0 value would be even more negative, but with such a large error that it is not possible to conclude.

Finally, if no prior is applied on Ω_M, one can conclude with some caution that $q_0 < 0$ at 80% C.L.

We can also address the problem of the determination of the value of the transition redshift z_t between deceleration and acceleration. Since the kinematical fit yielding the q_0 and q_1 values found in Ref.[2] is biased, one cannot be too confident with the advocated value $z_t = 0.46 \pm 0.13$.

Fig. 3 displays the function $q(z) = q_{bf}(z)$ computed from the best fit (bf) values of the parameters entering Eq.(7). The 1σ and 2σ intervals are computed by

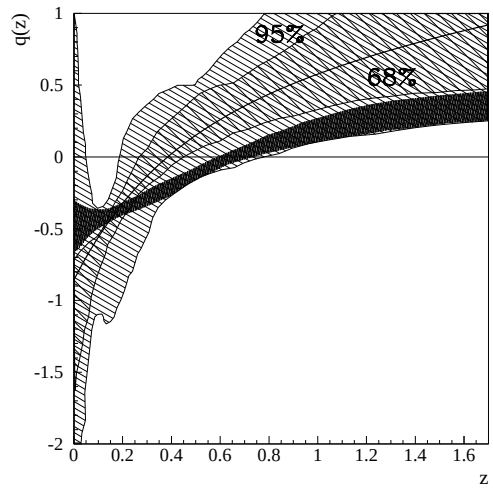

FIG. 3: best fit $q_{bf}(z)$ function computed with Eq.(7) using the best fit values of the cosmological parameters from the current data (heavy plain curve). The uncertainty contours are constructed from the probability density function around the best fit at a given redshift from one thousand simulated experiments. The dark stripe corresponds to the 1σ error on the fitted function from a simulation around the ΛCDM concordant model fiducial values, with the SNAP statistics.

taking the probability density of simulated experiments around the $q_{bf}(z)$ value in each redshift bin. We still use a weak prior $\Omega_M = 0.27 \pm 0.2$. The extracted z_t value corresponds to the point where $q_{bf}(z_t) = 0$ and $\frac{dq_{bf}}{dz}(z = z_t) > 0$. We find $z_t = 0.34^{+0.12}_{-0.06}$, where the errors are evaluated from the z_t probability density. With the dynamical $q(z)$, our errors are small because the transition is steeper than in the linear kinematical case $(q(z) = q_0 + q_1 z)$, as can be seen from Fig. 3.

Our central value is far from the "theoretical" z_t of the Concordance Model : $z_t = [\frac{2\Omega^0_\Lambda}{\Omega^0_M}]^{1/3} - 1 = 0.76$ (for $\Omega_M = 0.27$). Moreover, we have observed that, contrary to the extraction of q_0, the extraction of z_t is very much dependent upon the chosen parametrization for $w(z)$, a fact mentioned by various authors [20, 23]. Then, one can stay prudent and state that, with present data, the extracted z_t value is certainly much less robust than the extracted q_0 value, even in a dynamical approach. We should also point out that no systematical effects are taken into account in these evaluations, in particular a normalisation effect between low and high redshift supernovae would affect directly the determination of z_t.

It is interesting to evaluate what should be the best strategy to extract q_0 and z_t in the future. Using a biased method, as the linear kinematical one, will be worse when the statistical errors will decrease. A dynamical, although model-dependent approach, remains a good way to estimate the behavior of the acceleration. In Fig. 3,

we show the results for a statistical sample expected from the space-based SNAP mission [24] where the systematical uncertainties should be controlled at the same level of precision. Thanks to the rich sample of 2000 SNe and also to the large range of redshift ($0.2 \lesssim z \lesssim 1.7$) a very precise determination of the transition region should be allowed. For instance, with the ΛCDM fiducial values ($w_0 = -1, w_a = 0, \Omega_M = 0.27$), keeping a weak prior on Ω_M, one gets $z_t = 0.67^{+0.08}_{-0.06}$ and $q_0 = -0.55^{+0.26}_{-0.13}$.

We estimate that the correct procedure for the future is to avoid using priors on Ω_M. We prefer instead the combination of supernovae data with other probes such as the CMB, the Weak Gravitational Lensing and the galaxy power spectrum. This kind of method will not introduce external biases and some recent papers go in that direction (see e.g. [16, 21, 22, 25]).

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