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Projection effects in cluster mass estimates : the case of MS2137-23

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Abstract. We revisit the mass properties of the lensing cluster of galaxies MS2137-23 and assess the mutual agreement between cluster mass estimates based on strong/weak lensing, X-rays and stellar dynamics. We perform a thorough elliptical lens modelling using arcs and their counter-images in the range $20 \lesssim R \lesssim 100$ kpc and weak lensing ($100 \lesssim R \lesssim 1000$ kpc). We confirm that the dark matter distribution is well consistent with an NFW profile with high concentration $c \sim 11.7 \pm 0.6$.

We analyse the consequences of this model on the stellar kinematics of the central cD galaxy observed by Sand et al. (2002, 2004). We fully calculate the line-of-sight velocity distribution (LOSVD) of stars and show that the LOSVD is not gaussian leading to $\sim 15\%$ low-biased velocity dispersion measurements close to the center. Although we quantify/correct this source of systematic uncertainty, the NFW lens model is unable to properly fit kinematical data. Furthermore our NFW model predicts a factor of ~ 2 more massive cluster than the NFW model inferred from X-rays analysis (Allen et al. 2001), whereas this latter is consistent with kinematical constraints.

There is a discrepancy between projected (lensing) and tridimensional (X-rays,dynamics) mass estimates. We show that such discrepancies can be explained by assuming prolate (triaxial) halos with the major axis oriented toward the line-of-sight. As well, the high concentration and the misalignment between stellar and dark matter components ($\Delta\psi \sim 13^\circ$) support this hypothesis.

We then calculate the systematic and statistical uncertainties in the relative normalization between the cylindrical $M_2(< r)$ and spherical $M_3(< r)$ mass estimates for triaxial halos. These uncertainties prevent any attempt to couple 2D and 3D constraints without undertaking a complete tridimensional analysis. Such asphericity/projection effects should be a major concern for comparisons between lensing and X-rays/dynamics mass estimates.

Key words. Cosmology: Dark Matter – Galaxies: Clusters: General, MS2137 – Galaxies: Elliptical and Lenticular, cD – Cosmology: Gravitational Lensing

1. Introduction

The issue of the late non-linear evolution of cosmic structure is essentially addressed via large N-body numerical simulations. Consequently, it is important to check their validity by comparing the small scale matter distribution to numerical predictions. Two observations act as key-tests for the CDM paradigm: the mass distribution of dark matter halos (radial density profile and triaxiality) and the abundance of sub-halos within main halos. This work focuses on the former issue.

Most CDM simulations predict a universal profile of the general form:

$$\rho(r/r_s) = \rho_s(r/r_s)^{-\alpha} (1 + r/r_s)^{\alpha-3}, \quad (1)$$

with an inner slope α ranging between $\alpha = 1$ and $\alpha = 1.5$ (Navarro et al. 1997; Moore et al. 1998; Ghigna et al. 2000;

Jing & Suto 2000). The parameters r_s and ρ_s can be related to the halo mass (Bullock et al. 2001; Eke et al. 2001). The most recent simulations propose a slightly different universal analytical form:

$$\ln \frac{\rho(r)}{\rho(r_s)} = -\frac{2}{\mu} ((r/r_s)^\mu - 1), \quad (2)$$

with $\mu \sim 0.17$ (Navarro et al. 2004) (see also Stoehr et al. 2002; Stoehr 2004). Such profiles have a logarithmic slope $\frac{d \ln \rho}{d \ln r}$ which continuously flattens when $r \rightarrow 0$.

The global agreement between observations and simulations is subject to controversy. The inner slope of dark matter halos of low surface brightness (LSB) dwarf galaxies as inferred from rotation curves tends to favor soft cores with $\alpha \lesssim 0.2$ (de Blok & Bosma 2002; de Blok et al. 2003, and references therein), leading to the so-called cusp-core debate. Many observations have focused on LSB galaxies because their baryonic content can be neglected and the dark matter distribution in the halo shall match

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simulations. However, departures from axisymmetry (triaxial halos) make the interpretation of rotation curves more complex and could reconcile observations and CDM predictions (Hayashi et al. 2004). The question of the very central mass profile on dwarfs scales is still open...

Recently a similar discrepancy at clusters of galaxies scales is claimed by Sand et al. (2002, 2004, hereafter Sa04). Using HST images (allowing the modelling of strong gravitational lensing configurations) together with Keck spectroscopy (providing the radial velocity dispersion of stars in the central cD galaxy of the cluster, the BCG) on a sample of six clusters, these authors found that the inner slope of the dark matter halo must be significantly flatter than those measured in simulations. Typically, on a subsample of three clusters with radial arcs, they found an inner slope $\alpha = 0.52 \pm 0.05$ (68%CL). This result takes advantage of the joint constraints provided by lensing and stellar kinematics. However, the lensing part of the analysis of Sa04 has been independently discussed by Bartelmann & Meneghetti (2004) and Dalal & Keeton (2003) because they did not take into account the lens ellipticity when using the critical lines radii as a constraint on the density profile. These two latter authors found that the mass profile is consistent with a NFW model. The analysis of Sa04 couples 2D projected (from lensing that deal with mass enclosed in the cylinder of radius R) and 3D tridimensional (from stellar dynamics which project an indirect information on the mass enclosed in the sphere of radius r) mass estimates.

Comparing lensing and X-rays cluster mass estimates is another way to couple 2D and 3D mass constraints. The overall agreement between X mass and the mass enclosed in the Einstein radius of clusters are been addressed by various authors (Miralda-Escude & Babul 1995; Allen 1998; Wu 2000; Arabadjis et al. 2004; Smith et al. 2004). In most cases, slightly depending on the presence of cooling flows or the degree of relaxation of the cluster, lensing mass estimates are often larger by a factor $\gtrsim 1.5$.

In this paper, we focus the density profile modelling of the cluster MS2137-21 which is part of the Sa04 sample. We make a simultaneous modelling of the strong and weak lensing configuration, which builds on the previous analysis of Gavazzi et al. (2003, hereafter G03). In Sect. 2 we present the strong and weak lensing modelling of MS2137 with a NFW model and show that it is consistent with all the lensing data at hand from 10 kiloparsecs to 1 megaparsec. In Sect. 3 we develop a detailed method for the analysis of stellar kinematics and apply it to the best fit NFW model derived in the previous section. We then discuss the overall agreement between lensing mass estimates and the constraints from the stellar kinematics and X-rays observations of Allen et al. (2001). In Sect. 4 we investigate the origin on the systematic underestimate of cluster/galaxies mass estimates from 3D analyses as compared to that of lensing, and show that the tridimensional shape of halos (prolate, triaxial) is likely to explain such discrepancies. In Sect. 5 we calculate the statistical properties of the relative normalization between 2D and 3D

mass estimates of triaxial halos. We discuss our results and conclude in Sect. 6.

In the following, we assume a $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 70 h_{70} \text{ km s}^{-1} \text{ Mpc}$, leading to the scaling $1'' = 4.59 h_{70}^{-1} \text{ kpc}$.

2. Lens modelling

2.1. Optical data and χ^2 definition

In this section we focus on the density profile modelling using lensing constraints only. The lens properties of the cluster of galaxies have been extensively studied (Fort et al. 1992; Mellier et al. 1993; Miralda-Escudé 1995; Bartelmann 1996; Hammer et al. 1997; Gavazzi et al. 2003; Sand et al. 2002, 2004; Dalal & Keeton 2003). The cluster's redshift is $z_l = 0.313$ and both radial and tangential arcs lie at $z_s = 1.501$ (Sand et al. 2002), leading to the critical surface density $\Sigma_{\text{crit}} = 2.39 \times 10^9 h_{70} \text{ M}_\odot \text{ kpc}^{-2}$.

Our analysis builds on the previous work of Gavazzi et al. (2003) (Hereafter G03). More precisely, we use 26 multiple conjugate knots in the tangential and radial arcs systems. The method and the knots location are presented in G03. Here, we inflate the uncertainties on knot positions in order to account for possible bad associations. Basically, the positional error is raised to the conservative value $\sigma_x = 0''.3$. We exclude constraints from the fifth central demagnified image reported in G03 since its detection is marginal and is not confirmed by Sa04. We use a personal ray-tracing inversion code that includes many aspects of the `lensmodel` code (Keeton 2001a,b). In particular, we adopt the same source plane χ_{src}^2 definition.

Moreover, we simultaneously include weak lensing constraints also presented in G03. The catalogue of background “weakly lensed” galaxies comes from VLT/FORS and VLT/ISAAC images for which we were able to derive a good estimate of photometric redshifts using *UBVRIJK* bands. We fully compute the likelihood as a function of model parameters (Schneider et al. 2000; King & Schneider 2001).

$$\mathcal{L}_{\text{wl}} = \prod_{i=1}^{N_{\text{bg}}} p(e_i) \quad (3)$$

where e_i is the observed ellipticity of the background galaxy. We have

$$p(e_i) = p_s(e_s(e_i, g_i)) \left| \frac{de_s}{de_i} \right|, \quad (4)$$

e_s being the source ellipticity and $g_i = g(\theta_i, z_i)$ is the reduced shear. See Geiger & Schneider (1998) for the description of the relation $e_s(e_i, g)$ and for the corresponding transformation Jacobian. Ellipticities are measured on the I band image. We improved the previous analysis of G03 and built a new PSF smearing correction pipeline based on the KSB method (Kaiser et al. 1995) but leading to a better weighting scheme (Gavazzi et al. 2004). We fully

take into account redshift information, either photometric¹ for weak lensing or spectroscopic for strongly lensed arcs. The use of photometric redshift to select the sample of background galaxies avoids the problem of contamination by foreground unlensed galaxies (Broadhurst et al. 2005b). The global χ^2 for lensing is :

$$\chi_{\text{lens}}^2 = \chi_{\text{src}}^2 - 2 \ln \mathcal{L}_{\text{wl}}. \quad (5)$$

The model inversion is based on its minimisation, which is done using the `minuit` library developed at CERN². The error analysis is performed using both `minuit` facilities and Monte-Carlo Markov Chains (MCMC) based on the implementation developed by I. Tereno. See Tereno et al. (2005) for a full description of the method and an application to cosmological parameters constraints. We chose to use MCMCs because `minuit` has difficulties to draw $\Delta\chi^2$ contours in the parameter space when there are strong degeneracies. In order to fasten the convergence of the chains, we previously run many `minuit` optimizations starting from a broad domain of initial conditions. From the well defined best fit minimum position, we started up to five chains with each of them ending with 6×10^4 relevant iterations. The convergence assessment was done in the same way as in Tereno et al. (2005).

2.2. Luminosity profile

We fitted the central cD surface brightness on the F702/WFPC2 Hubble Space Telescope image (Hammer et al. 1997) with a general projected stellar density profile assuming that all stars have a constant mass-to-light ratio. The surface brightness profile was measured with the `ellipse/stsdas` IRAF tool. We first assumed the following analytic expression for the three-dimensional radial distribution of stars :

$$\rho_*(r) = \rho_{s*} x^{-\alpha} (1+x)^{\alpha-\beta}, \quad (6)$$

where $x = r/r_{s*}$ and r_{s*} is a scale radius. Our best fit parameter set is $\beta = 3.9$, $r_{s*} = 8.7 h_{70}^{-1}$ kpc and $\alpha \approx 0.86$. We also modified the 2 dimensional galaxy model fitting software `galfit` (Peng et al. 2002) to include Hernquist ($\alpha = 1, \beta = 4$) and Jaffe ($\alpha = 2, \beta = 4$) density profiles. The Hernquist fit yields an axis ratio $q_* = b/a = 0.83 \pm 0.12$, a scale radius $r_{s*} = 11.1 \pm 1.9 h_{70}^{-1}$ kpc and a reduced $\chi^2/\text{dof} = 10.2$. This latter value could be noticeably decreased by taking into account a rotation of the major axis position angle within the inner 3 arcsec (see Fig. 2 of G03) that cannot be modeled with a single elliptical component. We also tried to fit a Jaffe profile as proposed by Sa04 but we found a much worse fit with $\chi^2/\text{dof} = 99.0$. In the following we will consider that the stellar component is well modeled by a Hernquist density profile with $r_{s*} = 11.1 h_{70}^{-1}$ kpc = $2''.4 \pm 0''.1$ and a total rest frame V band luminosity $L_V = 4.77 \pm 0.40 \times 10^{11} h_{70}^{-2} L_{\odot}$ that come from Sa04. The mass content in stars is

$M_* = 2\pi\rho_{s*}r_{s*}^3 q_* \equiv \Upsilon_V L_V$ where Υ_V is the rest-frame V band stellar mass-to-light ratio.

2.3. NFW dark matter density profile

The lens is decomposed into two components. The stellar component is modeled by the elliptical Hernquist profile of the previous section. The stellar mass-to-light ratio Υ_V is treated as a free parameter with a broad uniform prior $1.5 \leq \Upsilon_V \leq 10$. The dark matter halo is modeled with an elliptical NFW density profile:

$$\rho_{\text{DM}}(r) = \rho_s(r/r_s)^{-1} (1 + r/r_s)^{-2}. \quad (7)$$

The lens properties of such a density profile are presented in (Bartelmann 1996). We used numerical integrations algorithms for the elliptical³ Hernquist and NFW density profiles (Keeton 2001a). The model has five free parameters :

- the dark halo scale radius : r_s ,
- the characteristic convergence : $\kappa_s \equiv \rho_s r_s / \Sigma_{\text{crit}}$,
- the dark halo axis ratio : $q = b/a$,
- the dark halo major axis position angle : θ_0 ,
- the stellar mass-to-light ratio : $\Upsilon_V \equiv M_*/L_V$.

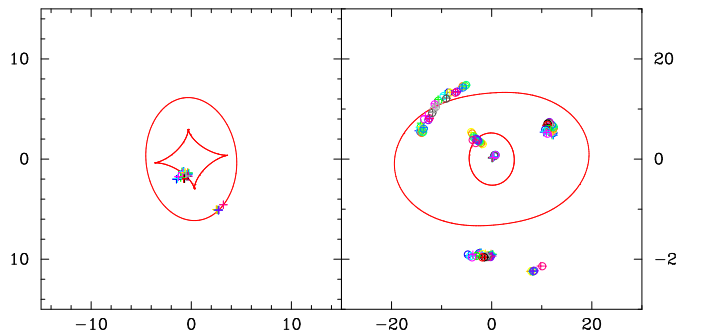


Fig. 1. Illustration of the quality of the NFW model to fit strong lensing constraints. *Left:* Caustic lines with the position of the 26 knots in the source plane. *Right:* Critical lines with the observed (resp. model) position of the 26 knots represented with circles (resp. + signs). Geometrical units are arcsec.

Fig. 1 details strong lensing results. One can see the source and image planes with caustic and critical lines together with the location of the 26 multiply images knots (see G03 for further details). Each observed point (circle) is well reproduced by the model (+ signs). The source associated to the tangential arc is clearly crossing the corresponding caustic line (inner astroid) whereas one can only see the part of the source associated to the radial arc that is inside the radial caustic (since the part outside the caustic is not multiply imaged and cannot be used). The central image associated to the tangential system is plotted but is not taken into account in the modelling. Besides the critical lines location (the only constraint used

³ which are not approximated by elliptical lens potentials (numerically faster but leading to unphysical surface mass density at large radius)

¹ using `hyperz` facilities (Bolzonella et al. 2000), see also G03

² <http://cernlib.web.cern.ch/cernlib/>

by Sa04), our model also remarkably explains the position of counter-images, the azimuthal configuration as a function of the position angle, and the length and width of arcs.

The model requires a rest frame V band stellar mass-to-light ratio $\Upsilon_V = 2.5 \pm 0.4$. This value is in good agreement with expectations of evolution of $2. \lesssim t \lesssim 4$ Gyr old stellar populations.

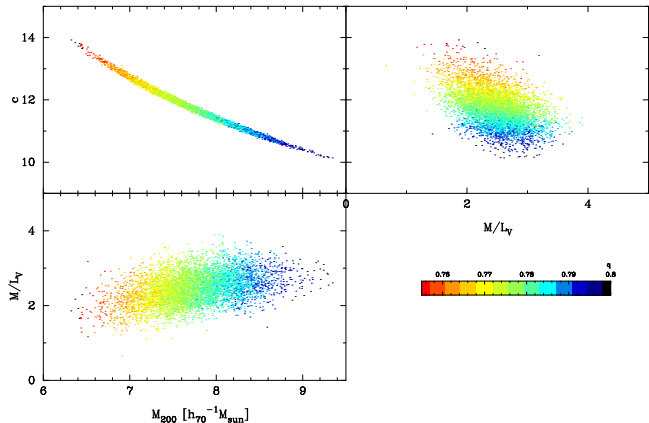


Fig. 2. Scatter plot showing the projection of MCMCs on some planes of the parameters space of the NFW lens modelling. The color codes for the ellipticity parameter according to the scale at the bottom right corner. The axes are the virial mass M_{200} , the concentration c and the rest-frame V band stellar mass-to-light ratio $\Upsilon_V \equiv M_*/L_V$.

In table 1 we present results in terms of more physical quantities like the virial radius r_{200} , the concentration parameter $c = r_{200}/r_s$ or the virial mass M_{200} that all derive from κ_s , r_s and q . At the best fit parameter set, the minimum χ^2 value is $\chi^2/\text{dof} = 4931.20/4965 = 0.993$. The (SL) and (SL+WL) columns detail how the best fit model is changed whether or not weak lensing constraints are added to the model. Basically, errors are just reduced and no significant change in the best fit parameters value is observed. These latter come from the analysis of MCMCs as already pointed out in the previous section. Fig. 2 shows the degeneracies between the concentration parameter, virial mass, stellar mass-to-light ratio and ellipticity (color-coded).

The quality of the fit is such that very few departs from the NFW we found are allowed. In order to check how restrictive is the analytical form (7), we also assumed the following profile for the dark matter component

$$\rho_{\text{DM}}(r) = \rho_s x^{-\alpha} (1 + x^2)^{(\alpha-\beta)/2}, \quad (8)$$

with two more free parameters: the asymptotic inner and outer slopes α and β respectively and x is the radius in units of a scale radius $r = xr_s$. We will refer to this profile as the “*gen*” profile in the following. We found $\Upsilon_V = 2.09 \pm 0.16$ also consistent with stellar evolution models, $\beta = 2.69^{+0.32}_{-0.22}$ and $\alpha = 1.262^{+0.013}_{-0.017}$. The constraints on α are very tight and show that lensing hampers any soft core with $\alpha < 1$. However, it does not contradict the NFW behavior $\rho \propto r^{-1}$ at small scales because the

fast transition $1 + x^2$ in the “*gen*” profile differs from that of the NFW case ($1 + x$). This can clearly be seen in Fig. 3 where the projected NFW and “*gen*” mass profiles match over a broad radius range ($r \lesssim 100$ kpc). The differences at larger scales are still consistent with the weak lensing uncertainties. Consequently, we can faithfully trust the radial behavior of the lensing deduced mass profile of the NFW model between $10 < R < 1000$ kpc.

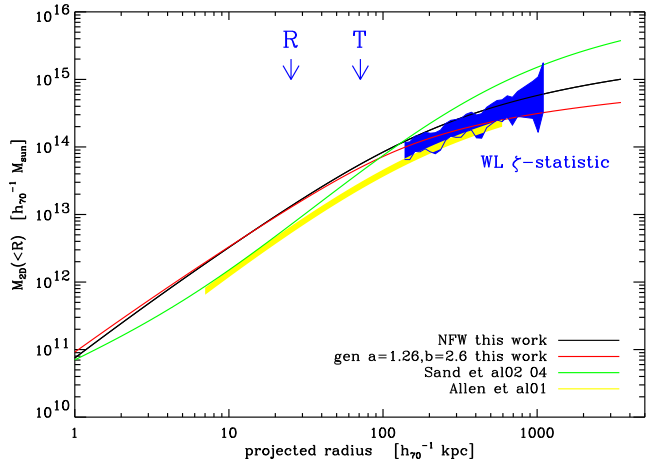


Fig. 3. Projected total mass (stars+DM) profile within cylinder of radius R for the best fit NFW model (black), the best fit general “*gen*” model with $\alpha = 1.26$ and $\beta = 2.6$ (red), the best fit model of Sand et al. (2002, 2004) (green) and the best fit NFW model from X-rays constraints (Allen et al. 2001) (yellow). The width on this latter curve is representative of the uncertainty reported by the authors. The blue filled region is the domain constrained by weak-lensing ζ -statistic. The location of the tangential (resp. radial) critical lines is reported by a “T” (resp. “R”) blue arrow. The “*gen*” and NFW models deduced from lensing match well from the center to the inner radius probed by weak-lensing. Above, they are consistent within the weak-lensing uncertainties. For $R \lesssim 40$ kpc both profiles present large (a factor ~ 2) discrepancies with the curves inferred by Sa04 and A101. See text.

Fig. 3 also shows the best fit model of Sa04 which presents strong discrepancies with both our NFW and “*gen*” models. Except the fact that the projected mass at the tangential arc radius (~ 100 kpc) matches our estimates, the model of Sa04 turns out to be inconsistent with most lensing constraints. They imposed the radial critical line to fit the observations but their model cannot reproduce the radial arc length and its counter-images, nor the tangential arc width and weak lensing at $R \gtrsim 200$ kpc. This can be understood by comparing the circularly averaged deflection α profile of these models in the upper panel of Fig. 4. This figure is usually used to graphically solve the lens equation. One can see that the tangential critical radius (intersection of curves $\alpha(r)$ and $y = r$) are consistent from one model another. As well the curves $\alpha(r)$ are tangent to the line $y = r + u$ at the same radial critical radius. But the intersection of $\alpha(r)$ and this line at the opposite side, which give the location of the counter-image of the radial arc, significantly differ (~ 2 arcsec). Moreover, we can see that the Sa04 model predicts a much more elongated radial arc that could extend very close to the lens center. This is clearly excluded by the data. Notice that

param.	(SL)	(SL+WL)	(1)	unit
κ_s	0.67 ± 0.05	0.66 ± 0.03	0.30 ± 0.15	
r_s	158^{+15}_{-13}	162^{+11}_{-9}	160 ± 30	h_{70}^{-1} kpc
r_{200}	1.88 ± 0.05	1.89 ± 0.04	$1.39^{+0.49}_{-0.38}$	h_{70}^{-1} Mpc
M_{200}	$7.56^{+0.63}_{-0.54}$	$7.72^{+0.47}_{-0.42}$	-	$10^{14} h_{70}^{-1} M_{\odot}$
c	$11.92^{+0.77}_{-0.74}$	11.73 ± 0.55	$8.7^{+1.2}_{-0.9}$	
q	0.774 ± 0.010	0.777 ± 0.007	-	
θ_0	5.86 ± 0.14	5.88 ± 0.13	-	deg
Υ_V	2.40 ± 0.45	2.48 ± 0.39	-	$h_{70} (M/L)_{\odot}$

Table 1. Best fit NFW model parameters and their 68% CL uncertainty (marginalized over all the other parameters). (SL) corresponds to a model in which weak lensing constraints are ignored whereas (SL+WL) takes both strong and weak lensing constraints into account. (1) refers to the CHANDRA X-rays values of Allen et al. (2001). The apparent disagreement between their estimates and ours is discussed in the text.

the Sa04 model predicts another radial critical line at the very center ($r \sim 0.1''$) and globally higher magnifications since $\alpha(r)$ is close to the bisectrix $y = r$. The only models consistent with all lensing constraints are the ones similar to the NFW and the “gen” models.

The column (1) of Table 1 also resumes the NFW model parameters deduced from Chandra X-rays observations of Allen et al. (2001, hereafter A101). The projected mass profile of their model is shown as the yellow thick curve on Fig. 3. One can see that the A101 model predicts a NFW mass profile twice as low as our NFW and “gen” models over a range $10 < R \lesssim 200$ kpc (*i.e.* the factor 1/2 in the value of κ_s in table 1). At larger scales $R \gtrsim 300$ kpc, the A101 mass profile becomes consistent with weak lensing and our models. It is worth noticing that X-rays (A101) and stellar dynamics (Sa04) mass estimates agree at small scales $R \lesssim 50$ kpc.

3. Dynamics of stars in the BCG

In this section, we focus on the kinematical properties of stars in the central cD galaxy without regard for lensing (either strong or weak). First we relate the gravitational potential and the line-of-sight velocity distribution LOSVD through a detailed dynamical analysis in order to check whether or not the mass discrepancies between our lensing-based models are due to oversimplified assumptions about LOSVD. We compare the velocity properties of the NFW, the Sa04 and A101 models of the previous section to the kinematical observations of Sa01.

3.1. From the overall potential to the velocity distribution

In the following r denotes the three-dimensional radial coordinate whereas R is the 2D projected radius. Usually, the Jeans equation is used to derive the gravitational potential Φ from the mean squared velocity of tracers v_r^2 , the density of tracers $\nu(r)$, which is here identified to the luminosity density. The Jeans equation reads (Binney & Tremaine 1987) :

$$\frac{1}{\nu} \frac{d(\nu \overline{v_r^2})}{dr} + 2 \frac{\beta \overline{v_r^2}}{r} = - \frac{d\Phi}{dr} = - \frac{GM_{\text{tot}}}{r^2}. \quad (9)$$

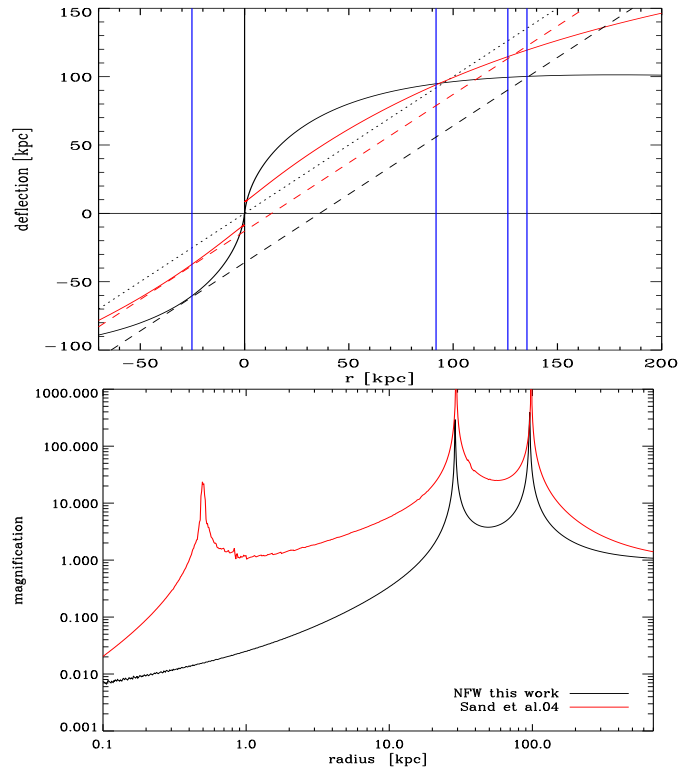


Fig. 4. Upper panel: Circularly averaged deflection angle for our best NFW fit (solid black curve) compared to that of Sa04 (solid red curve). From left to right, the blue vertical lines represent the radial and tangential critical radii, the Sa01 and our prediction of the radial arc counterimage. The critical radii match from one model to another but the radial length significantly differ. The dotted line represents the line $y = r$ and gives the solution of the tangential critical radius. The dashed lines $y = r + u$ (with u the source location) give the solution of the radial critical radius where it is tangent to the curve $y = r$. Lower panel: Magnification profile. The Sa04 model predicts high magnifications close to the center with another radial critical line at the very center ($r \sim 0.1''$).

However, the knowledge of the velocity tensor anisotropy $\beta(r) = 1 - \frac{v_r^2}{v_t^2}$ is necessary to infer the mass profile. The aim of this analysis is to derive the whole line-of-sight velocity distribution (LOSVD) of stars from the gravitational potential.

We assume that the distribution function DF $f(\mathbf{r}, \mathbf{v})$ can be modeled by Osipkov-Merritt (Osipkov 1979; Merritt 1985) distribution functions that depend on the reduced energy $\mathcal{E} = \Psi(r) - v^2/2$ and angular momentum

$L = rv \sin \zeta$ through the variable

$$Q = \mathcal{E} - \frac{L^2}{2r_a^2} = \Psi(r) - \frac{v^2}{2} \left(1 + \frac{r^2}{r_a^2} \sin^2 \zeta \right). \quad (10)$$

In these equations $\Psi(r) = \Phi(r_{\max}) - \Phi(r)$ is the reduced potential, $\mathcal{E} = \Psi(r) - v^2/2$ is the reduced energy and r_{\max} is the outermost radius at which a particle is bound to the system, *i.e.* satisfying $\mathcal{E} \geq 0$. Otherwise specified, we will set $r_{\max} = 2 h_{70}^{-1} \text{Mpc}$ in the following. ζ is the polar angle of the velocity direction with respect to \mathbf{r} . r_a is the anisotropy radius such that $\beta(r) = \frac{r^2}{r^2 + r_a^2}$. Orbits are nearly isotropic for $r < r_a$ and nearly radial above r_a . The isotropic case is recovered for $r_a \rightarrow \infty$.

The anisotropy profile of Osipkov-Merritt models has the generally observed shape in numerical simulations though recent analyses of N-body simulations propose a slower change between a completely isotropic to a mildly radial regime of the form $\beta(r) = \frac{1}{2} \frac{r}{r+r_a}$. See the discussion of Mamon & Lokas (2004) and references therein. However Osipkov-Merritt models have the appealing property that the distribution function $f(Q)$ can directly be calculated from the radial distribution ρ_* of tracers and Ψ , through the Eddington formula (Binney & Tremaine 1987)

$$f(Q) = \frac{1}{\sqrt{8\pi^2}} \left[\int_0^Q \frac{d^2 \tilde{\rho}_s}{d\Psi^2} \frac{d\Psi}{\sqrt{Q-\Psi}} + \frac{1}{\sqrt{Q}} \left(\frac{d\tilde{\rho}_s}{d\Psi} \right)_{\Psi=0} \right]. \quad (11)$$

where $\tilde{\rho}_s = (1 + \frac{r^2}{r_a^2})\rho_s$. Note that the potential includes the contribution of all the materials and satisfies: $\Delta\Psi = -4\pi G\rho$, where $\rho = \rho_{\text{DM}} + \rho_s$.

Once the Eq. (11) numerically integrated, it is possible to derive the LOSVD $p(R, v_{\parallel})$ as a function of the projected radius R by integrating over the line of sight coordinate z and over the perpendicular velocity v_{\perp} with $v^2 = v_{\parallel}^2 + v_{\perp}^2$.

$$p(R, v_{\parallel}) \propto \int_0^{z_m} dz \int_0^{v_{\perp, m}} v_{\perp} dv_{\perp} f(Q), \quad (12)$$

with z_m the maximum line-of-sight coordinate for a particle moving at velocity v located at the projected radius R and satisfying $\Psi(\sqrt{R^2 + z_m^2}) = v^2/2$. In the isotropic case, Eq. (12) can be simplified :

$$p_{\text{iso}}(R, v_{\parallel}) = 2\pi \int_{v_{\parallel}^2}^{2\Psi(R)} dv^2 \int_0^{z_m(v)} dz f(\mathcal{E}), \quad (13)$$

and a numerical integration is rather fast. However, in the general case, this is not possible and we present in the following a much faster method based on Monte-Carlo techniques.

3.2. Monte-Carlo Integration

The integration of Eq. (12) is done by randomly sampling the distribution function with a large number N of stars. Since the stellar density profile is known to a

scaling mass-to-light ratio, one can assign each star a radius r according to the cumulative Hernquist stellar mass profile $M_*(r) = M_* \left(\frac{r}{r+r_{s*}} \right)^2$. Each radius r_i can be projected onto the plane of sky yielding \mathbf{R}_i and z_i , the line-of-sight coordinate as before. At this point, it is trivial to incorporate observational conditions smearing like seeing by adding a random displacement $\mathbf{R}_i \rightarrow \mathbf{R}_i + \delta\mathbf{R}_i$ ⁴. Similarly, if the slit width Δ is negligible $R_i = |\mathbf{R}_i|$ can be identified to the position along the slit, otherwise, it is straightforward to split \mathbf{R}_i into (x_i, y_i) , only consider those points satisfying $2|y_i| \leq \Delta$ and then identify x_i as the position along the slit. This is the situation we will consider in the following.

This spatial sampling of the DF is thus independent of the potential Ψ nor on the anisotropy radius r_a and can be stored for further calculation. For a given $\Psi(r)$ and r_a , one must solve Eq. (11), assign a velocity v and a velocity orientation Ω using the calculated DF $f(Q)$. This is done with acceptance-rejection technics (*e.g.* Press et al. 1992). See also Kuijken & Dubinski (1994) or Kazantzidis et al. (2004) for similar applications of the method. We can write the conditional PDFs for the polar angle ζ and Q at radius r :

$$p(\zeta|r) = \frac{1}{2} \frac{\sin \zeta}{\left(1 + \frac{r^2}{r_a^2} \sin^2 \zeta \right)^{3/2}} \quad (14)$$

$$p(Q|r) \propto f(Q) \sqrt{2(\Psi(r) - Q)}.$$

Hence each star has a position x_i and a line-of-sight velocity $v_{\parallel, i}$. It is now possible to calculate $p(R, v_{\parallel})$ and the associated velocity dispersion $\sigma_{\text{los}}(R) = \sqrt{v_{\parallel}^2}$ and kurtosis $\kappa(R) = \frac{v_{\parallel}^4}{\sigma_{\text{los}}^4} - 3$, respectively related to the second and forth order moments of $p(R, v_{\parallel})$. Of course, the achieved accuracies for σ_{los} and κ depend on the number of stars N .

3.3. Application to MS2137

We now compute the LOSVD deduced from the best fit NFW model of lensing constraints only of Sect. 2.3 and compare the inferred velocity dispersion to the measurements of Sa04. These data were obtained by assuming Gaussian absorption lines profile due to Doppler broadening of stellar motion. van der Marel & Franx (1993) showed that departs from Gaussianity of the LOSVD imply a bias in any velocity dispersion measurement done assuming such a distribution. To the first order, the biased pseudo-velocity dispersion, that we will refer to as σ_{h_4} , can be expressed by :

$$\sigma_{\text{h}_4} = \frac{\sigma_{\text{los}}}{1 + \kappa/8}, \quad (15)$$

⁴ where $\delta\mathbf{R}$ may follow a 2D Gaussian distribution with standard deviation $\sigma_{\text{seeing}} = \text{FWHM}/2.35$.

showing that σ_{h_4} reduces to σ_{los} in the Gaussian case (*i.e.* when the kurtosis κ vanishes). Moreover, we consider observational conditions of Sa04, *i.e.* a slit width $\Delta = 1.25'' \simeq 5.8 h_{70}^{-1}$ kpc, and a gaussian seeing $0.6'' = 2.8 h_{70}^{-1}$ kpc FWHM.

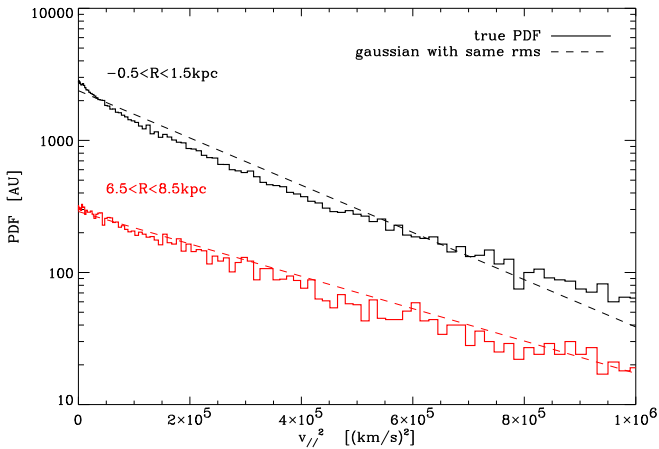


Fig. 5. Here we consider the the best fit NFW model of Sect. 2.3. Line-of-sight velocity probability distribution for stars with a projected radius in the innermost $-0.5 < R < 1.5 h_{70}^{-1}$ kpc (the upper black histogram) and outermost bins $6.5 < R < 8.5 h_{70}^{-1}$ kpc (the lower red histogram) of Sa04. In each case, we plot a gaussian distribution with the same dispersion for comparison. One can see that important non-gaussian tail for stars close to the center. In this example, orbits are assumed to be isotropic.

Fig. 5 shows the LOSVD as a function of the line-of-sight velocity $v_{||}$ for the innermost ($-0.5 < R < 1.5 h_{70}^{-1}$ kpc) and outermost ($6.5 < R < 8.5 h_{70}^{-1}$ kpc) radial bins of Sa04. Departs from gaussianity are clearly visible close to the center and decrease with increasing radius. Therefore, the velocity bias changes with projected radius as can be seen on the left panel of Fig. 6, in which we plot $\sigma_{\text{los}}(R)$ and $\sigma_{h_4}(R)$ for different values of the anisotropy radius $r_a = \infty$ and $r_a = 10 h_{70}^{-1}$ kpc. For isotropic velocity tensors, the bias is $(\sigma_{h_4} - \sigma_{\text{los}})/\sigma_{\text{los}} \sim -13\%$ at $R \sim 1$ kpc and then decreases whereas it can reach $\sim 30\%$ for anisotropic orbits. The agreement between the measurements of Sa04 and $\sigma_{h_4}(R)$ is better than with $\sigma_{\text{los}}(R)$ but introducing anisotropy cannot improve the fit quality for $R \simeq 10$ kpc: the NFW model σ_{h_4} curve raises too fast whereas measurements indicate a declining tendency. However, if kinematical data would extend to slightly larger scales, we expect the profile to start raising and get closer to the model beyond a few tens of kpc as observed in others cD galaxies (Dressler 1979; Kelson et al. 2002).

We attempted to couple lensing and kinematical constraints by to minimize the merit function $\chi_{\text{tot}}^2 = \chi_{\text{lens}}^2 + \chi_{\text{kin}}^2$, with

$$\chi_{\text{kin}}^2 = \sum_i^{N_{\text{bins}}} \frac{(\sigma_{h_4,i} - v_{\text{mes},i})^2}{\sigma_{\text{mes},i}^2}, \quad (16)$$

that accounts for kinematical constraints and χ_{lens}^2 defined in Eq. (5). $v_{\text{mes},i}$ and $\sigma_{\text{mes},i}$ are the measurements

and errors of Sa04, and $N_{\text{bins}} = 8$. The inferred NFW model is slightly changed as compared to the one found using lensing only. The NFW model is overconstrained by lensing and cannot fit the Sa04 data better. We find $\chi_{\text{kin}}^2/N_{\text{bins}} \sim 7$ for the best model⁵. There are no major changes in the best fit parameters that are still consistent with those of table 1. After χ^2 minimization, the NFW model is still unable to reproduce the velocity decline at $R \gtrsim 4$ kpc. We made the same exercise with the “gen” model which has more free parameters and found no significant changes when comparing to a pure lensing fit. More precisely, we found $\alpha = 1.250 \pm 0.011$ and $\Upsilon_V = 1.83 \pm 0.14$ which is a rather low value. In the case of model “gen” as well, there is no better agreement between lensing and stellar kinematics.

We have shown that departs from gaussian absorption lines induce a small bias (that starts being important for dynamical systems with radial orbits) but this latter is unable to explain the discrepancy between lensing and stellar kinematics. Furthermore, such a bias has no role in the discrepancy between lensing and X-rays mass estimates. We can see on the central and right panels of Fig. 6 that the mass model of Sa04 fairly reproduces kinematical data (it has been built to do so!), and that the X-rays A101 model provides a good fit to these data when adding the contribution of the central cD galaxy with $\Upsilon_V \sim 2.5$.

4. Discrepancies between mass estimates

At this level, let us resume the main problems that arise from the previous sections. A detailed lens modelling predicts a robust projected mass distribution that is consistent with NFW universal profiles. We have used a more general density profile for the dark matter halo in order to check that any other realistic mass profile should match our best fit NFW model over a broad range $20 \lesssim R \lesssim 1000$ kpc. This family of models turns out to be inconsistent with the X-rays and kinematical mass estimates that are basically indirect measurements of the 3D mass within radius r . These two latter estimates are mutually consistent for $R \lesssim 50$ kpc. Since lensing is sensitive to the integrated mass contrast along the line of sight, it is natural to expect overestimates due to fortuitous alignments with mass concentrations that are not physically related to the main halo of interest or that lie in its large scale neighborhood (like filaments). Such an hypothesis has been investigated by various authors in N-body simulations (Cen 1997; Reblinsky & Bartelmann 1999; Metzler et al. 2001; Clowe et al. 2004).

More likely is the possibility of departs from spherical symmetry. If dark matter halos are triaxial (or nearly oblate/prolate) as observed in N-body simulations (*eg* Jing & Suto 2002), the ratio of the mass enclosed in

⁵ We find $\chi_{\text{kin}}^2/N_{\text{bins}} \sim 8$ if we change σ_{h_4} by σ_{los} in Eq. 16, *ie* if we neglect the velocity bias due to non-gaussian LOSVD. Consequently, the correction has a weak effect on the modelling.

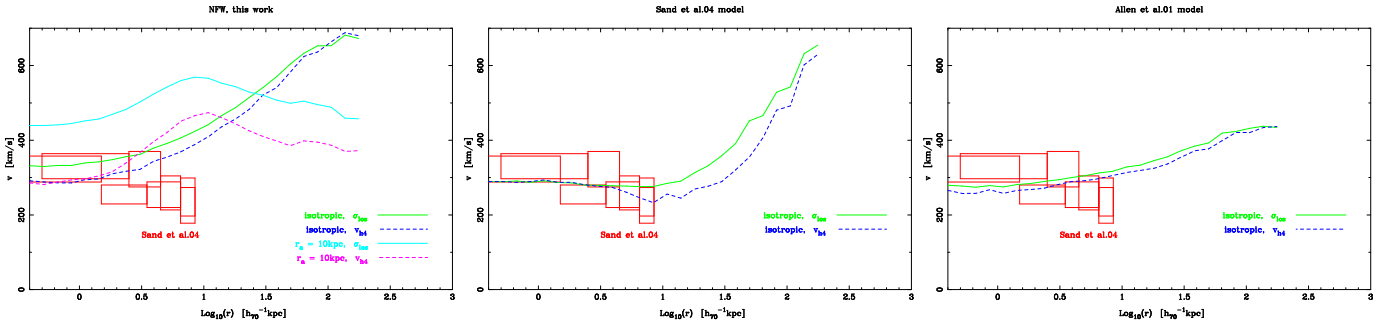


Fig. 6. Velocity dispersion $\sigma_{\text{los}}(R)$ (solid green line) and biased velocity $\sigma_{\text{h}_4}(R)$ (dashed blue line) profiles for isotropic orbits. The measurements of Sa04 are represented by red boxes. *Left:* Our best fit NFW model. In this panel we have also represented another couple of $(\sigma_{\text{los}}, \sigma_{\text{h}_4})$ curves (cyan and magenta respectively) which correspond to an anisotropic Osipkov-Merritt case with $r_a = 10$ kpc. The introduction of anisotropy does not improve the fit quality but leads to huge depart between σ_{los} and σ_{h_4} . *Center:* Best fit model of Sa04 which reproduces well the data. *Right:* Best fit model of A101, inferred from X-rays analysis. This model also matches the measurements of Sa01 whereas none of these latter profile are consistent with lensing observations.

the cylinder of radius R to the mass enclosed in the sphere of same radius will differ from that of a spherically symmetric situation. In order to illustrate projection effects, we consider an axisymmetric NFW density profile of the form :

$$\rho(m) = \frac{1}{m(1+m)^2} \quad , \text{ with } m^2 = R^2 + \frac{z^2}{q^2} . \quad (17)$$

The line of sight is along the z -axis and matches the major axis of a prolate halo when $q > 1$ or the minor axis of an oblate halo when $q < 1$. We can express $m \equiv r\lambda(q, \mu)$ with $\mu = \cos\theta$ and $\lambda^2 = 1 + \mu^2(1/q^2 - 1)$. Since we are interested in ratios between mass estimates we pay no attention to normalization constants and write the exact mass $M_{\text{true}}(r; q)$ enclosed by the sphere of radius r as

$$M_{\text{true}}(r; q) = \int_0^1 \frac{d\mu}{\lambda^3} M_{\text{true}}(\lambda r; 1) , \quad (18)$$

where $M_{\text{true}}(r; 1) = \ln(1+r) - \frac{r}{1+r}$ for a NFW profile. We now calculate the mass $M_{\text{lens}}(r; q)$ (resp. M_{dyn} , M_{X}) as it would be found from lensing (resp. stellar kinematics, X-rays) measurements performed assuming spherical symmetry.

4.1. Projection effect on lensing

Since lensing measures the projected mass along the line of sight and owing to the fact that the major/minor axis is aligned, the net effect of asphericity is to multiply the surface mass density by q . So, we can write

$$M_{\text{lens}}(r; q) = q M_{\text{true}}(r; 1) . \quad (19)$$

We plot the ratio $M_{\text{lens}}(r; q)/M_{\text{true}}(r; q)$ as a function of radius for various values of the axis ratio q on the left panel of Fig. 7. We can observe strong discrepancies for extreme values of the axis ratio. Departs between lensing and true masses tend to vanish at large scale. Thus, one expects a lower effect of asphericity on weak lensing based mass estimates.

4.2. Projection effect on kinematics

Regarding stellar kinematics, projection effects are much more complex because stars are expected to move faster along the major axis and enhance the mass estimate. Assuming a distribution function of stars of the form $f(E, L_z)$ and a reduced gravitational potential $\psi(R, z)$ the Euler/Jeans equations read :

$$\frac{1}{\nu} \partial_R (\nu \overline{v_R^2}) + \frac{\overline{v_R^2} - \overline{v_\phi^2}}{R} = \partial_R \psi \quad (20a)$$

$$\frac{1}{\nu} \partial_z (\nu \overline{v_z^2}) = \partial_z \psi . \quad (20b)$$

(Chandrasekhar 1960; Hunter 1977; Binney & Tremaine 1987). Thus, we can express the components of the velocity ellipsoid:

$$\nu \overline{v_R^2} = \nu \overline{v_z^2} = - \int_z^\infty dz' \nu \partial_z \psi \quad (21a)$$

$$\nu R \Omega^2 = - \int_z^\infty dz' [\partial_R \nu \partial_z \psi - \partial_z \nu \partial_R \psi] . \quad (21b)$$

with $R^2 \Omega^2 = \overline{v_\phi^2} - \overline{v_R^2}$. The main difficulty is to compute the potential and its derivatives generated by an ellipsoidal distribution of mass $\rho(m)$. To do so we use the formalism of Chandrasekhar (1969) (see also *eg* Qian et al. 1995) :

$$\psi(R, z) - \psi_0 = - 2\pi G q \int_0^\infty \frac{du}{\Delta(u)} \int_U^\infty dm m \rho(m) \quad (22a)$$

$$\partial_R \psi = - 2\pi G q R \int_0^\infty \frac{du}{\Delta(u)(1+u)} \rho(U) \quad (22b)$$

$$\partial_z \psi = - 2\pi G q z \int_0^\infty \frac{du}{\Delta(u)(q^2+u)} \rho(U) , \quad (22c)$$

with ψ_0 the central potential (which is not relevant for our purpose), $\Delta(u) = (1+u)\sqrt{q^2+u}$ and $U^2 = \frac{R^2}{1+u} + \frac{z^2}{q^2+u}$. For simplicity we assume that the density of tracers $\nu(R, z)$ does not contribute to the potential (massless). We also assume that the density of tracers is ellipsoidal $\nu(R, z) = \nu(m)$ with the same axis ratio as the dark halo.

We now calculate the observable luminosity-weighted line-of-light velocity dispersion

$$I\sigma_{\text{los}}(R) = \int_{-\infty}^{\infty} dz \nu(R, z) \overline{v_z^2}. \quad (23)$$

If this quantity is assumed to be due to a spherically symmetric system and is deprojected according to

$$\nu(r) = -\frac{1}{\pi} \int_r^{\infty} \frac{dI(R)}{dR} \frac{dR}{\sqrt{R^2 - r^2}} \quad (24a)$$

$$\overline{v_r^2}(r) = -\frac{1}{\pi} \int_r^{\infty} \frac{dI\sigma_{\text{los}}(R)}{dR} \frac{dR}{\sqrt{R^2 - r^2}}, \quad (24b)$$

one will calculate a biased radial velocity dispersion $\overline{v_r^2}$. The corresponding biased mass profile $M_{\text{kin}}(r)$ is given by the standard Jeans equation (with isotropic velocity tensor):

$$M_{\text{kin}}(r) = -\frac{\overline{v_r^2} r}{G} \frac{d \ln \overline{v_r^2}}{d \ln r}. \quad (25)$$

We plot the ratio $M_{\text{kin}}(r; q)/M_{\text{true}}(r; q)$ as a function of radius for various values of the axis ratio q on the central panel of Fig. 7. Here again projection effects can be huge for extreme values of q . Unfortunately, $M_{\text{kin}}(r; q)$ depends on the profile of tracers $\nu(r)$ and the details of this figure cannot be representative of a general oblate/prolate NFW halo. For instance the bump at $r \lesssim 1$ is due to our assumed density of tracers which here corresponds to our model of MS2137. However, departs between $M_{\text{kin}}(r; q)$ and M_{true} are always important for high q (or $1/q$).

4.3. Projection effect on X-rays

Similarly, we calculate the perturbation of asphericity on X-rays mass estimates. As compared to lensing or dynamics the effect is expected to be weaker since the gravitational potential is systematically rounder than the mass. For simplicity we assume that the gas (with density ρ_g) is isothermal and in hydrostatic equilibrium.

We have $\nabla \ln \rho_g = \nabla \psi / V_0^2$, with $V_0^2 = kT / \mu m_p$ and $\ln \rho_g \propto \exp[-\psi / V_0^2]$. The X-rays surface brightness of the optically thin gas distribution is

$$S_X(R) \propto \int \exp[-2\psi(R, z) / V_0^2] dz, \quad (26)$$

with $\psi(R, z)$ given by (22a). Here again, when interpreting this surface brightness distribution as arising from a spherically symmetric system, one will deproject $S_X(R)$, obtain a biased gas density ρ_g and use it in the following equation to obtain the biased mass profile $M_X(r)$.

$$M_X(r) = -\frac{V_0^2 r}{G} \frac{d \ln \rho_g}{d \ln r}. \quad (27)$$

We plot the ratio $M_X(r; q)/M_{\text{true}}(r; q)$ as a function of radius for various values of the axis ratio q on the right panel of Fig. 7. As expected $M_X(r; q)/M_{\text{true}}(r; q)$ exhibits much less scatter about unity as compared to the two previous mass estimates. The asymptotic divergence at very large scales ($r \gtrsim 10$) is a numerical artefact of our crude deprojection algorithm.

4.4. Comments and application to MS2137

Fig. 7 clearly shows that moderate values of the axis ratio q can lead to strong discrepancies between 2D and 3D mass estimates or between lensing and X-rays or stellar kinematics.

It is difficult to fully characterize the ratio $M_{\text{lens}}(r; q)/M_{\text{kin}}(r; q)$ because it depends on the distribution of tracers $\nu(R, z)$ and is severely sensitive to the orientation of the axis ratio relative to the line of sight. Moreover we did not take non gaussianity of LOSVD into account. Sect. 3 clearly showed that this effect may be important and projection may boost this trend. Therefore a direct comparison between lensing and dynamical mass estimates is hazardous. $M_{\text{lens}}(r; q)/M_{\text{kin}}(r; q)$ can have a different radial behavior as a function of radius for a given axis ratio. It can be either greater or less than unity. Reasonable values of q can lead to important departs from unity in $M_{\text{lens}}(r; q)/M_{\text{kin}}(r; q)$.

Comparing lensing and X-rays mass estimates is easier since the X-rays mass estimate turns out to be less sensitive to asphericity and projection effects. In this respect $M_{\text{lens}}(r; q)/M_X(r; q)$ will systematically be > 1 (resp. < 1) for prolate (resp. oblate) halos with a well known radial behavior. We do not expect important changes when the halo is not aligned with the line of sight.

For the MS2137 purpose, a prolate halo with $q \sim 0.3 - 0.5$ could explain most discrepancies between our best fit models and the results of Sa04 and A101. A prolate halo aligned toward the line of sight is a natural explanation for the high concentration parameter we found $c = 11.73 \pm 0.55$ and may also explain the high values of Broadhurst et al. (2005b) : $c = 13.7_{-1.1}^{+1.4}$ in A1689 and Kneib et al. (2003) : $c \approx 22$ in CL0024 (see also Clowe et al. 2004).

At this level, it is not possible to simply refine the modelling of MS2137, since our prolate model is idealized. It should be triaxial and/or not perfectly aligned with the line of sight because the projected density profile is elliptical. However the hypothesis of a projected triaxial halo also provides a direct explanation of the misalignment between the projected diffuse stellar component of the cD and the projected dark matter halo $\Delta\text{PA} = 13.0 \pm 0.5$ deg (G03). Binney (1985) and Romanowsky & Kochanek (1998) give the necessary formalism to infer the position angle and projected ellipticity of both dark and luminous halos from their tridimensional triaxial shape and orientation, provided their principal axes are aligned (see also Buote et al. 2002, for a similar application). The information that can be derived from the geometry of projected light and dark matter densities is detailed in appendix A. Basically, these independent constraints give the following results for the orientation θ (polar angle of the major axis with respect to the line of sight), the minor axis ratios c_{DM} and c_* of dark matter and stellar components respectively: $\theta = 27.4 \pm 5.1$, $c_{\text{DM}} = 0.55 \pm 0.08$, and $c_* = 0.52 \pm 0.12$. It is remarkable that this geometrical information is fairly consistent with the value of $c_{\text{DM}} \sim 0.3$

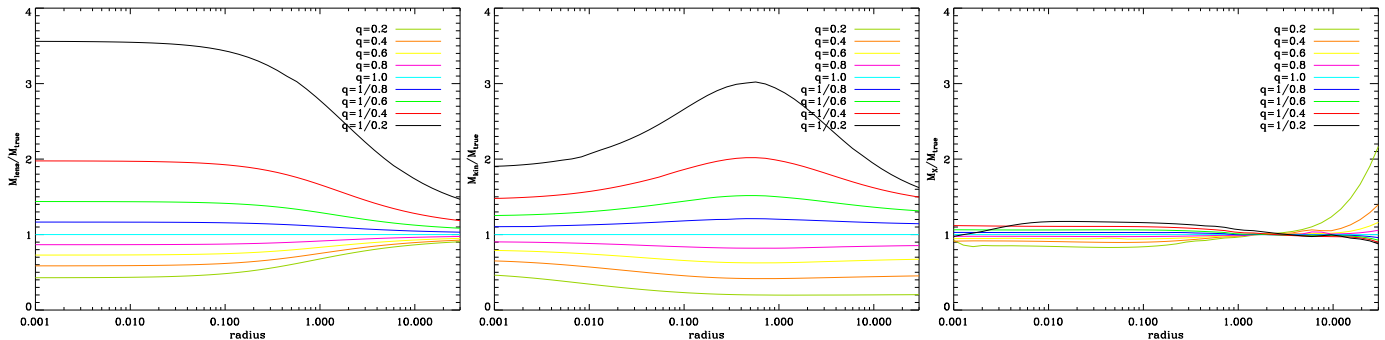


Fig. 7. Radial behavior of ratio between mass estimates for various values of the axis ratio $q = 0.2, 0.4, 0.6, 0.8, 1, 1/0.8, 1/0.6, 1/0.4, 1/0.2$. *Left Panel:* Ratio $M_{\text{lens}}(r; q)/M_{\text{true}}(r; q)$. *Middle Panel:* Ratio $M_{\text{kin}}(r; q)/M_{\text{true}}(r; q)$. *Right Panel:* Ratio $M_X(r; q)/M_{\text{true}}(r; q)$. In this latter case, one can see much lower values whatever the axis ratio. The asymptotic behavior is slightly affected by numerical instability for $r \gtrsim 10$ and extreme values of q .

and a perfect alignment ($\theta = 0$) we assumed to explain the mass discrepancies.

There is sufficient material to be convinced that no simple coupling between 2D and 3D mass estimates is possible. Consequently, we expect that most of the previous analyses based on such a coupling should be considered with caution, either in terms of significance or in terms of possibly biased results. The aim of the next section is to predict the statistics of such mass discrepancies between lensing and any other mass estimate which is not much sensitive to asphericity effect like X-rays.

5. Discrepancies : an expected general trend

Let us now consider a more general situation with a triaxial halo $\rho(m)$ where $m^2 = x^2 + y^2/b^2 + z^2/c^2$ with $0 < b < c \leq 1$ and an orientation relative to the line of sight parameterized by the polar angles (θ, ϕ) or the unit vector \mathbf{n} .

The mass within the sphere of radius r is independent on the halo orientation and reads

$$M_{\text{true}}(r; b, c) = \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^\pi \sin\vartheta d\vartheta \frac{M_3(\nu r; 1, 1)}{\nu^3}, \quad (28)$$

with $\nu^2 = \sin^2\vartheta (\sin^2\varphi/b^2 + \cos^2\varphi/c^2) + \cos^2\vartheta$ and again $M_{\text{true}}(R; 1, 1)$ is simply the mass within radius r for a spherically symmetric halo.

The mass M_2 within cylinder of radius r will depend on their axis ratios and the orientation \mathbf{n} but the system is equivalent to an elliptical projected mass distribution with axis ratio q and position angle ψ . Thus we can express M_2 as:

$$M_2(r; b, c, \mathbf{n}) = \frac{\tilde{q}_x^2}{\sqrt{f}} \int_0^{2\pi} \frac{d\varphi}{2\pi} \frac{M_2(\tau r/\tilde{q}_x, 1)}{\tau^2} \quad (29)$$

where $\tau^2 = \sin^2\varphi + \cos^2\varphi/\tilde{q}^2$ and \tilde{q} , \tilde{q}_x and f are given by Eqs. (A.7) in the appendix. They depend on the intrinsic axis ratios and orientation. $M_2(r; 1, 1)$ is simply the cylindrical mass within radius r for a spherically symmetric halo.

As before, an observer measuring the 3D mass profile within radius r will find a different normalization as compared to an observer interested in the cylindrical mass of

radius r . They will differ by a factor

$$\eta(R; b, c, \mathbf{n}) = \frac{M_2(r; b, c, \mathbf{n})/M_2(r; 1, 1)}{M_{\text{true}}(r; b, c)/M_{\text{true}}(r; 1, 1)}. \quad (30)$$

We can now calculate the statistical properties of this ratio by averaging over the b and c PDFs of Jing & Suto (2002) given by (A.8) and the orientation of the major axis (assumed isotropic). This can be expressed as :

$$p_R(\eta) = \frac{1}{4\pi} \int_0^1 dc p(c) \int_0^1 db p(b|c) \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \delta[\eta - \eta(R; c, b, \mathbf{n})]. \quad (31)$$

We plot on the upper panel of Fig. 8 the distribution of η for three fiducial values of $R = 0.03, 0.3$ and 3 which are relevant for strong lensing/stellar dynamics, strong lensing/X-rays and weak lensing/X-rays comparisons respectively. We clearly see a broad, shifted and skewed distribution that converges toward unity with increasing radius. However, at small scales, the median value of η is not unity and generously extends toward high values $\eta \gtrsim 1.5$ with typically : $\eta = 1.24_{-0.18}^{+0.23}$ (resp. $1.19_{-0.14}^{+0.17}$, 1.07 ± 0.06) for $R = 0.03$ (resp. $0.3, 3.0$). Thus, important departs between M_2 and M_{true} are naturally expected if halos are effectively triaxial.

Moreover, there must be a correlation between the observed projected axis ratio q and η since the apparently rounder halos are likely to be elongated along the line of sight. This effect can be seen on Fig. 9 where we plot the conditional PDFs $p(\eta)$, $p(\eta|q > 0.9)$, $p(\eta|q > 0.7)$ and $p(\eta|q < 0.7)$ for a radius $R = 0.3$. The highest values of η are due to the roundest projected halos. For instance, given $q > 0.7$ we have $\eta = 1.25_{-0.13}^{+0.17}$.

Finally projection effects of triaxial halos have the appealing properties to explain the fact that weak lensing and X-rays measurements generally match since $\eta \sim 1$ for $r \gtrsim$ a few r_s . Likewise, the general trend that strong lensing mass estimates are generally greater (by a factor of $1 - 3$) than X-rays since they occur at scales $0.1 \lesssim r \lesssim 1$ (Allen et al. 2001; Wu 2000). The relative normalization between lensing and stellar kinematics is more complex

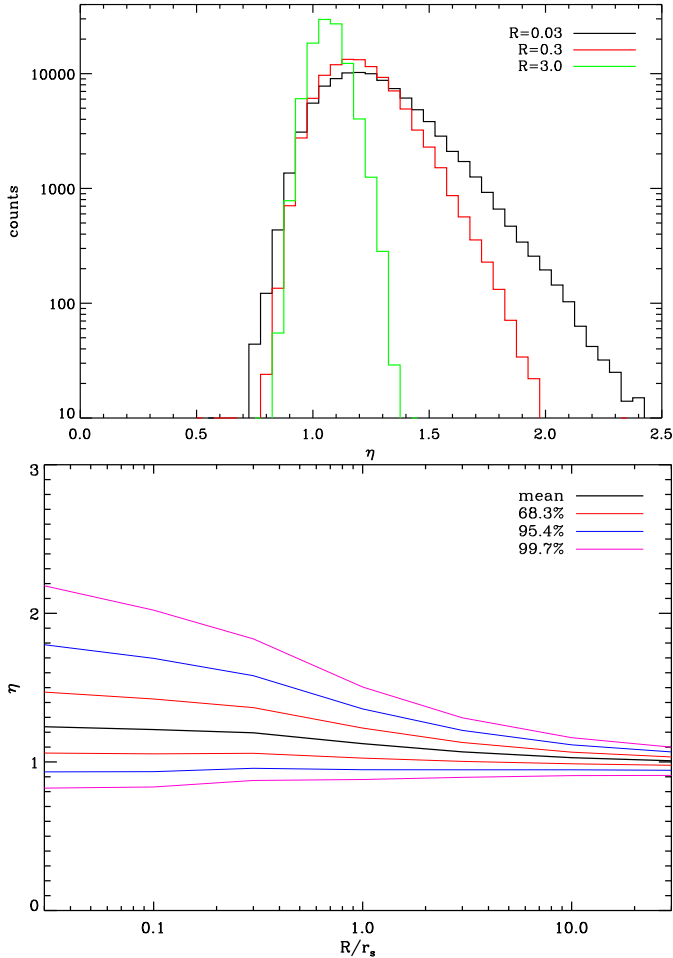


Fig. 8. *Upper panel* : Distribution for the mass ratio η for three different radii $r = 0.03$ (the broader black curve), $r = 0.3$ (the intermediate red curve) and $r = 3.0$ (the narrower green curve). At small scales, the distribution is broad and clearly not centered on $\eta = 1$, leading to unreliable direct normalization between 2D and 3D mass estimates like in strong lensing and X-rays/stellar dynamics comparisons. With increasing radius ($r \gtrsim 3.0$), the distribution significantly narrows and explains the overall agreement between large scale weak lensing and X-rays mass estimates. *Lower panel* : Mean (thick black curve) and 68.3, 95.4 and 99.7 % quantiles (thin red, blue and magenta curves respectively) of the η distribution as a function of radius.

and cannot be represented by the η statistic. However we expect a similar scatter and a strong dependence on the major axis orientation.

6. Discussion & Conclusion

Regarding the particular case of MS2137, using a detailed modelling of both strong and weak lensing data, we have shown that the dark matter density profile must be close to NFW. See also (Miralda-Escudé 1995; Bartelmann 1996; Gavazzi et al. 2003; Bartelmann & Meneghetti 2004; Dalal & Keeton 2003) for similar conclusions. We have explained the reason why the Sa04 lens model is inconsistent with lensing data (radial arc counter image, weak lensing...). We have highlighted a possible discrepancy between our lens model and other mass estimates from stellar kinematics in the central cD galaxy and X-rays.

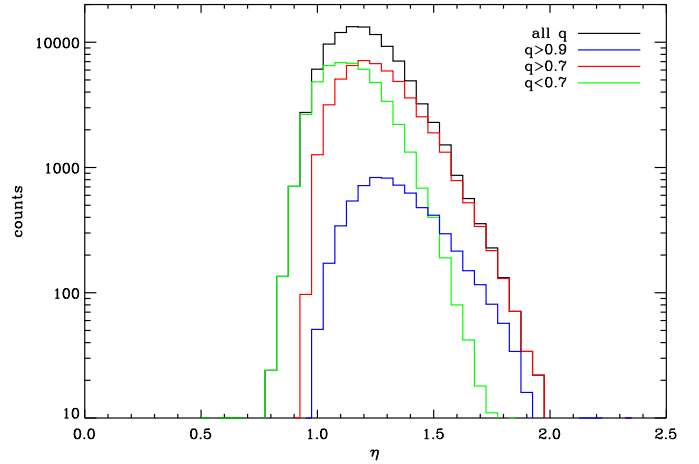


Fig. 9. For $r = 0.3$, we show the dependence of η on the projected axis ratio q . The conditional PDFs $p(\eta|q > 0.9)$ (blue) $p(\eta|q > 0.7)$ (red) and $p(\eta|q < 0.7)$ (green) are compared to the overall distribution $p(\eta)$ (black). The rounder the halo is, the more likely is the major axis of the halo oriented toward the line of sight and the more important is η .

We have undertaken a thorough dynamical analysis of the line-of-sight velocity distribution of stars in the cD in order to check whether or not departs from gaussianity may explain the relative inconsistency between our lens models and stellar kinematics. The effect of non-gaussian absorption lines is to slightly lower ($\sim 15\%$) the measured velocity dispersion estimates of Sa04 but does not greatly improve the fit quality of our lens model to the bias-corrected data. Moreover such a bias cannot explain the disagreement between our lens model and Al01 X-rays mass estimates. This latter 3D mass estimates turns out to be consistent with stellar dynamics, showing that there must exist some problem in the relative normalization of 2D and 3D mass estimates.

These discrepancies can be alleviated if one considers the possibility of departs from spherical symmetry for the dark matter and stellar components. More precisely, we have shown that a prolate halo with its major axis oriented close to the line of sight and an axis ratio ~ 0.4 is likely to explain the discrepancies. This hypothesis is supported by the misalignment ($\Delta\psi \sim 13^\circ$) between projected DM and stellar distributions (see appendix A). Furthermore such a geometrical configuration well explains the concentration parameter we infer from lensing analysis $c = 11.73 \pm 0.55$. A preferred elongated halo toward the LOS boosts lensing efficiency and may explain the high concentration of some strong lensing clusters (Kneib et al. 2003; Broadhurst et al. 2005a,b). Studying the lensing properties of simulated clusters, Clowe et al. (2004) also point out this general trend.

We have shown that triaxiality is a general problem that hampers any attempt to simply couple 2D and 3D mass estimates assuming spherical symmetry. Once projected, triaxial halos are elliptical and lens modelling is able to take ellipticity into account. Usually dynamical or X-rays analyses do not fully incorporate such a complexity. This should be an important work to do before compari-

son to (or coupling with) lensing. However, if we assume that spherically symmetric 3D analyses yield a reasonable estimate of the 3D mass⁶, we still expect discrepancies between 2D and 3D mass estimates that simply arise from projection effects. In Sect. 5, we have assumed the statistical distribution of axis ratios proposed by Jing & Suto (2002) in order to calculate the mass $M_2(r)$ within cylinder of radius r and the mass $M_{\text{true}}(r)$ within the sphere of same radius. The difference is important and can lead to a significant discrepancy in the relative normalization between 2D and 3D mass estimates.

The statistic of $\eta(r)$ shows that, at small scales $R < 1$, in average a systematic depart from unity is expected for η with an important scatter and skewness toward high values of η . Therefore the relative normalization at small scales is biased and highly uncertain if one neglects projection effects. At larger scales, the distribution of η converges to unity and explains why weak lensing mass estimates are generally in better agreement with X-rays or dynamics of galaxies in clusters (eg Allen 1998; Wu 2000; Arabadjis et al. 2004). Similarly, the coupling between stellar kinematics and strong lensing at clusters scales (Sand et al. 2002, 2004) or at galaxies scales (eg Koopmans & Treu 2002; Treu & Koopmans 2004; Rusin et al. 2003) may be oversimplistic since they do not take asphericity into account. First, the mean value $\eta \sim 1.2$ for $r \lesssim 0.3r_s$ leads to a expected systematic bias, but also the $\sim 20\%$ scatter in the distribution of η will increase the uncertainty in the mass normalization and prevent the appealing temptation to couple these independent mass estimates.

In conclusion, the density profile of the dark matter halo of MS2137-23 is well consistent with NFW and previous claimed discrepancies may be due to the spherical symmetry assumption. Indeed, it turns out that when coupling lensing to other mass estimates we cannot avoid a detailed (and cumbersome) 3D triaxial modelling of X-rays and dynamical properties. It is worth noticing that such a level of refinement is already achieved in lensing studies that assume elliptical symmetry. The triaxiality of dark matter halos (and stellar components) is a major concern for joint modelling and should systematically be taken into account for future analyses.

A less pessimistic conclusion is that the triaxiality of dark matter halos, as confirmed in this analysis at scales of a few kiloparsecs, is inconsistent with the hypothesis of self-interacting dark matter (Miralda-Escudé 2002).

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⁶ this is true for X-rays but difficulty controlable for dynamics.

Appendix A: Further evidence for triaxiality

In the section we follow the formalism of Binney (1985) and Romanowsky & Kochanek (1998) and calculate the orientation and axis ratio of a projected triaxial mass (or light) distribution as a function of its intrinsic 3D axis ratios $0 \leq c \leq b \leq 1$ such that the density $\rho(\mathbf{r}) = \rho(m)$ with $m^2 = x^2 + y^2/b^2 + z^2/c^2$. We express the orientation of the minor axis with the polar angles (θ, ϕ) relative to the line of sight. The projected density reads :

$$\Sigma(x, y) = \frac{2}{\sqrt{f}} \int_0^\infty \rho(u^2 + m^2) du, \quad (\text{A.1})$$

where

$$f = \sin^2 \theta \left(\cos^2 \phi + \frac{\sin^2 \phi}{b^2} \right) + \frac{\cos^2 \theta}{c^2}, \quad (\text{A.2})$$

$$m^2 = \frac{1}{f} (Ax^2 + Bxy + Cy^2), \quad (\text{A.3})$$

$$A = \frac{\cos^2 \theta}{c^2} \left(\sin^2 \phi + \frac{\cos^2 \phi}{b^2} \right) + \frac{\sin^2 \theta}{b^2}, \quad (\text{A.4})$$

$$B = \cos \theta \sin 2\phi (1 - 1/b^2) \frac{1}{c^2}, \quad (\text{A.5})$$

$$C = \left(\frac{\sin^2 \phi}{b^2} + \cos^2 \phi \right) \frac{1}{c^2}. \quad (\text{A.6})$$

The projected distribution is elliptical with an axis ratio \tilde{q} and a position angle ψ given by :

$$\tilde{q}_{x/y}^2 = \frac{2f}{A + C \mp \sqrt{B^2 + (A - C)^2}}, \quad (\text{A.7a})$$

$$\tilde{q} = \tilde{q}_y / \tilde{q}_x, \quad (\text{A.7b})$$

$$\tan 2\psi = \frac{B}{A - C}. \quad (\text{A.7c})$$

These equations are verified by the dark matter and the stellar components which have their own axis ratios c_{DM} , b_{DM} , c_* and b_* but their principal axes are assumed to match. In general different values of c_i and b_i lead to different values of \tilde{q}_i and ψ_i . This is what we observe in MS2137 where the light satisfies $\tilde{q}_* = 0.83 \pm 0.12$, $\psi_* = (18 \pm 1)$ deg and our NFW lens modelling yields $\tilde{q}_{\text{DM}} = 0.750 \pm 0.005$ and $\psi_{\text{DM}} = (4.90 \pm 0.15)$ deg. Hence we can define infer the parameters $\theta, \phi, c_{\text{DM}}, b_{\text{DM}}, c_*, b_*$ from these constraints and some additional priors since the problem is underconstrained. We can use the axis ratio distribution found in cosmological simulations by Jing & Suto (2002):

$$p(c) = \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{(c - \bar{c})^2}{2\sigma_c^2}\right), \quad (\text{A.8a})$$

$$p(b|c) = \frac{3}{2(1 - \max(c, 1/2))} \left[1 - \left(\frac{2b - 1 - \max(c, 1/2)}{1 - \max(c, 1/2)} \right)^2 \right], \quad (\text{A.8b})$$

with $\sigma_c \sim 0.113$ and $\bar{c} \sim 0.54$. However, since the number of constraints is not sufficient we force the intermediate

axis ratios b_* and b_{DM} to equal the most probable value. In other word, we have :

$$p(b|c) = \delta\left(b - \frac{1 + \max(1/2, c)}{2}\right). \quad (\text{A.9})$$

The best fit with priors yields : $\theta = 27.4 \pm 5.1$, $c_{\text{DM}} = 0.55 \pm 0.08$ and $c_* = 0.52 \pm 0.12$. This analysis gives strong indications on the reliability of triaxial dark matter and stellar distribution with the major axis relatively close to the line-of-sight and a value of c_{DM} close to that inferred to explain the discrepancy between 2D and 3D mass estimates in Sect. 4.1.