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DECOMPOSITION METHOD FOR CAMASSA-HOLM EQUATION

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ABSTRACT. The Adomian decomposition method is applied to the Camassa-Holm equation with smooth initial data, providing an explicit solution for comparison with other existing procedures.

Key Words: Camassa-Holm equation, Adomian decomposition method.

1. THE ADM FOR CAMASSA-HOLM EQUATION

The Camassa-Holm equation [5] for real $u(x, t)$

$$(1) \quad u_t - \frac{1}{4}u_{xxt} + \frac{3}{2}(u^2)_x - \frac{1}{8}(u_x^2)_x - \frac{1}{4}(uu_{xx})_x = 0$$

is written as

$$(2) \quad L_t \left(u - \frac{1}{4}u_{xx} \right) = L_x \left(-\frac{3}{2}(u^2)_x + \frac{1}{8}u_x^2 + \frac{1}{4}uu_{xx} \right)$$

where $L_t = \frac{\partial}{\partial t}$ and $L_x = \frac{\partial}{\partial x}$. Then $L_x^{-1}(\cdot) = \int_0^x (\cdot) dx$ and $L_t^{-1}(\cdot) = \int_0^t (\cdot) dt$. If we operating the two sides of (2) with L_t^{-1} , we have

$$(3) \quad u = u(x, 0) - \frac{1}{4}u_{xx}(x, 0) + \frac{1}{4}u_{xx} + L_t^{-1}L_x \left(-\frac{3}{2}(u^2)_x + \frac{1}{8}u_x^2 + \frac{1}{4}uu_{xx} \right)$$

By decomposition Adomian [1], $u_0 = u(x, 0) - \frac{1}{4}u_{xx}(x, 0)$, $u = \sum_{n=0}^{\infty} u_n$ and the nonlinear terms are written in terms of the A_n polynomials (see Adomian [1] for details)

$$(4) \quad u = u_0 + L_t^{-1}L_x \left(-\frac{3}{2} \sum_{n=0}^{\infty} A_n \{(u^2)_x\} + \frac{1}{8} \sum_{n=0}^{\infty} A_n \{u_x^2\} + \frac{1}{4} \sum_{n=0}^{\infty} A_n \{uu_{xx}\} \right) + \frac{1}{4} \sum_{n=0}^{\infty} u_{nxx}.$$

where $u_{nxx} = \frac{\partial^2 u_n}{\partial x^2}$.

Since the $u = \sum_{n=0}^{\infty} u_n$, we can write

$$(5) \quad u_1 = L_t^{-1}L_x \left(-\frac{3}{2}A_0 \{(u^2)_x\} + \frac{1}{8}A_0 \{u_x^2\} + \frac{1}{4}A_0 \{uu_{xx}\} \right) + \frac{1}{4}u_{0xx}$$

$$(6) \quad u_2 = L_t^{-1}L_x \left(-\frac{3}{2}A_1 \{(u^2)_x\} + \frac{1}{8}A_1 \{u_x^2\} + \frac{1}{4}A_1 \{uu_{xx}\} \right) + \frac{1}{4}u_{1xx},$$

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and recursively we have that

$$(7) \quad u_{n+1} = L_t^{-1} L_x \left(-\frac{3}{2} A_n \{(u^2)_x\} + \frac{1}{8} A_n \{u_x^2\} + \frac{1}{4} A_n \{uu_{xx}\} \right) + \frac{1}{4} u_{nxx}.$$

Once the A_n for u_x^2 , $(u^2)_x$ and for uu_{xx} are generated and u_0 is specified, components can be calculated to some m-term approximation

$$(8) \quad \phi_m[u] = \sum_{n=0}^{m-1} u_n,$$

which converges to u (See [1] for more details).

When we consider a problem of determining the solution of the functional equation

$$(9) \quad u - w = NL(u) + L(u),$$

where the unknown u belongs to some appropriate functional space S , w is a given element of S , NL is a nonlinear operator from X onto itself, and L is a linear operator from X into X . According to the ADM, we seek a solution of Eq. (9) in the form $u = \sum_{n=0}^{\infty} u_n$, then we approximate the nonlinear operator F as

$$(10) \quad NL(u) = \sum_{n=0}^{\infty} A_n.$$

The functions A_n are the so-called Adomian's polynomials, and a general algorithms for the A_n is given by

$$(11) \quad A_n = \frac{1}{n!} \left[\frac{d}{d\lambda^n} NL(u_\lambda) \right]_{\lambda=0}, \quad n = 0, 1, 2, \dots$$

where we write

$$(12) \quad u_\lambda = \sum_{i=0}^{\infty} \lambda^i u_i.$$

Note that $u_1 = u$.

In our case $w = 0$, $L = \frac{1}{4} \frac{\partial^2}{\partial x^2}$ and the algorithms for the A_n are given by

$$\begin{aligned} A_0 \{u_x^2\} &= u_{0x}^2 \\ A_1 \{u_x^2\} &= 2 \frac{\partial}{\partial x} u_0 u_1 \\ A_2 \{u_x^2\} &= \frac{\partial}{\partial x} u_1^2 + 2 \frac{\partial}{\partial x} u_0 u_2 \\ &\vdots \end{aligned}$$

and

$$\begin{aligned} A_0 \{uu_{xx}\} &= u_0 u_{0xx} \\ A_1 \{uu_{xx}\} &= u_0 u_{1xx} + u_1 u_{0xx} \\ A_2 \{uu_{xx}\} &= u_0 u_{2xx} + u_1 u_{1xx} + u_2 u_{0xx} \\ &\vdots \end{aligned}$$

and

$$\begin{aligned} A_0 \{(u^2)_x\} &= (u_0^2)_x \\ A_1 \{(u^2)_x\} &= 2(u_0 u_1)_x \\ A_2 \{(u^2)_x\} &= (u_2 u_0)_x \\ &\vdots \end{aligned}$$

can now calculate the u_n . The solution is $u = \sum_{n=0}^{\infty} u_n$ which is approximated by

$$(13) \quad \phi_m[u] = \sum_{n=0}^{m-1} u_n,$$

in an m -term rapidly convergent series so that m is very small. This has been investigated in numerous works (see [4], [2], [3] for more details.).

Remark 1.1. *It straightforward to see that the preceded procedure will be applicable to the following general equation:*

$$(14) \quad u_t + a u_{xxt} + b (u^2)_x + c (u_x^2)_x + d (uu_{xx})_x = \gamma(x, t)$$

where a, b, c, d are reals and constants and the function γ is sufficiently smooth.

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