



HAL
open science

Quantum/classical transition in matter-wave interferometers through the scattering of gravitational wave backgrounds

Brahim Lamine, Rémy Hervé, Astrid Lambrecht, Serge Reynaud

► **To cite this version:**

Brahim Lamine, Rémy Hervé, Astrid Lambrecht, Serge Reynaud. Quantum/classical transition in matter-wave interferometers through the scattering of gravitational wave backgrounds. 2005. hal-00004899v1

HAL Id: hal-00004899

<https://hal.science/hal-00004899v1>

Preprint submitted on 10 May 2005 (v1), last revised 10 Feb 2006 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Quantum/classical transition in matter-wave interferometers through the scattering of gravitational wave backgrounds

Brahim Lamine,^{*} Rémy Hervé, Astrid Lambrecht, and Serge Reynaud

*Laboratoire Kastler Brossel, [†] Université Pierre et Marie Curie,
case 74, Campus Jussieu, F-75252 Paris cedex 05*

(Dated: May 10, 2005)

Stochastic backgrounds of gravitational waves correspond to intrinsic fluctuations of spacetime and lead to a universal decoherence mechanism. This mechanism defines an ultimate limit for matter-wave interferometry which sets a natural borderline between classical and quantum worlds. The decoherence border is characterized by figures which involve the gravitational environment as well as the quadrupole of the matter probe. We discuss the feasibility of experiments aimed at its observation in the context of ongoing progresses towards highly sensitive matter-wave interferometry.

PACS numbers: 03.65.Yz, 03.75.-b, 04.30.-w

The idea that intrinsic fluctuations of gravity might draw a borderline between classical and quantum world was already present in Feynman's lectures on gravitation [1]. It is quite natural since the Planck mass $m_P = \sqrt{\hbar c/G}$ built on the Planck constant \hbar , the velocity of light c and the Newton constant G , has a value $m_P \simeq 22 \mu\text{g}$ which lies on the borderland between microscopic and macroscopic masses. An object with a mass m larger than m_P is thus associated to a Compton wavelength \hbar/mc smaller than the Planck length $\ell_P = \hbar/m_P c$ typical of quantum fuzziness of spacetime. Even if this length scale $\ell_P \sim 10^{-35} \text{m}$ is not accessible to experiments, one may wonder whether fluctuation behaviours are modified when m crosses the mass scale m_P [2].

This question can be answered in a precise manner when considering the coupling of a quantum probe to the stochastic gravitational waves which are the intrinsic fluctuations of spacetime predicted by General Relativity. Such waves are scattered by any physical system leading to a universal decoherence mechanism which is the dominant source of decoherence for macroscopic probes [3]. This mechanism however tends to be negligible for the microscopic probes usually involved in matter-wave interferometers [4]. The difference between microscopic and macroscopic probes is clearly reminiscent of the argument on the mass m of the probe crossing the Planck mass m_P but the precise position of the quantum/classical border depends not only on the mass ratio but also on the underlying parameters controlling decoherence, in particular the geometry of the interferometer and the noise spectrum describing the gravitational environment.

In fact, the decoherence process can be thought of as revealing the large number of scattering processes of gravitational waves by the matter waves. It results from a long-term diffusion of the dephasing which can be observable though the sensitivity of the interferometer is

not sufficient to see single scattering events. Decoherence here plays a role analogous to that of Brownian motion which has the ability to reveal collisions of light molecules with a heavier particle even in cases where single collisions or single molecules are not observable [5].

In this letter, we give a discussion of the quantum/classical transition for matter-wave interferometers coupled to stochastic gravitational waves (GW). Within this fully calculable problem, the coupling of the probe field to GW is treated by standard techniques and a natural measure of decoherence in the interferometer is given by the reduction of the fringe contrast. Precisely, we will extract the relevant figures characterizing this transition from the quantitative study given in [6]. We will then discuss the case of more and more sensitive experiments which are being developed with higher and higher mass molecules [7]. At the moment, the decoherence of these interferometers stems from collisions with the residual gas, emission of thermal radiation by the molecules or instrumental dephasings produced for example by vibrations of the gratings. These noise sources can be reduced by using higher vacuum, lower temperature, improved velocity selection and, more generally, a better controlled and quieter environment available in particular in space experiments [8]. With the ongoing progress in the control of these noise sources, more fundamental limits may eventually be reached such as the unavoidable gravitational decoherence. It is precisely these intrinsic borderlands of the quantum world which are investigated for matter-wave interferometers in the present letter.

Besides the bursts of GW which are looked for by existing interferometric detectors [9], there exist stochastic GW backgrounds extending over a large frequency range. A part of these backgrounds is originating from the gravitational emission of binary systems in our galaxy and its vicinity [10]. The stochastic character thus comes from our lack of knowledge on the precise parameters associated with the enormous number of unresolved binaries. It does not have a quantum origin but is nevertheless unavoidable since neither can the associated GW

[†]Unité du CNRS, de l'ENS et de l'UPMC.

be monitored nor can the probes be protected against their influence. There also exists a cosmological background corresponding to relic GW originating from the primordial era of the cosmic evolution. These waves were produced through the amplification during the expansion of the Universe of the primordial vacuum fluctuations of the gravitational field [11].

For the sake of simplicity, we suppose here the background to be gaussian, stationary, unpolarized and isotropic. The latter assumption is not true for the binary confusion background because of the anisotropy of the galactic contribution but it is anyway sufficient for giving rough estimates of the decoherence. Note that more general background can be dealt with [6]. Within these simplifying assumptions, the backgrounds are described by a spectral density $S_h[\omega]$ of strain fluctuations giving the gravitational noise at a given frequency ω . This spectral density determines the whole correlation function of the metric in a TT coordinate system, at a fixed position taken here to be the center of the coordinate system :

$$\langle h_{ij}(t)h_{kl}(0) \rangle = \delta_{ijkl} \int \frac{d\omega}{2\pi} S_h[\omega] e^{-i\omega t} \quad (1)$$

$$\delta_{ijkl} \equiv \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}$$

Informations on the spectral density $S_h[\omega]$ can be found in [12]. The binary confusion background, deduced from our knowledge on the distribution of binaries in the galaxy, shows a roughly flat plateau between the μHz and mHz and drops rapidly on both sides on this plateau. The cosmological background shows a $1/\omega^3$ dependence and should dominate at low frequencies. It depends on a poorly known parameter Ω_{gw} measuring the GW energy density compared to the critical cosmic density.

The phase difference between the two arms of the interferometer is a gauge-invariant quantity which can therefore be evaluated in an arbitrary coordinate system. We chose here a TT coordinate system comoving with the center of mass of the interferometer. This choice is convenient because the metric is thus transverse and traceless with only purely spatial components differing from zero. We then evaluate the sensitivity to GW of this dephasing by well-known techniques. For the sake of evaluating decoherence, the simplest treatment using the eikonal approximation [13] is sufficient. We also make the simplifying assumption of a nearly freely fall of the mechanical setup defining the geometry of the interferometer.

The gravitational perturbation of the interferometer can in fact be represented as a Lagrangian density [14] proportional on one hand to the energy momentum tensor $T^{\mu\nu}$ of the probe and on the other hand to the metric perturbations $h_{\mu\nu}$. Using the realistic assumption that the size of the interferometer is small compared to the gravitational wavelength, we can use the quadrupole approximation which is analogous to the dipole approximation used in electromagnetism to describe the coupling on

an atom having a size much smaller than the wavelength. The perturbation can thus be written as an action term S which couples the spatial part h_{ij} of the metric to the second time derivative of the quadrupole Q^{ij} associated with the probe [15] :

$$S = \frac{1}{4} \int dt h_{ij}(t) \frac{d^2 Q^{ij}(t)}{dt^2} \quad (2)$$

$$Q^{ij}(t) = \frac{1}{c^2} \int d^3\mathbf{x} \left(x^i x^j - \frac{1}{3} \delta^{ij} x^k x_k \right) T^{00}(t, \mathbf{x})$$

We also use the eikonal approximation, so that Hamilton-Jacobi theory leads to an identification of the matter-wave phase to the action S divided by \hbar . The dephasing $\Delta\Phi$ between the two arms of the interferometer is then given by the difference ΔS between the two action integrals and, therefore, by the expression (2) where Q^{ij} is replaced by the difference ΔQ^{ij} between the quadrupoles evaluated on the two arms. We write the resulting expression in the frequency domain as :

$$\Delta\Phi(t) = \frac{\Delta S(t)}{\hbar} \equiv \int \frac{d\omega}{2\pi} h_{ij}[\omega] a^{ij}[\omega] e^{-i\omega t} \quad (3)$$

$$a^{ij}[\omega] = \frac{i}{4\hbar} \int \frac{d\omega'}{2\pi} \omega'^2 \Delta Q^{ij}[\omega'] \frac{1 - e^{-i(\omega+\omega')\tau}}{\omega + \omega'}$$

The apparatus function $a^{ij}[\omega]$ describes the geometry of the interferometer which has here been assumed to have a rhombic form with τ the time of flight along each arm.

The expression (3) of the dephasing includes the whole frequency spectrum. Some frequency components lie inside the detection window of the interferometer taken as a GW detector and are thus considered as contributing to a GW signal. Other frequencies lying outside the detection window act as an uncontrolled noise and are responsible for decoherence. To be more precise, we denote $\delta\varphi(t)$ the uncontrolled noise written as a convolution of the dephasing $\Delta\Phi(t)$ by a filter function f . Again the expression is conveniently written in the frequency domain :

$$\delta\varphi(t) = \int \frac{d\omega}{2\pi} h_{ij}[\omega] a^{ij}[\omega] f[\omega] e^{-i\omega t} \quad (4)$$

In a simple signal processing strategy, the function $f[\omega]$ is a high pass filter defining uncontrolled noise as frequencies larger than the inverse of the averaging time T .

The decoherence can then be characterized by the value of the fringe contrast \mathcal{V} deduced, within a gaussian description of the fluctuations, as the exponential of the variance of the uncontrolled noise :

$$\mathcal{V} = \langle \exp(i\delta\varphi) \rangle = \exp\left(-\frac{\Delta\varphi^2}{2}\right), \quad \Delta\varphi^2 = \langle \delta\varphi^2 \rangle \quad (5)$$

Using expressions (3-4) of the phase noises as well as the correlation functions (1) of the metric perturbation, we finally rewrite the variance $\Delta\varphi^2$ as an integral in the

frequency domain [4] :

$$\Delta\varphi^2 = \int \frac{d\omega}{2\pi} S_h[\omega] \mathcal{A}[\omega] F[\omega] \quad (6)$$

$$F[\omega] = |f[\omega]|^2, \quad \mathcal{A}[\omega] = \delta_{ijkl} a^{ij}[\omega] a^{kl}[-\omega]$$

The integrand is the product of three terms, the gravitational noise spectrum S_h , the apparatus response function \mathcal{A} and the filter function F .

The apparatus function $\mathcal{A}[\omega]$ has a complicated expression for an arbitrary geometry with however simple asymptotic properties : (i) it goes to zero at the high frequency limit; (ii) its static limit at zero frequency is determined by the phase difference evaluated for a perturbation constant in space; if the interferometer has a center of symmetry in particular, the static response is null because the GW have equal effects in both arms. For forthcoming discussions, we will consider the commonly studied case [8] of a Mach-Zehnder geometry with rhombic symmetry in the limits of small aperture angle ($\alpha \ll 1$) and non relativistic velocity ($v \ll c$). The apparatus function is thus given by the following expression which captures the main ingredients of the physical description of decoherence [4] :

$$\mathcal{A}_{MZ}[\omega] = (4\Omega \sin \alpha)^2 \left(\frac{1 - \cos(\omega\tau)}{\omega} \right)^2, \quad \Omega = \frac{mv^2}{2\hbar} \quad (7)$$

This response function scales as the square of the kinetic energy Ω of the probe field measured as a frequency. Note that this scaling law, deduced from the quadrupole expression (3) of the dephasing, is radically different from that associated with atomic interferometers considered as inertial sensors, with the response determined by the rest mass energy mc^2 . We also remark that the function $\mathcal{A}_{MZ}[\omega]$ goes to zero with the angular separation since the two arms are thus exposed to the same perturbation. Through its last term finally, it selects a frequency band in the gravitational spectrum which is essentially determined by the inverse of the time of flight 2τ of the probe field inside the interferometer.

These features are sufficient to give an estimate of decoherence deduced from the preceding calculations :

$$\Delta\varphi^2 \sim (2\Omega \sin \alpha)^2 \overline{S_h} 2\tau \quad (8)$$

$$\overline{S_h} \equiv \int \frac{d\omega}{\pi} S_h[\omega] F[\omega] \frac{(1 - \cos(\omega\tau))^2}{\omega^2\tau}$$

Assuming that the averaging time T is much larger than τ and that the filter F cuts off the potential divergence of the integral at its low frequency side, $\overline{S_h}$ may essentially be interpreted as the average of S_h over the bandwidth of the interferometer, the latter being roughly given by the inverse of the time of flight.

For a first estimation of the orders of magnitude, we can consider the simple assumption of a bandwidth lying within the plateau of the binary confusion background.

This entails that S_h is roughly flat so that the averaged value $\overline{S_h}$ is simply given by the noise level on the plateau, that is $S_h \simeq 10^{-34}$ s. In particular, this value is independent on τ so that the variance (8) effectively reproduces a Brownian-like diffusion of the phase characterized by a linear dependance of $\Delta\varphi^2$ in the time of exposition τ to the perturbation. In a more general case, the averaged value $\overline{S_h}$ also depends on the filter function and, therefore, on the parameter τ .

We now discuss the numbers coming out of expression (8) for specific experimental configurations. Our main purpose is to investigate the possibility for a matter wave interferometer to approach the quantum/classical transition now characterized by the quantitative condition $\Delta\varphi^2 \sim 1$. Starting from the known fact that $\Delta\varphi^2$ is usually much smaller than unity for microscopic probes [4], we see on formula (8) that approaching $\Delta\varphi^2 \sim 1$ requires two kinds of condition. Considering first the point of view of geometry, we need to have an interferometer combining large angular separation and large time of flight. This condition is different from the large area condition helping the atomic interferometer to be used as an inertial sensor. The difference is due to the fact that GW vary in space and time so that even a null area interferometer could be sensitive to them. As already noticed, sensitivity to GW is determined by the kinetic energy of the probe and not to its rest mass energy. This entails that rapid probes should be preferred to slow ones, a condition clearly different from the one looked for with atomic interferometers used as inertial sensors.

The geometrical and energetical conditions are certainly conflicting with each other : a high velocity of the matter beams decreases the time of flight for a given spatial size; meanwhile, it decreases the angular separation between interfering paths if the momentum transfer is fixed, which is the case for beam splitters built up on Raman scattering processes [17]. This last argument can be made more precise by introducing the velocity v of the probe along the arms and the momentum $\Delta p \equiv \Delta K$ transferred on the beam splitters. As a matter of fact, the factor $(\Omega \sin \alpha)$ appearing in (8) is just the product $v\Delta K$. For interferometers using Raman scattering processes for beam splitting, the transferred momentum is equal to twice the momentum of one photon. The variance $\Delta\varphi^2$ is thus read as $8v^2\Delta K^2\tau\overline{S_h}$ and it has an extremely small value, when taking as an example the design of the HYPER project [18]. The transferred momentum ΔK can be increased by using multiple Raman scattering, up to 140 emissions/absorptions [19], but this is not enough for approaching $\Delta\varphi^2 \sim 1$.

Other experimental configurations are conveniently discussed by noticing that the previous expression $v\Delta K$ may also be written as $\Delta\Omega = \Delta E_k/\hbar$ where E_k is the kinetic energy of the probe and $\Delta E_k = v\Delta p$ its variation on the beam splitter, if we consider the case of a transferred momentum orthogonal to the velocity. With

this notation, the variance $\Delta\varphi^2$ is read as $8(\Delta\Omega)^2\overline{S_h\tau}$ which shows that the most relevant parameter for characterizing the beam splitting is ΔE_k . With the figures corresponding to HYPER, ΔE_k has a very small value of the order of 10^{-9} eV.

The value of ΔE_k can be enlarged for example by using magnetic interaction guiding [20]. In this case, it is only limited by the depth of the guiding well and can go up to a few 10^{-7} eV. Even larger values of the potential depth (1 – 100 meV) are obtained with non resonant dipole interaction [21]. Deflection from material gratings also allow high kinetic energy transfer. For a slit of width a and a diffraction order n , the transferred momentum ΔK has a value of the order of $2\pi n/a$ which increases when a decreases. Another promising solution is the inelastic scattering of metastable molecules by nano-slit transmission gratings [22] in which Van Der Waals interactions lead to energy transfer of the order $\Delta E_k \sim 1$ eV. For all these configurations however, the energy transfers are still too small to approach the transition $\Delta\varphi^2 \sim 1$.

In order to show how challenging is the objective of seeing the quantum/classical transition associated with gravitational decoherence, let us consider at this point an hypothetical matter-wave interferometer with a wide angle aperture $\sin\alpha \sim 1$ and, therefore, a large kinetic energy transfer $\Delta E_k \simeq E_k$. This interferometer can approach the transition $\Delta\varphi^2 \sim 1$ if we suppose the beam to consist of molecules with a mass of 8×10^8 amu circulating at a velocity 1 km.s^{-1} in arms with one meter size. These numbers are calculated with the binary confusion background used as the source of fluctuations. They have to be contrasted with advanced projects of interferometers aiming at large molecules with mass in the 10^5 amu scale [23]. Meanwhile, they correspond to supersonic molecular beams with a kinetic energy transfer of the order of 300 keV, far above the splitting capabilities of the previously mentioned configurations.

These numbers show that approaching the quantum/classical transition associated with gravitational decoherence is out of reach for the presently developed molecular interferometers. An attractive idea would be to use Bose Einstein condensates (BEC) instead of large molecules. Should the BEC respond to the gravitational perturbation as a rigid object containing a large number N of atoms, the parameter ΔE_k would have to be multiplied by the factor N leading to an amplification by a factor N^2 of the decoherence rate. A first characterization of this rigidity condition is that the motion of the center of mass of the BEC induced by gravitational waves should correspond to frequencies lying well below the excitation spectrum of internal resonances. Detailed calculations are underway to make this characterization more precise and, thus, to discuss the feasibility of this kind of interference experiments.

Anyway, this argument pleads for BEC used as an interferometric probe in space experiments where the in-

strumental and environmental noises can be more efficiently controlled. This could be the best way to test the borderland between quantum and classical worlds, due to the ultimate decoherence mechanism associated with the scattering of the intrinsic fluctuations of spacetime by quantum matter [1, 2].

The authors would like to thank Gert-Ludwig Ingold, Marc-Thierry Jaekel and Paulo Maia-Neto for many stimulating discussions.

* Electronic address: lamine@spectro.jussieu.fr

- [1] Feynman R.P. *et al*, *Feynman lectures on gravitation*, ed. Penguin (1999).
- [2] Karolyhazy F., *Nuovo Cim.* **42A**, 390 (1966); Jaekel M.-T. and Reynaud S., *Phys. Lett.* **A185**, 143 (1994); Penrose R., *Gen. Rel. Grav.* **28**, 581 (1996).
- [3] Reynaud S., Maia-Neto P.A., Lambrecht A. *et al*, *Europhys. Lett.* **54**, 135 (2001); Reynaud S., Lambrecht A., Maia-Neto P. *et al*, *Int. J. Mod. Phys.* **A17**, 1003 (2002).
- [4] Lamine B., Jaekel M.-T. and Reynaud S., *Eur. Phys. J.* **D20**, 165 (2002).
- [5] Percival I.C., *Phys. World* **10**, 48 (1997).
- [6] Lamine B., *PhD thesis*, (2004), <http://tel.ccsd.cnrs.fr/>
- [7] Hornberger K., Uttenthaler S., Brezger B. *et al*, *Phys. Rev. Lett.* **90**, 160401 (2003); Hackermueller L., Hornberger K., Brezger B. *et al*, *Nature* **427**, 711 (2004).
- [8] *HYPER, Hyper-precision cold atom interferometry in space*, ed. Assessment study report ESA-SCI (2000).
- [9] A list of links is given on the VIRGO website <http://www.virgo.infn.it/facilities/othergw.html>
- [10] Bender P.L. and Hils D., *Class. Quantum Grav.* **14**, 1439 (1997).
- [11] Grishchuk L.P., *Sov. Phys. JETP* **40**, 409 (1975).
- [12] Schutz B., *Class. Quantum Grav.* **16**, A131 (1999); Maggiore M., *Phys. Reports* **331**, 283 (2000); Ungarelli C. and Vecchio A., *Phys. Rev.* **D63**, 064030 (2001); Grishchuk L.P., Lipunov V.M., Postnov K.A. *et al*, *Phys. Uspekhi* **44**, 1 (2001).
- [13] Linet B. and Tourrenc P., *Can. J. Phys.* **54**, 1129 (1976)
- [14] Landau L. and Lifchitz E., *Field theory*, ed. Mir (1970).
- [15] Weinberg S., *Gravitation and cosmology*, ed. Wiley (1972).
- [16] Stern A., Aharonov Y. and Imry Y., *Phys. Rev.* **A41**, 3436 (1990).
- [17] Bordé C.J., *C.-Rendus Acad. Sci. Paris IV* **2**, 509 (2001); Peters A., Chung K.Y. and Chu S., *Metrologia* **38**, 25 (2001).
- [18] Reynaud S., Lamine B., Lambrecht A. *et al*, *Gen. Rel. Grav.* **36**, 2271 (2004).
- [19] Weitz M., Young B. C. and Chu S., *Phys. Rev. Lett.* **73**, 2563 (1994)
- [20] Andersson E., Calarco T., Folman R. *et al*, *Phys. Rev. Lett.* **88**, 100401 (2002).
- [21] Pfau T., Kurtsiefer C., Adams C.S. *et al*, *Phys. Rev. Lett.* **71**, 3427 (1993); Houde O., Kadio D. and Pruvost L., *Phys. Rev. Lett.* **85**, 5543 (2000).
- [22] Boustimi M., Baudon J., Ducloy M. *et al*, *Eur. Phys. J.* **D17**, 141 (2001).
- [23] Arndt M., Private discussion.