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# Minkowski vacuum in background independent quantum gravity

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We consider a local formalism in quantum field theory, in which no reference is made to infinitely extended spacial surfaces, infinite past or infinite future. This can be obtained in terms of a functional $W[\varphi,\Sigma]$ of the field $\varphi$ on a closed 3d surface $\Sigma$ that bounds a finite region $\mathcal{R}$ of Minkowski spacetime. The dependence of $W[\varphi,\Sigma]$ on $\Sigma$ is governed by a local covariant generalization of the Schrödinger equation. Particles' scattering amplitudes that describe experiments conducted in the finite region $\mathcal{R}$ –the laboratory during a finite time– can be expressed in terms of $W[\varphi,\Sigma]$. The dependence of $W[\varphi,\Sigma]$ on the geometry of $\Sigma$ expresses the dependence of the transition amplitudes on the relative location of the particle detectors.
In a gravitational theory, background independence implies that $W[\varphi,\Sigma]$ is independent from $\Sigma$. However, the detectors' relative location is still coded in the argument of $W[\varphi]$, because the geometry of the boundary surface is determined by the boundary value $\varphi$ of the gravitational field. This observation clarifies the physical meaning of the functional $W[\varphi]$ defined by non perturbative formulations of quantum gravity, such as the spinfoam formalism. In particular, it suggests a way to derive particles' scattering amplitudes from a spinfoam model.
In particular, we discuss the notion of vacuum in a generally covariant context. We distinguish the nonperturbative vacuum $|0_\Sigma\rangle$, which codes the dynamics, from the Minkowski vacuum $|0_M\rangle$, which is the state with no particles and is recovered by taking appropriate large values of the boundary metric. We derive a relation between the two vacuum states. We propose an explicit expression for computing the Minkowski vacuum from a spinfoam model.

## I. INTRODUCTION

To understand quantum gravity, we have to understand how to formulate quantum field theory (QFT) in a background-independent manner. In the presence of a background, QFT yields scattering amplitudes and cross sections for asymptotic particle states, and these are compared with data obtained in the laboratory. The conventional theoretical definition of these amplitudes involves infinitely extended spacetime regions and relies on symmetry properties of the background. In a background independent context this procedure becomes problematic. For instance, consider the 2-point function

$$W(x,y) = \langle 0|\phi(x)\phi(y)|0\rangle. \tag{1}$$

In QFT over a background, the independent variables $x$ and $y$ can be related to the spacetime location of particle detectors. In a background independent context, general covariance implies immediately that $W(x,y)$ is constant for $x \neq y$, and therefore it is not clear how the formalism can control the localitazion of the detectors. (See for instance [1].)

Indeed, current efforts to define a quantum theory of gravity nonperturbatively, such as loop gravity [2,3], may claim remarkable theoretical progress, but the problem of deriving scattering amplitudes remains open. The effort to develop a covariant version of loop gravity lead to the spinfoam techniques [4]. These provide well defined expressions for a Misner-Hawking "sum over 4-geometries" [5,6], where finiteness results from the discreteness of space revealed by loop gravity. The spinfoam formalism provides an amplitude for quantum states of gravity and matter on a 3d boundary [6,7]. But, as far as we know, no formalism is yet available for deriving particles' scattering amplitudes from these boundary amplitudes. Here we indicate a direction to construct such formalism.

The key ingredient for developing this formalism is the Minkowski vacuum state, namely the "no-particle" state, or the coherent semiclassical state associated to the classical Minkowski solution. The construction of this state

is considered a major open problem in nonperturbative quantum gravity, and it is being studied using a variety of different techniques. See for instance [8] and references therein. Here, we propose a tentative explicit expression for computing the Minkowski vacuum from a spinfoam formalism.

We begin by introducing a certain number of general tools, in the context of the quantum field theory of a simple free massive scalar field. The euclidean functional integral over a finite spacetime region $\mathcal{R}$ of spacetime defines a functional $W[\varphi, \Sigma]$ that depends on the field boundary value $\varphi$ and the geometry of the 3d surface $\Sigma$ that bounds $\mathcal{R}$. We argue that all physical predictions on measurements performed in the region $\mathcal{R}$, including scattering amplitudes between particles detected in the lab, can be expressed in terms of $W[\varphi, \Sigma]$. The geometry of $\Sigma$ codes the relative spacetime localization of the particle detectors. The functional satisfies a local Schrödinger equation. This defines a covariant formalism for QFT entirely in terms of boundary data. For a general formulation of classical and quantum mechanics along these lines, see [9,10] and [11].

Next, we consider the application of this formalism to the gravitational context. In the gravitational context, if $W[\varphi, \Sigma]$ is well defined, then background independence implies that it is independent from local variations of the location of $\Sigma$. At first sight, this seems to leave us in the characteristic interpretative obscurity of background independent QFT: the independence of $W[\varphi, \Sigma]$ from $\Sigma$ is equivalent to the independence of $W(x, y)$ from $x$ and $y$, mentioned above. But at a closer look, it is not so: in this context the boundary field $\varphi$ includes the gravitational field, which is the metric, and therefore the argument of $W[\varphi, \Sigma] = W[\varphi]$ *still* describes the relative spacetime location of the detectors! This fact should allow us to express scattering amplitudes directly in terms of $W[\varphi]$ even in the background-independent context.

We distinguish two distinct notions of vacuum. The first is the nonperturbative vacuum state $|0_\Sigma\rangle$ that the functional integral on the bulk defines on the (kinematical) Hilbert space associated to the boundary surface $\Sigma$. If the metric on $\Sigma$ is chosen to be to be spacelike, this is the Hartle-Hawking state [7]. In the context we are considering, instead, $\Sigma$ is the boundary of a finite 4d region of spacetime, and $|0_\Sigma\rangle$ is a background-independent way of coding quantum dynamics [11]. The second notion of vacuum is (the local approximation to) the Minkowski vacuum state. Here we denote the Minkowski vacuum state as $|0_M\rangle$ (except in equation (1), where it was denoted $|0\rangle$). We shall argue that this state is recovered for appropriate values of the boundary metric.

A main result of this work is an equation connecting the two vacuum states, and an explicit formula for the Minkowski vacuum state $|0_M\rangle$, in terms of a spinfoam model. Here we present only the key ideas and the main results, detailed derivations will appear elsewhere.

## II. LOCAL TOOLS IN QFT

### A. Field to field propagator

Consider a real massive scalar field $\phi(x)$ on Minkowski space. To start with assume it is a free field. We write $x = (t, \vec{x})$. Denote by $\varphi(\vec{x})$ the classical field configuration at time zero: $\varphi(\vec{x}) = \phi(\vec{x}, 0)$. The state space at time zero, $\mathcal{H}_{t=0}$, is Fock space, where the (distributional) field operator $\varphi(\vec{x})$ and the hamiltonian $H$ are defined. The lowest eigenstate of $H$ is the vacuum state $|0_M\rangle$, and its energy $E_0$ is zero. Fock space admits countable bases. Choose a basis $|n\rangle$ of eigenstates of $H$ with eigenvalues $E_n$, and consider the operator

$$W(T) = \sum_n e^{-TE_n} |n\rangle\langle n|. \tag{2}$$

In the large $T$ limit, this becomes the projector on the vacuum

$$\lim_{T\to\infty} W(T) = |0_M\rangle\langle 0_M|. \tag{3}$$

We now move to a functional Schrödinger representation. Given a classical field configuration $\varphi$ at time zero, let $|\varphi\rangle$ be the (generalized) eigenstate of the operator $\varphi(\vec{x})$ with eigenvalue $\varphi$. We can express any state $|\Psi\rangle$ of Fock space in this field basis

$$\Psi[\varphi] = \langle\varphi|\Psi\rangle. \tag{4}$$

In this representation, the operator (2) reads

$$W[\varphi_1, \varphi_2, T] = \langle\varphi_1|W(T)|\varphi_2\rangle. \tag{5}$$

It satisfies the euclidean Schrödinger equation (in both variables)

$$-\frac{\partial}{\partial T}\, W[\varphi_1, \varphi_2, T] = H_{\varphi_1}\, W[\varphi_1, \varphi_2, T]. \tag{6}$$

From (3), we can obtain the vacuum (up to normalization) as

$$\Psi_{0_M}[\varphi] = \langle\varphi|0_M\rangle = \lim_{T\to\infty} W[\varphi, 0, T]. \tag{7}$$

Particles' scattering amplitudes can be derived from $W[\varphi_1, \varphi_2, T]$. For instance the 2-point function can be obtained as the analytic continuation of the Schwinger function

$$S(x_1, x_2) = \lim_{T\to\infty} \int D\varphi_1 D\varphi_2\; W[0, \varphi_1, T]\; \varphi_1(\vec{x}_1)\; W[\varphi_1, \varphi_2, (t_1 - t_2)]\; \varphi_2(\vec{x}_2)\; W[\varphi_2, 0, T]. \tag{8}$$

This can be generalized to any $n$-point function where the times $t_1, \dots t_n$ are on the $t = 0$ and the $t = T$ surfaces; these in turn, are sufficient to compute all scattering amplitudes, since time dependence of asymptotic states is trivial.

$W[\varphi_1, \varphi_2, T]$ admits the well-defined functional integral representation

$$W[\varphi_1, \varphi_2, T] = \int_{\substack{\phi|_{t=T}=\varphi_1 \\ \phi|_{t=0}=\varphi_2}} D\phi \;\; e^{-S_T^E[\phi]}. \tag{9}$$

Here the integral is over all fields $\phi$ on the strip $\mathcal{R}$ bounded by the two surfaces $t=0$ and $t=T$, with fixed boundary value. The action $S_T^E[\phi]$ is the Euclidean action. Notice that using this functional integral representation the expression (9) for the Schwinger function becomes the well known expression

$$S(x_1, x_2) = \int D\phi \; \phi(x_1) \; \phi(x_2) \; e^{-S^E[\phi]}, \tag{10}$$

obtained by joining at the two boundaries the three functional integrals in the regions $t<t_2$, $t_2<t<t_1$ and $t_1<t$.

The functional $W[\varphi_1, \varphi_2, T]$ can be computed explicitly in the free field theory. Its expression in terms of the Fourier transform $\tilde{\varphi}$ of $\varphi$ is (here $\omega = \sqrt{\vec{k}^2 + m^2}$)

$$W[\varphi_1, \varphi_2, T] = \mathcal{N} \; e^{-\frac{1}{2}\int \frac{d^3k}{(2\pi)^3} \, \omega \left( \frac{|\tilde{\varphi}_1|^2 + |\tilde{\varphi}_2|^2}{\tanh(\omega T)} - \frac{2\tilde{\varphi}_1 \overline{\tilde{\varphi}_2}}{\sinh(\omega T)} \right)}. \tag{11}$$

### B. Kinematical Hilbert space and nonperturbative vacuum

Consider the 3d surface $\Sigma_T = \partial\mathcal{R}$, namely the boundary of the strip $\mathcal{R}$. This surface is composed by the two disconnected components $t=0$ and $t=T$. Define a "kinematical" Hilbert space $\mathcal{K}_{\Sigma_T}$, associated to the *entire* surface $\Sigma_T$, as the tensor product

$$\mathcal{K}_{\Sigma_T} = \mathcal{H}^*_{t=T} \otimes \mathcal{H}_{t=0}. \tag{12}$$

The notation $\mathcal{H}^*$, indicates the dual of the Hilbert space $\mathcal{H}$ (which is of course canonically isomorphic to $\mathcal{H}$ itself). Denote as $\varphi = (\varphi_1, \varphi_2)$ a field on $\Sigma_T$. The field basis of the Fock space induces the basis

$$|\varphi\rangle = |\varphi_1, \varphi_2\rangle \equiv \langle\varphi_1|_{t=T} \; \otimes \; |\varphi_2\rangle_{t=0} \tag{13}$$

in $\mathcal{K}_{\Sigma_T}$; the vectors $|\Psi\rangle$ of $H_{\Sigma_T}$ are written in this basis as functionals

$$\Psi[\varphi] = \Psi[\varphi_1, \varphi_2] \equiv \langle\varphi_1, \varphi_2|\Psi\rangle. \tag{14}$$

The functional $W[\varphi_1, \varphi_2, T]$ defines the preferred (bra) state

$$\langle 0_{\Sigma_T}|\varphi\rangle \equiv W[\varphi_1, \varphi_2, T]. \tag{15}$$

in this Hilbert space. This corresponding to the functional $\rho$ of [11]. We call the state $|0_{\Sigma_T}\rangle$, the "nonperturbative vacuum", or "covariant vacuum". This state expresses the dynamics from $t=0$ to $t=T$. A state in the tensor product of two Hilbert spaces defines a linear mapping between the two spaces. The linear mapping from $\mathcal{H}_{t=0}$ to $\mathcal{H}_{t=T}$ defined by $\langle 0_{\Sigma_T}|$ is precisely the (imaginary time) evolution $e^{-TH}$. Indeed, we have by construction

$$\langle 0_{\Sigma_T}| \left( \langle\psi_{out}| \otimes |\psi_{in}\rangle \right) = \langle\psi_{out}|e^{-TH}|\psi_{in}\rangle. \tag{16}$$

Or

$$\langle 0_{\Sigma_T}|\psi_{in}\rangle = e^{-TH} \; |\psi_{in}\rangle. \tag{17}$$

Notice that the bra/ket mismatch is apparent only, as the three states live in different Hilbert spaces.

Equation (3) shows that in the limit $T \to \infty$ we have the projector on the vacuum

$$\lim_{T\to\infty} \langle 0_{\Sigma_T}| \left( \langle\psi_{out}| \otimes |\psi_{in}\rangle \right) = \langle\psi_{out}|0_M\rangle \; \langle 0_M|\psi_{in}\rangle. \tag{18}$$

We can therefore write the relation between the two notions of vacuum that we have defined as

$$\lim_{T\to\infty} |0_{\Sigma_T}\rangle = |0_M\rangle \otimes \langle 0_M|. \tag{19}$$

This is a key equation for what follows. Again, the bra/ket mismatch is apparent only, as the three states are in different Hilbert spaces.

The tensor product of two quantum state spaces describes the ensemble of the measurements described by the two factors. Therefore $\mathcal{K}_{\Sigma_T}$ is the space of the possible results of all measurements performed at time 0 *and* at time $t$ [9–11]. Observations at two different times are correlated by the dynamics. Hence $\mathcal{K}_{\Sigma_T}$ is a "kinematical" state space, in the sense that it describes more outcomes than the physically realizable ones. Dynamics is then a restriction on the possible outcome of observations [9–11]. It expresses the fact that measurement outcomes are correlated. The state $\langle 0_{\Sigma_T}|$, seen as a linear functional on $\mathcal{K}_{\Sigma_T}$, assigns an amplitude to any outcome of observations. This amplitude gives us the correlation between outcomes at time 0 and outcomes at time $T$. Therefore the theory can be represented as follows. The Hilbert space $\mathcal{K}^*_{\Sigma_T}$ describes all possible outcomes of measurements made on $\Sigma_T$. Dynamics is given by the single linear functional

$$\begin{aligned} \rho: \quad \mathcal{K}_T \; &\to \; C, \\ |\Psi\rangle \; &\mapsto \; \langle 0_{\Sigma_T}|\Psi\rangle. \end{aligned} \tag{20}$$

For a given collection of measurement outcomes described by a state $|\Psi\rangle$, the quantity $\langle 0_{\Sigma_T}|\Psi\rangle$ gives the correlation probability amplitude between these measurements.

### C. The functional $W[\varphi, \Sigma]$

We consider the extension this formalism to the case where $\mathcal{R}$, instead of being the strip between two planes,

is an arbitrary *finite* regions of spacetime. Let $\Sigma$ be the boundary of $\mathcal{R}$, that is a closed, connected 3d surface with the topology (but in general not the geometry) of a 3-sphere. Let $\varphi$ be a scalar field on $\Sigma$ and consider the functional

$$W[\varphi,\Sigma]=\int_{\phi|_\Sigma=\varphi} D\phi\ e^{-S^E_{\mathcal{R}}[\phi]}. \tag{21}$$

The integral is over all 4d fields on $\mathcal{R}$ that take the value $\varphi$ on $\Sigma$, and the action in the exponent is the Euclidean action where the 4d integral is over $\mathcal{R}$. In the free theory the integral is a well defined Gaussian integral and can be evaluated. The classical equations of motion with boundary value $\varphi$ on $\Sigma$ form an elliptic system and in general has a solution $\phi_{cl}[\varphi]$, which can be obtained by integration from the Green function for the shape $\mathcal{R}$. A change of variable in the integral reduces it to a trivial Gaussian integration times $e^{-S^E_{\mathcal{R}}[\varphi]}$. Here $S^E_{\mathcal{R}}[\varphi]$ is the field theoretical Hamilton function: the action of the bulk field determined by the boundary condition $\varphi$. This function satisfies a local Hamilton-Jacobi functional equation and solves the classical field theoretical dynamics [10,12].

### D. Local Schrödinger equation

$W[\varphi,\Sigma]$ satisfies a local functional equation that governs its dependence on $\Sigma$. Let $\vec\tau$ be arbitrary coordinates on $\Sigma$. Represent the surface and the boundary fields as $\Sigma:\vec\tau\mapsto x^\mu(\vec\tau)$ and $\varphi:\vec\tau\mapsto\varphi(\vec\tau)$. Let $n^\mu(\vec\tau)$ be the unit length normal to $\Sigma$. Then

$$n^\mu(\vec\tau)\frac{\delta}{\delta x^\mu(\vec\tau)}W[\varphi,\Sigma]=H(\vec\tau)\ W[\varphi,\Sigma] \tag{22}$$

where $H(\vec x)$ is an operator obtained by replacing $\pi(\vec x)$ by $-i\frac{\delta}{\delta\varphi(x)}$ in the hamiltonian density

$$H(\vec x)=g^{-\frac12}\pi^2(\vec x)+g^{\frac12}\ (|\vec\nabla\varphi|^2+m^2\varphi^2); \tag{23}$$

$g$ is the determinant of the induced metric on $\Sigma$ and the norm is taken in this metric. Since $W$ is independent from the parametrization

$$\frac{\partial x^\mu(\vec\tau)}{\partial\vec\tau}\frac{\delta}{\delta x^\mu(\vec\tau)}W[\varphi,\Sigma]=\vec P(\vec\tau)\ W[\varphi,\Sigma] \tag{24}$$

where the linear momentum is $\vec P(\vec\tau)=\vec\nabla\phi(\vec\tau)\ \delta/\delta\varphi(\vec\tau)$. Details will be given elsewhere. If $\Sigma$ is spacelike, (22) is the euclidean Tomonaga-Schwinger equation [13]. See also the cautionary remarks in [14].

### E. Relation with the propagator

Choose now $\Sigma$ to be a cylinder $\Sigma_{RT}$, with radius $R$ and height $T$, with the two bases on the surfaces $t=0$ and $t=T$. Given two compact support functions $\varphi_1$ and $\varphi_2$, defined on $t=0$ and $t=T$ respectively, we can always choose $R$ large enough for the two compact supports to be included in the bases of the cylinder. Then we expect that

$$W[\varphi_1,\varphi_2,T]=\lim_{R\to\infty}\ W[\varphi_1,\varphi_2,\Sigma_{RT}] \tag{25}$$

because the euclidean Green function decays rapidly and the effect of having the side of the cylinder at finite distance goes rapidly to zero as $R$ increases. Eq (8) illustrates how scattering amplitudes can be computed from $W[\varphi_1,\varphi_2,T]$. In turn, eq (25) indicates how $W[\varphi_1,\varphi_2,T]$ can be obtained from $W[\varphi,\Sigma]$, where $\Sigma$ is the boundary of a finite region. Therefore knowledge of $W[\varphi,\Sigma]$ allows us to compute physical scattering amplitudes. We expect that this should remain true in the perturbative expansion of an interacting field theory as well, where $\mathcal{R}$ includes the interaction region.

$W[\varphi,\Sigma]$ can be directly defined in the Minkowski regime as well. For a cylindrical box in Minkowski space, let $\varphi=(\varphi_{out},\varphi_{in},\varphi_{side})$ be the components of the field on the spacelike bases and timelike side. Consider the field theory defined in the box, with time dependent boundary conditions $\varphi_{side}$, and let $U[\varphi_{side}]$ be the evolution operator from $t=0$ to $t=T$ generated by the (time dependent) hamiltonian of the theory. Then $W[\varphi,\Sigma]\equiv\langle\varphi_{out}|U[\varphi_{side}]|\varphi_{in}\rangle$. When $\varphi_{side}$ is constant in time, this can be obtained by analytic continuation from the Euclidean functional.

### F. How far is infinity?

At first sight, the limits $T,R\to\infty$ seem to indicate that arbitrarily large surfaces $\Sigma$ are needed to compute vacuum and scattering amplitudes. Notice however that the convergence of $W[\varphi_1,\varphi_2,T]$ to the vacuum projector is dictated by (2): it is exponential in the mass gap $E_1$, or the Compton frequency of the particle. Thus $T$ at laboratory scales is largely sufficient to guarantee arbitrarily accurate convergence. In the Euclidean, rotational symmetry suggests the same to hold for the $R\to\infty$ limit. Thus the limits can be replaced by fixing $R$ and $T$ at laboratory scales. Problems could arise for the analytical continuation, which might not commute with the limits, but these problems do not affect the determination of the vacuum state, where no analytical continuation is required.

The fact that we can define the vacuum state, or particle states, locally seems to contradict the fact that the notions of vacuum and particle states are global. Let us therefore comment on this delicate point. The conventional notions of vacuum and particle states are global, but particle detectors are *finitely* extended. In facts, we may distinguish two distinct notions of particle [15]. Fock particle states are "global", while states detected by a localized detector (eigenstates of local operators

describing detection) are "local" particles states. Local particle states are very close to (in norm), but distinct from, the corresponding "global" particle states. On a flat background, we conveniently approximate the local particle state detected by the detectors, using global particle states, which are far easier to deal with. The global nature of the conventional definition of vacuum and particles is therefore an approximation adopted for convenience, it is not dictated by physical properties of particles we detect.

Replacing the limits $R \to \infty$ and $T \to \infty$ with finite macroscopic $R$ and $T$ we miss the *exact* global vacuum or $n$-particle state, but we can nevertheless describe local experiments. The restriction of QFT to a finite region of spacetime must describe completely experiments confined to this region and states detected by finitely extended particle detectors.

In conclusion, QFT can be formulated in terms of a state space $\mathcal{K}_\Sigma$ associated to the boundary of a finite region $\mathcal{R}$. States in $\mathcal{K}_\Sigma$ represent measurement outcomes on $\Sigma$. Dynamics is expressed by the single state $\langle 0_\Sigma|$ in $\mathcal{K}_\Sigma$, which gives the amplitude for any complete set of measurements. This can be computed as a functional integral over the interior region $\mathcal{R}$. Measurement of the boundary *field* are represented by the basis $|\varphi\rangle$. The Minkowski vacuum state $|0_M\rangle$ is obtained by propagation in imaginary time for a laboratory scale. *Particle* detection determines particle states in $\mathcal{K}_\Sigma$, which can be obtained acting with field operator on $|0_M\rangle$. More details on boundary particle states will be given elsewhere.

## III. QUANTUM GRAVITY

In quantum gravity, making the formulation described above concrete is a complex task. The problem that we consider here is only how to interpret a functional integral for quantum gravity defining a functional of the boundary states, assuming this is given to us. Concrete definitions of $W[\varphi, \Sigma]$ are rather well developed in the context of the spinfoam formalism. Lorentzian and Riemannian version of the formalism have been studied, and some finiteness results have been proven to all orders in a perturbative expansion [16].

Background independence implies immediately that the gravitational functional $W[\varphi, \Sigma]$ defined by an appropriate version of (21) is independent from any local variation of $\Sigma$. Fixing the topology of $\Sigma$, we have therefore

$$W[\varphi, \Sigma] = W[\varphi]. \tag{26}$$

At first sight, this seems the sort of independence from position and time, that renders background-independent QFT difficult to interpret. The independence of $W[\varphi, \Sigma]$ is indeed analogous to the independence of $W(x, y)$ from $x$ and $y$ mentioned at the beginning of this paper.

However, the property of $\Sigma$ that codes the relative spacetime location of the detectors is the *metric* of $\Sigma$. In the gravitational case, the metric of $\Sigma$ is not coded in the location of $\Sigma$ on a manifold: it is coded in the boundary value of the gravitational field on $\Sigma$. Therefore the relative location of the detectors, lost with $\Sigma$ because of general covariance, comes back with $\varphi$, as this includes the boundary value of the gravitational field. Therefore, if we are given a functional integral for gravity, we can interpret it exactly as we did for the scalar field! The boundary value of the gravitational field plays the double role previously played by $\varphi$ and $\Sigma$. In fact, this is precisely the core of the conceptual novelty of general relativity: there is no a priori distinction between localization measurements and measurements of dynamical variables.

$W[\varphi]$ determines a preferred state $|0_\Sigma\rangle$, defined by $\langle 0_\Sigma|\varphi\rangle = W[\varphi]$ in the kinematical state space $\mathcal{K}$ associated with the boundary. This is the covariant vacuum, and codes the dynamics. It satisfies a dynamical equation analogous to equation (22), where $H(\vec{\tau})$ is now the hamiltonian constraint density operator. But since $W$ is independent from $\Sigma$ by general covariance, the left hand side of (22) vanishes, leaving

$$H(\vec{\tau})\ W[\varphi] = 0, \tag{27}$$

which is the (lorentzian) Wheeler-DeWitt equation [6].

### A. Minkowski vacuum in quantum gravity

The quantum state $|0_M\rangle$ that describes the Minkowski vacuum is not singled out by the dynamics alone in quantum gravity. Rather, it is singled out as the lowest eigenstate of an energy $H_T$ which is the variable canonically conjugate to a nonlocal function $T$ of the gravitational field defined as the proper time along a given worldline.

This situation has an analogy in the simple quantum system formed by a single a relativistic particle. In the Hilbert space of such a system there is no preferred vacuum state. But we can choose a preferred Lorentz frame, and therefore a preferred Lorentz time $x^0$. The conjugate variable to $x^0$ is the momentum $p_0$, and there is a (generalized) state of minimum $p_0$.

To find the Minkowski vacuum state, we can repeat the very same procedure used above. The only difference is that the bulk functional integral is not over the bulk matter fields, but also over the bulk metric. This difference has no bearing on the above formulas, which regard the boundary metric, which, in the two cases, is an independent variable.

As a first example, a boundary metric can be defined as follows. Consider a three-sphere formed by two "polar" *in* and *out* regions and one "equatorial" *side* region. Let the matter+gravity field on the three-sphere be split as

$$\varphi = (\varphi_{out}, \varphi_{in}, \varphi_{side}). \tag{28}$$

Fix the equatorial field $\varphi_{side}$ to take the special value $\varphi_{RT}$ defined as follows. Consider a cylindrical surface $\Sigma_{RT}$ of radius $R$ and height $T$ in $R^4$, as defined above. Let $\Sigma_{in}$ (and $\Sigma_{out}$) be a (3d) disk located within the lower (and upper) basis of $\Sigma_{RT}$, and let $\Sigma_{side}$ the part of $\Sigma_{RT}$ outside those disks, so that

$$\Sigma_{RT} = \Sigma_{in} \cup \Sigma_{out} \cup \Sigma_{side}. \tag{29}$$

Let $g_{RT}$ be the metric of $\Sigma_{side}$ and let $\varphi_{RT} = (g_{RT}, 0)$ be the boundary field on $\Sigma_{side}$ determined by the metric being $g_{RT}$ and all other fields being zero. Given arbitrary values $\varphi_{out}$ and $\varphi_{in}$ of all the fields, included the metric, in the two disks, consider $W[(\varphi_{out}, \varphi_{in}, \varphi_{RT})]$. In writing the boundary field as composed by three parts as $\varphi(\varphi_{out}, \varphi_{in}, \varphi_{side})$ we are in fact splitting $\mathcal{K}$ as

$$\mathcal{K} = H_{out} \otimes H^*_{in} \otimes H_{side}. \tag{30}$$

Fixing $\varphi_{side} = \varphi_{RT}$ means contracting the covariant vacuum state $|0_\Sigma\rangle$ in $\mathcal{K}$ with the bra state $\langle\varphi_{RT}|$ in $H_{side}$. For large enough $R$ and $T$, we expect the resulting state in $H_{out} \otimes H^*_{in}$ to reduce to the Minkowski vacuum. That is

$$\lim_{R,T\to\infty} \langle\varphi_{RT}|0_\Sigma\rangle = |0_M\rangle \otimes \langle 0_M|. \tag{31}$$

Therefore for a generic *in* configuration, and up to normalization

$$\Psi_M[\varphi] = \lim_{R,T\to\infty} W[(\varphi, \varphi_{in}, \varphi_{RT})]. \tag{32}$$

gives the vacuum functional for large $R$ and $T$. (Below we shall use a simpler geometry for the boundary.)

One may hope that the convergence in $R$ and $T$ is fast. These formulas allow us to extract the Minkowski vacuum state from a euclidean gravitational functional integral. $n$-particle scattering states can then be obtained by generalizations of the flat space formalism, and, if it is well defined, by analytic continuation in the single variable $T$. Notice that we are precisely in the case of time independent $\varphi_{side}$, where analytical continuation may be well defined.

### B. Spinnetworks and spinfoams

The argument of $W$ is not a classical field: it is an element of the eigenbasis of the field operator. In the gravitational case, (functions of) the gravitational field operator can be diagonalized, but eigenvalues are not continuous fields. In loop quantum gravity, eigenstates of the metric are spin network states $|s\rangle$. Therefore the quantum gravitational $W$ must be a function of spin network states $W[s]$ on $\Sigma$, and not of continuous gravitational fields on $\Sigma$. In fact, this is precisely what a spin foam model provides.

A spinfoam sum where the degrees of freedom are not cut off by the choice of a fixed triangulation is defined by the Feynman expansion of the QFT over a group, studied in [17]. Let us recall here the basic equations of the formulations, referring to [17] and [10] for motivations and details. Let $\Phi(g_1, \dots, g_4)$ be a field on $[SO(4)]^4$, satisfying

$$\phi(g_1, g_2, g_3, g_4) = \phi(g_1 g, g_2 g, g_3 g, g_4 g), \tag{33}$$

for all $g \in$ SO(4). Consider the action

$$S[\phi] = \frac{1}{2}\int (\phi)^2 \; + \frac{\lambda}{5!}\int (P_{_H}\phi)^5. \tag{34}$$

Here $P_{_H}$ is defined by

$$P_{_H}\phi(g_1, g_2, g_3, g_4) = \int_{H^4} dh_1 \dots dh_4 \; \phi(g_1 h_1, g_2 h_2, g_3 h_3, g_4 h_4). \tag{35}$$

where $H$ is a fixed $SO(3)$ subgroup of $SO(4)$, and $\int \phi^5$ is a short hand notation for

$$\int \phi^5 = \int \prod_{i=1}^{10} dg_i \;\; \phi(g_1, g_2, g_3, g_4)\phi(g_4, g_5, g_6, g_7) \phi(g_7, g_3, g_8, g_9)\phi(g_9, g_6, g_2, g_{10}) \; \phi(g_{10}, g_8, g_5, g_1). \tag{36}$$

The Feynman expansion of this theory is a sum over spinfoams and can be interpreted as a well-defined version of the Misner-Hawking sum over geometries. Transition amplitudes between quantum states of space can be computed as expectation values of $SO(4)$ invariant operators in the group field theory. In particular, the boundary amplitude of a 4-valent spin network $s$ can be computed as

$$W[s] = \int D\Phi \; f_s[\Phi] \; e^{-S[\phi]}, \tag{37}$$

The spinfoam polynomial is defined as

$$f_s[\phi] = \prod_n \int dg_{n_1} \dots dg_{n_4} \; R^{(j_{n_1})\beta_{n_1}}_{\alpha_{n_1}}(g_{n_1}) \dots R^{(j_{n_4})\beta_{n_4}}_{\alpha_{n_1}}(g_{n_4}) \; v^{i_n}_{\beta_{n_1}\dots\beta_{n_4}} \prod_l \delta^{l_1 l_2} \tag{38}$$

where $n_1, \dots, n_4$ indicate four links adjacent to the node $n$, and $n_i = l_1$ (or $n_i = l_2$) if the $i$-th link of the node $n$ is the outgoing (or ingoing) link $l$.

We can now implement equation (32) in this theory. Instead of the cylindrical boundary consider above, we can choose a simpler geometry. Let the spin network $s'$ be composed by two parts connected to each other, $s' = s\#s_T$. Let $s$ be arbitrary and $s_T$ to be is a weave state [18] for the three-metric $g_T$ defined as follows. Take a 3-sphere of radius $T$ in $R^4$. Remove a spherical 3-ball of unit radius. $g_T$ is the three-metric of the three-dimensional surface (with boundary) formed by the sphere with removed ball. I recall that a weave state for a metric $g$ is an eigenstate of (functions of the smeared)

metric operator, whose eigenvalues approximate (functions of the smeared) $g$ at distances large compared to the Planck length.

The quantity

$$\Psi_M[s] = \langle s|0_M\rangle = \lim_{T\to\infty} \int D\Phi \; f_{s\#s_T}[\Phi] \; e^{-S[\Phi]}. \quad (39)$$

is then a tentative ansatz for the quantum state describing the Minkowski vacuum in a ball of unit radius. This quantity can be computed explicitly [17] and may be finite at all orders in $\lambda$ [16].

## IV. CONCLUSIONS

In this paper we have sketched several general ideas on the physical interpretation of the formalism in background independent QFT. The main ideas we have considered are the following

(i) In QFT, the functional integral over a finite region defines the functional $W[\varphi, \Sigma]$ of the boundary field, which expresses the physical content of the theory.

(ii) This functional can be used to compute the vacuum state $|0_M\rangle$, taking choosing $\Sigma$ appropriately.

(iii) In a background independent theory, $n$ particle functions $W(x_1, \ldots, x_n)$ become meaningless, because they are independent from the coordinates; while $W[\varphi, \Sigma]$ maintains its physical meaning, in spite of the fact that it is independent from $\Sigma$. This is because in a gravitational theory the relative location of the detectors is coded in $\varphi$ and not in $\Sigma$. Localization measurements are on the same footing as the dynamical variables measurements.

(iv) The functional $W[\varphi]$ defines a state $|0_\Sigma\rangle$ that codes the dynamics of the theory by determining the correlation amplitudes between boundary measurements.

(v) The Minkowski vacuum state $|0_M\rangle$ can be computed from nonperturbative quantum gravity by choosing appropriate boundary values of the gravitational field.

(vi) A tentative formula giving the Minkowski vacuum state in terms of a spinfoam model is given by equation (39).

(vii) Relevant analytical continuation is in the proper length of the boundary, not in the time coordinate.

Much remains to be done and many points are far from clear. The most urgent of these problems is the following. The spinfoam model we have referred to in the text is Riemannian, not Euclidean. Namely its amplitudes correspond to the complex quantity $e^{iS_E}$, where $S_E$ is the Euclidean action, and not to a real exponential. The relation between the Euclidean, Riemannian and Lorentzian spinfoam models has not yet completely clear, we believe.

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