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An electronic Mach-Zehnder interferometer in the Fractional Quantum Hall effect

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We compute the interference pattern of a Mach-Zehnder interferometer operating in the fractional quantum Hall effect. Our theoretical proposal is inspired by a remarkable experiment on edge states in the Integer Quantum Hall effect (IQHE) [1]. The Luttinger liquid model is solved via two independent methods: refermionization at $\nu = 1/2$ and the Bethe Ansatz solution available for Laughlin fractions. The current differs strongly from that of single electrons in the strong backscattering regime. The Fano factor is periodic in the flux, and it exhibits a sharp transition from sub-Poissonian (charge $e/2$) to Poissonian (charge e) in the neighborhood of destructive interferences.

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One of the most fascinating aspects of mesoscopic physics lies in the possibility of building analogs of optical devices with the help of nanostructures. In many situations both phenomena can be understood with the same language [2]. However, photons propagate in vacuum and therefore interact weakly, except during their generation/detection processes. On the opposite, interactions between electrons are manifest in one dimensional systems as well as in quantum dots. Here we want to inquire how electronic interactions affect the interference pattern of a classic optical device analog, a Mach-Zehnder (MZ) interferometer [3].

Recently, such an analog was achieved with edge states of the integral quantum Hall effect (IQHE) [1]. Interference visibilities as high as $\sim 60\%$ were observed. Edge states of the IQHE can be understood in principle at the single electron level, but at higher magnetic fields electronic interactions are explicit in the fractional quantum Hall effect (FQHE). The latter offers the opportunity to investigate the unique properties of interacting electrons in one dimension, such as fractional charge [4][5] and fractional statistics [6]. Interferometry in the FQHE was previously studied with regard to fractional charge detection [7] using perturbation theory. Here we report on MZ interferometry using exact models: refermionization at $\nu = 1/2$ [8] and the Bethe Ansatz solution [9]. In the strong backscattering regime, the interference pattern displays a dramatic effect of the interactions: the signal is not sinusoidal, and its amplitude at the output departs from the single electron expectations.

The MZ setup [1] is depicted in Fig. 1 (a): an edge state is injected at voltage V_0 , meets a first tunable quantum point contact (QPC) where it is either transmitted or reflected. The two resultant states combine at a second QPC, giving two outgoing edge states 1 and 2. A magnetic field B threads the surface S enclosed by the 2 edges between the 2 QPC, leading to an Aharonov-Bohm (AB)

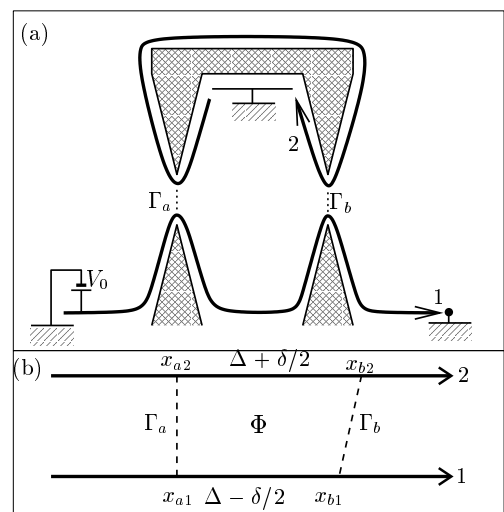


FIG. 1: (a) Mach Zehnder geometry in the quantum Hall effect: counter-propagating edge states at a QPC are made to meet again at a second QPC. (b) Edge state configuration equivalent to (a). Γ_a (Γ_b) is the tunneling amplitude at the first (second) QPC, at $x = x_{a1}$ and $x = x_{a2}$ ($x = x_{b1}$ and $x = x_{b2}$) for edge state 1 (2). The mean distance between the two QPC is Δ and the path difference is δ . Φ is the AB phase due to the magnetic flux.

flux and to a corresponding phase $\Phi = SB/\Phi_0^*$, where $\Phi_0^* = hc/e^*$ is the flux quantum for excitations with fractional charge $e^* = \nu e$ (ν is the filling factor). This setup is topologically equivalent to the one of Fig. 1 (b), 2 chiral states propagating in the same direction meeting successively two QPC. This geometry is thus different from the simple Hall bar described in [7] which allows for multiple round trips in the confined region.

We wish to calculate the outgoing current in edge 1 and 2 for arbitrary tunneling amplitudes Γ_a, Γ_b – thus non-perturbatively – as a function of the AB phase Φ ,

the applied voltage $V_0 = \hbar\omega_0/e^*$, the mean distance Δ between the two QPC and the path difference δ . The AB phase is a key ingredient to the problem, as it modulates the interferences between the two paths from the first to the second QPC. It is included in the tunneling amplitudes Γ_a and Γ_b . As this phase is present only for cross terms with one tunneling event at QPC 1 and another at QPC 2, this amounts to multiplying the $\Gamma_a\Gamma_b^*$ terms by the phase $e^{i\Phi}$. The bosonized Hamiltonian reads (as in [8], $\hbar = e = v_F = 1$, except in important results):

$$H = H_{\phi_1}^0 + H_{\phi_2}^0 + \sum_{q=a,b} \left(\Gamma_q e^{-i\omega_0 t} e^{\frac{i}{\sqrt{m}}(\phi_1(x_q,t) - \phi_2(x_q,t))} + \text{h.c.} \right). \quad (1)$$

The first two terms in H are the free Hamiltonians for edge states 1 and 2. The next two are the tunneling terms through the two QPC. We consider the case of equal distances between the two QPC: the first (second) QPC is located at x_a (x_b) for both edge states. Unequal distances tend to reduce the interferences, but do not change the results qualitatively.

At $\nu = 1/2$, it is natural to introduce new bosonic fields $\phi_{\pm}(x) = (1/\sqrt{2})(\phi_1(x) \pm \phi_2(x))$. The tunneling operators in Eq. (1) can be represented by a fermionic field $\eta(x) = e^{i\phi_{-}(x)}$. The Hamiltonian contains a trivial free part for ϕ_{+} , and a non-trivial part for ϕ_{-} . It is then possible to obtain a Hamiltonian which is quadratic in fermionic variables, provided that new fermionics fields are introduced such that $\psi(x,t) = \eta(x,t)f$, where f is a Majorana fermion ($f = C + C^{\dagger}$ and $\{C, C^{\dagger}\} = 1$):

$$H_{-} = \int dx \left[\psi^{\dagger}(x) (-i\partial_x - \omega_0) \psi(x) + \sum_{q=a,b} \sqrt{2\pi} \delta(x - x_q) (\Gamma_q \psi(x) f + \Gamma_q^* f \psi^{\dagger}(x)) \right]. \quad (2)$$

The $\psi^{\dagger}(x)(-\omega_0)\psi(x)$ term comes from a unitary transform made to get rid of the $e^{-i\omega_0 t}$ terms of Eq. (1) (this amounts to redefine the chemical potential). From this Hamiltonian, it is clear that $\psi(x)$ is propagating in ballistic manner, except at positions $x = x_a, x_b$ where it gets scattered. The Heisenberg equations for $\psi(x,t)$ are solved by introducing plane wave solutions: $\psi(x,t) = \sum_{\omega} u_{\omega} e^{i\omega_0 x} e^{i\omega(x-t)}$, with coefficients $u_{\omega} = A_{\omega}$ (C_{ω}) for the incoming (outgoing) field at the left (right) of the two QPC. The boundary conditions at the QPCs give:

$$C_{\omega} = D^{-1} \left[\left(i\omega - 4\pi\tilde{\Gamma}_a\tilde{\Gamma}_b^* 2i \sin(\omega\Delta) \right) A_{\omega} - 4\pi \left(\left(\tilde{\Gamma}_a^* \right)^2 + \left(\tilde{\Gamma}_b^* \right)^2 + 2\tilde{\Gamma}_a^*\tilde{\Gamma}_b^* \cos(\omega\Delta) \right) A_{-\omega}^{\dagger} \right], \quad (3)$$

with: $D = i\omega - 4\pi[|\Gamma_a|^2 + |\Gamma_b|^2 + (\tilde{\Gamma}_a\tilde{\Gamma}_b^* + \tilde{\Gamma}_a^*\tilde{\Gamma}_b)e^{i\omega\Delta}]$ and the tunneling amplitudes are redefined as $\tilde{\Gamma}_{a,b} =$

$\Gamma_{a,b} e^{i\omega_0 x_{a,b}} e^{\pm i\Phi/2}$. Equation (3) can be seen as the solution of a scattering problem. Writing $C_{\omega} = r_{\omega} A_{\omega} + t_{\omega} A_{-\omega}^{\dagger}$, with reflection (r_{ω}) and transmission (t_{ω}) coefficients, one can check that the flux is conserved ($|r_{\omega}|^2 + |t_{\omega}|^2 = 1$).

From the solution Eq. (3), we can proceed to the calculation of the current I_2 outgoing in edge state 2:

$$I_2 = \frac{e}{4\pi} \int_{-\omega_0}^{\omega_0} d\omega |t_{\omega}|^2. \quad (4)$$

The outgoing current in edge state 1 is simply $I_1 = e\omega_0/(2\pi) - I_2$, where $e\omega_0/(2\pi) = \nu e^2 V_0/h$ is the incoming Hall current. It is convenient to introduce the geometric mean modulus amplitude $\Gamma = \sqrt{|\Gamma_a||\Gamma_b|}$. The deviation from equal amplitudes is described with the parameter λ ($|\Gamma_a| = \lambda\Gamma$, $|\Gamma_b| = (1/\lambda)\Gamma$). The transmission becomes:

$$\begin{aligned} |t_{\omega}|^2 &= N(u)/D(u), \quad u = \omega/(4\pi\Gamma^2), \\ N(u) &= [(\lambda^2 + \lambda^{-2}) \cos(\omega_0\Delta + \Phi) + 2 \cos(4\pi\Gamma^2 u \Delta)]^2 \\ &\quad + [(\lambda^2 - \lambda^{-2}) \sin(\omega_0\Delta + \Phi)]^2, \\ D(u) &= [u - 2 \cos(\omega_0\Delta + \Phi) \sin(4\pi\Gamma^2 u \Delta)]^2 \\ &\quad + [(\lambda^2 + \lambda^{-2}) + 2 \cos(\omega_0\Delta + \Phi) \cos(4\pi\Gamma^2 u \Delta)]^2. \end{aligned} \quad (5)$$

The relevant regime for observing interference fringes is either weak pinchoff ($\Gamma \rightarrow 0$) or when $(\omega_0\Delta/v_F) < 1$ at strong pinchoff. This corresponds to the case of ‘‘coherent coupling’’ between the two QPC. At strong pinchoff, in the opposite regime, $(\omega_0\Delta/v_F) \gg 1$, the above integral gives $e\omega_0/(4\pi)$, and thus $I_1 = I_2 = e\omega_0/(4\pi)$, where all interferences are lost. For $(\omega_0\Delta/v_F) < 1$, the integral gives:

$$\begin{aligned} I_2 &\simeq (2\Gamma^2) \frac{\frac{1}{2}(\lambda^2 + 1/\lambda^2) + \cos(\omega_0\Delta + \Phi)}{1/2 - 4\pi\Gamma^2\Delta \cos(\omega_0\Delta + \Phi)} \\ &\times \tan^{-1} \left(\omega_0 \frac{\frac{1}{2} - 4\pi\Gamma^2\Delta \cos(\omega_0\Delta + \Phi)}{4\pi\Gamma^2 (\frac{1}{2}(\lambda^2 + 1/\lambda^2) + \cos(\omega_0\Delta + \Phi))} \right). \end{aligned} \quad (6)$$

Comparing this to the transmitted current for one QPC only, with tunneling amplitude Γ :

$$I_2|_{1QPC} = 2\Gamma^2 \tan^{-1} \left(\frac{\omega_0}{4\pi\Gamma^2} \right), \quad (7)$$

we see that the current in Eq. (6) can be expressed as the current for a single QPC, with an effective tunnel amplitude Γ_{eff} :

$$4\pi\Gamma_{eff}^2 = \frac{4\pi \left(\frac{1}{2}(|\Gamma_a|^2 + |\Gamma_b|^2) + |\Gamma_a||\Gamma_b| \cos \Phi \right)}{1/2 - 4\pi|\Gamma_a||\Gamma_b| \Delta \cos \Phi}. \quad (8)$$

This is a central result: as far as the current is concerned, the MZ setup behaves, for fractionally charged excitations, as a single QPC with an effective amplitude Γ_{eff} which is modulated by the AB phase. Technically speaking, the difference between this behavior and

the one of non-interacting electrons (as observed experimentally in the IQHE [1]) can be traced back to the Hamiltonian. In the FQHE, the fermionic Hamiltonian of Eq. (2) couples $\psi(x, t)$ to the auxiliary fermion f for both scattering events at x_a and at x_b : these two scatterings are thus strongly linked. On the other hand, in the IQHE, the tunneling part of the Hamiltonian $H_T = \int dx \sum_{q=a,b} \delta(x-x_q) \left(\Gamma_q \psi_1(x) \psi_2^\dagger(x) + \text{h.c.} \right)$ couples $\psi_1(x, t)$ to $\psi_2(x, t)$ at the same location, and each QPC is described independently by a scattering matrix. This different behavior leads to dramatically different re-

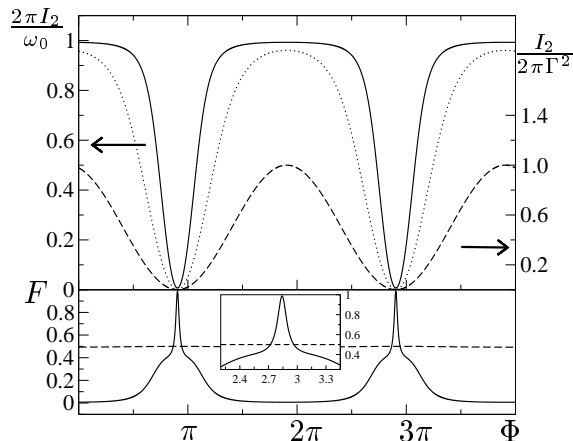


FIG. 2: Upper part: transmitted current I_2 as a function of the AB phase Φ , for 2 QPC with $\nu = 1/2$ and equal tunneling amplitudes Γ , with $\omega_0\Delta/v_F = 0.3$, and $8\pi\Gamma^2/\omega_0 = 0.01$ (dashed curve, right y axis), 1 (dotted curve, left axis), 100 (full curve, left axis). Lower part: Fano factor $F = S_2/(2eI_2)$ as a function of Φ for the same parameters, $8\pi\Gamma^2/\omega_0 = 100$ (full curve), 0.01 (dashed curve, extremely narrow peaks have been removed for the sake of visibility). Inset: zoom on one of the narrow peaks of F for $8\pi\Gamma^2/\omega_0 = 100$.

sults for the interferences in the transmitted current I_2 as a function of the AB phase Φ . Consider for simplicity the case where $|\Gamma_a| = |\Gamma_b| = \Gamma$. Indeed, $\lambda = 1$ merely ensures a maximum visibility for the interferences. In the IQHE, one has:

$$I_2|_{IQHE} = \omega_0 T(1-T)(1 + \cos(\omega_0\Delta + \Phi)) , \quad (9)$$

where $T = \Gamma^2/(1 + \Gamma^2/4)^2$ is the transmission of each QPC, and Φ is here SB/Φ_0 ($\Phi_0 = hc/e$). It shows that the maximum transmitted current in edge 2 is obtained for $T = 1/2$, while it goes to 0 for $T \rightarrow 0$ ($\Gamma \rightarrow 0$) or $T \rightarrow 1$ ($\Gamma \rightarrow 2$, which is the strong coupling limit in this case). For all values of T , I_2 shows sinusoidal oscillations as a function of Φ . Considering now the results for fractionally charged excitations, Eqs. (6) and (8), one can distinguish two different regimes. First, the tunneling regime, corresponding to $\Gamma \rightarrow 0$. We have then $\Gamma_{eff}^2 = 2\Gamma^2(1 + \cos(\omega_0\Delta + \Phi)) \ll 1$, and we recover results similar to the non-interacting case in the tunneling limit: sinusoidal oscillations of the current I_2 as a function of

the AB phase Φ , with $I_2 \sim \Gamma^2 \ll 1$. This is easily understood: when one keeps only the lowest order in Γ , the coupling between the two scattering events disappears and the results obtained for non-interacting electrons are recovered. The opposite limit is obtained when $\Gamma \rightarrow \infty$. As shown on Fig. 2, the current I_2 is nearly constant, with the value $e\omega_0/(2\pi)$, except near $\Phi = (2n+1)\pi$ where it shows narrow dips going to zero. For very large Γ , the width of the dips scales as $\sqrt{\omega_0\Delta/v_F} < 1$. This means that for large Γ , all the incoming current gets transmitted to edge 2, except for special values of the AB phase Φ where destructive interference happens. Note that this is totally different from the non interacting electron case where the incoming current in edge 1 gets scattered to edge 2 at the first QPC, then gets mostly scattered back in edge 1 at the second QPC. Because of electronic correlations, this picture is not valid in the FQHE, and the two QPCs must be considered as a whole.

The noise also has unique features. Its analytic expression is identical to that of non-interacting electrons:

$$S_2 = \frac{e^2}{2\pi} \int_{-\omega_0}^{\omega_0} d\omega |t_\omega|^2 (1 - |t_\omega|^2) , \quad (10)$$

a mere consequence of the fact that the transmission in 2 is described by a scattering process for the reformed field. Here, however, the energy dependence of $|t_\omega|^2$ reflects the electronic correlations. The Fano factor $F \equiv S_2/(2eI_2)$ is shown on the lower part of Fig. 2 for the two regimes discussed above. In the tunneling regime, $F \simeq 1/2 = \nu$: the small current outgoing in edge 2 is carried by quasiparticles of charge νe , and these can either tunnel at the first or at the second QPC. For arbitrary coupling, F is a periodic function of flux: in the regime of strong coupling, the lowering of the Fano factor is due to the factor $1 - |t_\omega|^2$ in Eq. (10), when the current is close to its maximal value. When destructive interference occurs (near $\Phi = (2p+1)\pi$, p integer), I_2 is suppressed and a peculiar behavior appears. The global shape of the Fano factor suggests a value of $1/2$ (sub-Poissonian) in this region, although backscattering is strong. In the close vicinity of $\Phi = (2p+1)\pi$ there is a sharp peak, and the Fano factor reaches 1 (Fig. 2). For AB phases corresponding to this narrow peak, the noise is Poissonian, and the current is carried by pairs of quasiparticles of charge νe , here electrons. This peak is in fact present for any value of Γ , but its width decreases with Γ which makes it invisible in the small Γ limit. For the large Γ regime, and with $\omega_0\Delta/v_F \simeq 0.3$, this peak could be seen if currents of a few percent of the incoming Hall current can be measured experimentally. All of the above results are robust up to $\omega_0\Delta/v_F \simeq 1$: beyond this value, the visibility of the current oscillations decreases rapidly and the ‘‘Poissonian’’ peak of the Fano factor is reduced.

So far, we worked with a filling factor $\nu = 1/2$.

Yet the underlying chiral Luttinger liquid description is valid only for simple Laughlin fractions of the FQHE $\nu = 1/(2p+1)$. It is therefore important to check whether these results are not specific to this approach, and that they could be observed with the experimentally accessible filling factors such as $\nu = 1/3$. In fact, the results for $\nu = 1/2$ suggest that it is possible to obtain non-perturbative results for arbitrary values of ν : the 2 QPC behave as one effective QPC (Eq. (8)), and there exists a general (non-perturbative) solution [9] for the single QPC. To this aim, we start with an imaginary time action formalism and for simplicity we consider the case $\Gamma_a = \Gamma_b = \Gamma$. Without loss of generality, in Luttinger liquid problems, all quadratic terms of the action can be integrated out: the only terms which survive are the terms associated with the tunneling locations $x = x_a$ and $x = x_b$. Following Ref.[10], we introduce the fields:

$$\begin{aligned}\bar{\phi}(\omega) &= \frac{1}{\sqrt{2}} (\phi_-(x_a, \omega) + \phi_-(x_b, \omega)) , \\ \tilde{\phi}(\omega) &= \frac{1}{\sqrt{2}} (\phi_-(x_a, \omega) - \phi_-(x_b, \omega)) .\end{aligned}\quad (11)$$

These fields are the only degrees of freedom which are left after integration of the quadratic part of the action associated with the Hamiltonian of Eq. (1). Assuming $\omega_0 \Delta / v_F < 1$, the effective action reads:

$$\begin{aligned}\mathcal{S} &= \frac{1}{2\pi\nu\beta} \left(\sum_{\omega} \frac{v_F}{\Delta} |\tilde{\phi}(\omega)|^2 + \frac{|\omega|}{2} |\bar{\phi}(\omega)|^2 \right) \\ &+ 4\Gamma \int_0^{\hbar\beta} d\tau \cos(\bar{\phi}(\tau) + \Phi/2) \cos(\tilde{\phi}(\tau) - \Phi/2) .\end{aligned}\quad (12)$$

where $\beta = 1/(k_B T)$. Note that in Eq. (12), the field $\tilde{\phi}(\omega)$ is massive. For $\Delta < \hbar v_F / \Gamma$ we can neglect its fluctuations, so that the field $\tilde{\phi}(\omega)$ is pinned to zero. The leading corrections to this approximation are computed elsewhere [11]. One is then left with the field $\bar{\phi}$ only. Shifting this field by $\Phi/2$, we get a new action:

$$\mathcal{S} = \frac{1}{4\pi\nu\hbar\beta} \sum_{\omega} |\omega| |\phi(\omega)|^2 + 4\Gamma \cos(\Phi/2) \int_0^{\hbar\beta} d\tau \cos(\phi(\tau)) .\quad (13)$$

This action is identical to the zero-mass limit of the Sine-Gordon model and the problem can be solved exactly as done in [9]. The transmitted current I_2 follows the scaling law

$$I_2 = \frac{\nu}{2\pi} \left(c_0 \Gamma \cos\left(\frac{\Phi}{2}\right) \right)^{\frac{1}{1-\nu}} \mathcal{F} \left(\frac{\omega_0}{\nu (c_0 \Gamma \cos(\frac{\Phi}{2}))^{\frac{1}{1-\nu}}} \right) ,\quad (14)$$

with $c_0 = 4\sqrt{2\pi}$. \mathcal{F} is the scaling function, with $\mathcal{F}(x) \sim x^{2\nu-1}$ for $x \gg 1$ and $\mathcal{F}(x) = x$ for $x \ll 1$. This proves that the 2 QPCs behave as 1 QPC with an effective coupling $\nu (4\sqrt{2\pi}\Gamma \cos(\Phi/2))$ which is modulated by the

AB phase. The results previously obtained for $\nu = 1/2$ can therefore be extended to describe Laughlin fractions, such as $\nu = 1/3$. For $\nu = 1/2$, $\mathcal{F}(x)$ is simply $\tan^{-1}(x)$, and the effective coupling is $\Gamma_{eff}^2 = 8\pi\Gamma^2(1+\cos\Phi)$. This is in agreement with Eq. (8), since by neglecting the massive field $\tilde{\phi}$ we have supposed that $\Delta \rightarrow 0$. The results for the current I_2 when $\nu = 1/3$ (not shown) are in precise correspondence with those obtained for $\nu = 1/2$.

To conclude, we have provided the first non-perturbative treatment of the electronic analog of an optical interferometer operating with strongly correlated fermions. The most dramatic effect occurs when both QPCs are close to pinchoff (large Γ), where the whole current exits in edge 2 (except at special values of the AB flux), contrarily to the case of the IQHE and classical optics. This departure from the single particle behavior is due to the statistics of the quasiparticles which tunnel from 1 to 2 and back. We predict a Fano factor which is periodic in the AB phase. At strong pinchoff, the noise undergoes a sharp transition from sub-Poissonian to Poissonian in a narrow AB phase range around the destructive AB interferences. Our predictions could be tested experimentally with the same ‘‘air bridge’’ setup as in Ref. [1], using higher magnetic field. With $v_F \simeq 10^5$ m/s [12], the important condition $\omega_0 \Delta / v_F \leq 1$ can be reached with state of the art techniques: temperature of a few tens of mK, Δ a few μm and V_0 a few tens of μV .

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