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Electron injection in a nanotube with leads: finite frequency noise-correlations and anomalous charges

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The non-equilibrium transport properties of a carbon nanotube which is connected to Fermi liquid leads, where electrons are injected in the bulk, are computed. A previous work which considered an infinite nanotube showed that the zero frequency noise correlations, measured at opposite ends of the nanotube, could be used to extract the anomalous charges of the chiral excitations which propagate in the nanotube. Here, the presence of the leads have the effect that such-noise cross-correlations vanish at zero frequency. Nevertheless, information concerning the anomalous charges can be recovered when considering the spectral density of noise correlations at finite frequencies, which is computed perturbatively in the tunneling amplitude. The spectrum of the noise cross-correlations is shown to depend crucially on the ratio of the time of flight of quasiparticles traveling in the nanotube to the "voltage" time which defines the width of the quasiparticle wave-packets injected when an electron tunnels. Potential applications toward the measurement of such anomalous charges in non-chiral Luttinger liquids (nanotubes or semiconductor quantum wires) are discussed.

PACS numbers:

I. INTRODUCTION

Over the years nano-electronics has been dealing mostly with transport through artificial nano-structures, made from semiconductor or from metallic material. Nowadays, attention has also focused on individual nano objects connected to leads: molecules, conjugated polymers, carbon nanotubes among others. Carbon nanotubes are especially interesting because, depending on their helicity, they constitute a near ideal one-dimensional metal[1]. Indeed, the dispersion of metallic nanotubes at the Fermi level is linear, and one can therefore expect that electron correlations play a dominant role[2]. Evidence for Luttinger liquid behavior has been seen in tunneling experiments[3]. A non-linear current-voltage characteristic[4] was predicted for both tunneling in the bulk of the nanotube and to the end of a nanotube. The tunneling density of states exponent found in these experiments suggest that the Luttinger liquid interaction parameter describing the total charge modes $K_{c+} \simeq 0.3$, which should be compared to the non-interacting value $K_{c+} = 1$: the effect of interactions is strong. A nanotube therefore "does not like" to accommodate electrons, because its elementary excitations do not resemble electrons as in a Fermi Liquid (FL): they consist of the collective bosonic excitations which are known to occur in one-dimensional correlated electron systems[5].

In chiral Luttinger liquids, which are believed to describe the physics of the fractional quantum Hall edge states, a measurement of the backscattering noise is sufficient to identify the fractional charge[6] of the quasiparticle. No straightforward analogy exists for a quantum wire or for a nanotube. Recent work have shown[7,8] that although left and right moving electrons mix in such Non-Chiral Luttinger Liquids (NCLL), their elementary excitations can be decomposed in right and left moving chiral bosonic modes which carry a non integer electron charge, where the latter depends on the interaction parameter identified below as K_{c+}.

Recently a theoretical suggestion to measure these anomalous charges using both current noise auto and cross-correlations was proposed[9]. Contrary to the fractional quantum Hall effect, an autocorrelation noise measurement alone is not sufficient to isolate the anomalous charge of a NCLL. In Ref. 9, it was assumed that electrons were injected into the bulk of the nanotube (Fig. 1), while current was being measured at both ends. The electron is then split into two chiral quasi-particle modes which move in opposite directions. Each pair of modes carries either of two anomalous charges $Q_\pm = (1 \pm K_{c+})/2$ attached to the right/left movers. Yet because an electron is injected locally, it has an equal probability to have $Q_+(Q_-)$ on the right or on the left (Fig. 2): the wave function describing the injection of an electron has entangled quasi-particle degree of freedom, with quantum numbers, or states, specified by the anomalous charges $Q_\pm$.

Nevertheless, one drawback is that injection was studied in an infinite nanotube. In the last ten years, it has been argued that when a Luttinger liquid is connected to normal metal leads, one looses the possibility for detecting anomalous charges in NCLL. One of such models consists of a Luttinger liquid with an interaction parameter which varies as a function of distance: $K_{c+} = 1$ in the leads while $K_{c+} < 1$ in the Luttinger liquid (Fig. 3). The purpose of the present work is threefold: first, the results for an infinite nanotube at zero frequencies will be extended to the finite frequency domain. Second, it will be shown explicitly that the zero frequency noise cross correlations vanish when one includes the non interacting leads described above. This result thus suggests that anomalous charges cannot be

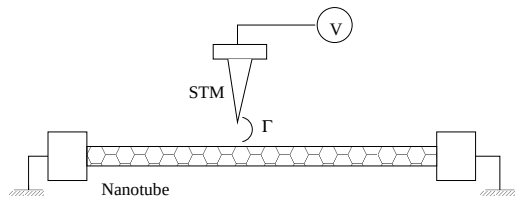

FIG. 1: Nanotube connected to leads, with electrons injected in the bulk via a STM

detected by the above correlation method. Third, are the good news: an analysis of finite frequency autocorrelation noise and noise cross-correlations – in the presence of Fermi liquid leads – allows to recover crucial information about the anomalous charges. The time of flight $\tau_{\rm L} = L/2v_{c+}$ for excitations in the nanotube and the voltage time scale $\tau_{\rm V} = \hbar/eV$ are the sole parameters which specify the behavior of the noise cross-correlations as a function of frequency.

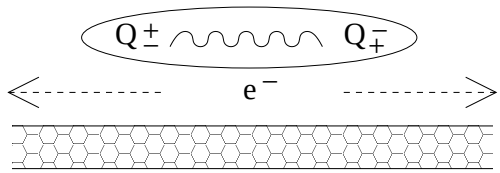

FIG. 2: Entanglement of quasiparticle excitations in an infinite Luttinger liquid.

II. MODEL HAMILTONIAN

Consider the following experimental setup: a nanotube connected to a two FL leads. An STM tip is put in contact with the bulk nanotube at $x = 0$. A bias voltage V is applied between the STM tip and the nanotube allowing electron transfer. Subsequently, quasi-particle excitations propagate along the nanotube towards the FL leads. Following Ref. 10, the electron field $\Psi_{r\alpha\sigma}(x,t)$ in the nanotube, describing the electron moving along the direction $r = \pm 1$, from the mode $\alpha = \pm 1$ with a spin $\sigma = \pm 1$, can be written in terms of the bosonic field $\varphi_{r\alpha\sigma}(x,t)$ as

$$\Psi_{r\alpha\sigma}(x,t) = \frac{F_{r\alpha\sigma}}{\sqrt{2\pi a}}\, e^{ik_{\rm F} rx + iq_{\rm F}\alpha x + i\varphi_{r\alpha\sigma}(x,t)}, \tag{1}$$

where a is an ultraviolet cutoff of the Luttinger Liquid (LL) model, $F_{r\alpha\sigma}$ are the Klein factors, $k_{\rm F}$ the Fermi momentum and $q_{\rm F}$ is the momentum mismatch associated with the two modes. For further calculation purposes it is convenient to rewrite the bosonic field $\varphi_{r\alpha\sigma}(x,t)$ in terms of the non-chiral bosonic fields $\theta_{j\delta}$ and $\phi_{j\delta}$, with index $j\delta \in \{c+, c-, s+, s-\}$ identifying the charge/spin, total/relative fields:

$$\varphi_{r\alpha\sigma}(x,t) = \frac{\sqrt{\pi}}{2}\sum_{j\delta} h_{\alpha\sigma j\delta}\left[\phi_{j\delta}(x,t) + r\theta_{j\delta}(x,t)\right], \tag{2}$$

with factors $h_{\alpha\sigma c+} = 1$, $h_{\alpha\sigma c-} = \alpha$, $h_{\alpha\sigma s+} = \sigma$ and $h_{\alpha\sigma s-} = \alpha\sigma$, and bosonic fields obeying the equal time commutation relations $[\phi_{j\delta}(x), \theta_{j'\delta'}(x')] = -(i/2)\delta_{jj'}\delta_{\delta\delta'}\, sgn(x - x')$. The Hamiltonian describing the nanotube connected to FL leads assumes the form:

$$\begin{aligned} H \;=\; \frac{1}{2}\sum_{j\delta}\int_{-\infty}^{\infty} dx\; v_{j\delta}(x)\Big[K_{j\delta}(x)(\partial_x\phi_{j\delta})^2 \\ +K_{j\delta}^{-1}(x)(\partial_x\theta_{j\delta})^2\Big], \end{aligned} \tag{3}$$

where the interaction parameters $K_{j\delta}(x)$ is now assumed to depend on position and the velocities $v_{j\delta}(x)$ satisfy $v_{j\delta}(x) = v_{\rm F}/K_{j\delta}(x)$.

The electrons in STM tip are assumed to be non-interacting. For convenience[9], the electron field $c_\sigma(t)$ in the STM tip can be described in terms of a semi-infinite LL with interaction parameter equal to 1:

$$c_\sigma(t) = \frac{f_\sigma}{\sqrt{2\pi a}}\, e^{i\tilde{\varphi}_\sigma(t)}, \tag{4}$$

where $\tilde{\varphi}_\sigma(t)$ is the chiral bosonic field, whose Keldysh Green's function at $x = 0$ is given by[11]:

$$\begin{aligned} g^{\sigma}_{\eta\mu}(t_1 - t_2) &= \Big\langle T_{\rm K}\big\{\tilde{\varphi}_\sigma(t_1^\eta)\tilde{\varphi}_\sigma(t_2^\mu)\big\}\Big\rangle \\ &= -\ln\left[1 + i(\eta{+}\mu)\frac{v_{\rm F}|t_1{-}t_2|}{2a} - i(\eta{-}\mu)\frac{v_{\rm F}(t_1{-}t_2)}{2a}\right], \end{aligned} \tag{5}$$

where $\eta, \mu = \pm 1$ denotes the upper/lower branch of the Keldysh contour.

The tunneling Hamiltonian describing the electron tunneling from STM tip to the nanotube has a standard form:

$$H_{\rm T}(t) = \sum_{r\alpha\sigma\epsilon} \Gamma^{(\epsilon)}(t)\big[\Psi^\dagger_{r\alpha\sigma}(0,t)c_\sigma(t)\big]^{(\epsilon)}. \tag{6}$$

The voltage is taken into account via a time dependence of the tunneling amplitude (Peierls substitution) $\Gamma(t) = \Gamma\, e^{i\omega_0 t}$ where $\omega_0 = eV/\hbar$ is the voltage frequency. The superscript (ϵ) leaves either operator unchanged $\epsilon = +$, or transforms it into its Hermitian conjugate $\epsilon = -$.

The total electric current in the nanotube and FL leads can be expressed through the bosonic field ϕ_{c+}:

$$\hat{I}(x,t) = 2ev_{\rm F}\,\frac{\partial_x \phi_{c+}(x,t)}{\sqrt{\pi}}. \tag{7}$$

In Ref. 9, the nanotube current was computed in terms of the bosonic Green's functions, and was found to have the expected non-linear behavior with voltage. The calculation of this current was indeed necessary there, because the diagnosis of anomalous charges in the infinite nanotube required to compute two ratios: the ratio between the auto-correlation and the nanotube current, as well as the ratio between the cross-correlation and the nanotube current. In the present finite frequency scheme which is used to extract anomalous charges, the current in both branches of the nanotube – which is constant because of the stationary bias, and which equals half of the tunneling current in a symmetric setup – is simply not needed. Below, we thus compute the noise auto-correlation and the noise cross-correlations.

III. NANOTUBE NOISE

A. Unsymmetrized correlator

The main quantity of our interest will be the nanotube current-current correlator $S_{xx'}(t,t') = \langle \hat{I}(x,t)\hat{I}(x',t')\rangle$ and using the Keldysh formalism this quantity can be written as

$$S_{xx'}(t,t') = \Big\langle T_{\rm K}\Big\{\hat{I}(x,t^-)\hat{I}(x',t'^+)e^{-i\int_{\rm K} H_{\rm T}(\tau)d\tau}\Big\}\Big\rangle. \tag{8}$$

Note that here we have purposely chosen the unsymmetrized noise. The motivation is as follows. When one considers the measurement of the current auto- and cross-correlations one has to be careful to extract the quantity which is measurable in experiments, for this correlator. The "textbook recipe"[12] is to take the symmetrized current-current correlator: this insures that the measured quantity is real. This is indeed the convention which is chosen in many reviews about noise[13].

For simplicity, let us forget here the spatial dependence of the correlator by considering $x = x'$. Considers the spectral power of current fluctuations (the Fourier transform):

$$S(\omega) = \int S(t)\, e^{i\omega t}\, dt\ , \tag{9}$$

symmetrized correlations can be expressed as:

$$S_{sym}(\omega) \equiv \frac{1}{2}(S(\omega) + S(-\omega))\ . \tag{10}$$

However, since the second half of the nineties, the point of view – about which noise correlator is more appropriate – has shifted. When taking a closer insight on this problem one is faced with the fact that the answer strictly depends on the measurement apparatus which is used to measure the noise.

In Ref. 14 a LC circuit was connected inductively to the mesoscopic device which emits noise, playing the role of a detector. There, it was shown that for a passive detector (i.e a detector which is in the ground state, which is such that it cannot provide any excitations to a measured system) the really measurable quantity is the spectral power of current auto–correlations at either positive or negative frequencies, depending on the correlator which is considered: $\langle \hat{I}(0)\hat{I}(t)\rangle$ or $\langle \hat{I}(t)\hat{I}(0)\rangle$. This results implies that the LC contour (detector) can adsorb the energy from the measured current but can not excite the measured system (the later processes correspond to a spectral power at negative frequencies for the correlator $\langle \hat{I}(0)\hat{I}(t)\rangle$).

This point of view has been further emphasized in two recent theoretical works. The first of these proposals considers a noisy mesoscopic circuit which is capacitively coupled to a double dot system[15], where information on the noise is extracted from the measurement inelastic current in the double dot, via the transimpedance of the two circuits. The second[16] considers a general system composed of an antenna (emitter of noise) and a detector, in the context of linear response theory. Incidentally, both Refs. 15 and 16 appeared after the noise review[13]. Since these theoretical works, there have been contributions[17,18] which point out that in the conditions mentioned in[14] (i.e. a passive detector), it is the non symmetrized contribution which is experimentally measured.

Ref.[17] used a superconductor-insulator-superconductor junction SIS capacitively coupled to the mesoscopic circuit which emits noise. We emphasize this particular work because it chooses the same convention (as in the present work) for the the noise correlator $\langle \hat{I}(t)\hat{I}(0)\rangle$, whose Fourier transform is only relevant at negative frequencies.

B. Zero point fluctuations

We are interested in measuring the current-current fluctuations in the nanotube due to a tunneling current from the STM tip into the nanotube. However, it should be emphasized that the spectral noise power has finite frequency contributions even if the tip is totally decoupled from the nanotube. This point has been noticed in a recent paper[19] where the two-terminal noise is computed. There it is explicitly shown that although these equilibrium fluctuations (zero point fluctuations in our case) vanish at zero frequency, there is a finite contribution for the spectral power of the symmetrized noise. In thus section we compute these correlations and address their relevance for our non-symmetrized correlator.

In the absence of coupling to the tip ($\Gamma = 0$), we use the expression of the current operator Eq. (7):

$$S^0(x-x', t-t') = \partial_x \partial'_x G^{\phi\phi}_{-+}(x, x', t, t') \tag{11}$$

In this equation, the Green's function corresponds to the field ϕ_{c+}. In principle, one needs the full expression for the Green's function in the presence of the leads. As a starter, let us consider an infinite nanotube without leads. The Green's function has been computed in Ref.[9]:

$$G^{\phi\phi}_{-+}(x,t) = -\frac{1}{8\pi K_{c+}} \sum_r \ln\left(1 + i v_F \frac{t}{a} + i r K_{c+} \frac{x}{a}\right) \tag{12}$$

In this case,

$$S^0(x,t) = -\frac{K_{c+}}{8\pi a^2}\left[\left(v_F \frac{t}{a} + K_{c+}\frac{x}{a} - i\right)^{-2} + \left(v_F \frac{t}{a} - K_{c+}\frac{x}{a} - i\right)^{-2}\right] \tag{13}$$

Consider for simplicity $x = 0$. Taking the Fourier transform, we use Cauchy's theorem and recognize that we pick up a pole in the upper half plane when frequencies are positive. We thus conclude that for zero or negative frequencies ($\omega \leq 0$) there is no contribution to this noise correlator. In a similar manner, we find that the correlator $\langle \hat{I}(0)\hat{I}(t)\rangle$ has non-zero contributions only for negative frequencies. Note that the above argument can be generalized for the Green's function of a nanotube with leads, which are analyzed in detail in the rest of this paper.

We thus have a complete agreement with the claims of Ref.[19]: outside $\omega = 0$, there is a finite contribution for the symmetrized noise in the absence of the STM tip, or, by the same token, in the absence of an applied voltage between the tip and the nanotube.

However, we turn back to the discussion of Ref.[14], and emphasize that for the correlator $\langle \hat{I}(t)\hat{I}(0)\rangle$, only negative frequencies have a physical meaning. From the discussion above, we found that this correlator is zero at negative frequencies. As a result, there is strictly no need to consider the effect of zero point fluctuations in the present problem.

This is a consequence of our choice of setting the temperature to zero in our problem. Note that this discussion could have been avoided from the start by saying that we are considering the contributions to the noise which constitute deviations with respect to the zero point result - the excess noise.

C. Excess noise calculation

To get the lowest non trivial contribution to the nanotube current noise, one should expand the exponent in perturbation series in the tunneling amplitude up to the second order in Γ. Then expressing the electrons operators in terms of bosonic fields, one finds that current fluctuations depends only on the time difference $t-t'$ because of time translational invariance: $S_{xx'}(t,t')=S_{xx'}(t-t')$. The current fluctuations in the nanotube become:

$$\begin{aligned}S_{xx'}(t)=&-\frac{e^2v_{\rm F}^2\Gamma^2}{2(\pi a)^2}\sum_{\eta_1\eta_2 r\sigma}\eta_1\eta_2\int dt_1dt_2\,A^{r\sigma}_{\eta_1\eta_2}(t_1-t_2)\\&\times\big[B^{r\sigma}_{-,\eta_1}(x,0,t-t_1)-B^{r\sigma}_{-,\eta_2}(x,0,t-t_2)\big]\\&\times\big[B^{r\sigma}_{+,\eta_1}(x',0,-t_1)-B^{r\sigma}_{+,\eta_2}(x',0,-t_2)\big].\end{aligned}\tag{14}$$

A real time correlator associated with the tunneling event at $x=0$ has been introduced:

$$\begin{aligned}A^{r\sigma}_{\eta\mu}(t)=&\cos\omega_0 t\,e^{g^{\sigma}_{\eta\mu}(t)}\exp\Big[\frac{\pi}{4}\sum_{j\delta}\tilde{G}^{\phi\phi}_{j\delta,\eta\mu}(0,0,t)\\&+r\tilde{G}^{\phi\theta}_{j\delta,\eta\mu}(0,0,t)+r\tilde{G}^{\theta\phi}_{j\delta,\eta\mu}(0,0,t)+\tilde{G}^{\theta\theta}_{j\delta,\eta\mu}(0,0,t)\Big]\,,\end{aligned}\tag{15}$$

together with a correlator associated with propagation of excitations along the nanotube:

$$B^{r\sigma}_{\eta\mu}(x,0,t)=\partial_x\Big[\tilde{G}^{\phi\phi}_{c+,\eta\mu}(x,0,t)+r\tilde{G}^{\phi\theta}_{c+,\eta\mu}(x,0,t)\Big].\tag{16}$$

Here $\tilde{G}^{\phi\phi}_{j\delta,\eta\mu}(x,x',t-t')=\big\langle T_{\rm K}\big\{\phi_{j\delta}(x,t^{\eta})\phi_{j\delta}(x',t'^{\mu})\big\}\big\rangle-\frac{1}{2}\langle\phi^2(x,t)\rangle-\frac{1}{2}\langle\phi^2(x',t')\rangle$ is the Keldysh Green's function for the nanotube bosonic field $\phi_{j\delta}(x,t)$ (similar definitions hold for other combinations of bosonic fields), $g^{\sigma}_{\eta\mu}(t)$ is the Keldysh Green's function for STM tip bosonic field defined in Eq. (5).

We now define the spectral power of current fluctuations as $S_{xx'}(\omega)=\int S_{xx'}(t)\,e^{i\omega t}\,dt$, i.e. including the dependence on spatial coordinates. Taking the Fourier transform of Eq. (14), which is a convolution, can be done assuming that the Fourier transform of $A^{r\sigma}_{\eta\mu}(t)$ and $B^{r\sigma}_{\eta\mu}(x,0,t)$ are known. One therefore gets:

$$\begin{aligned}S_{xx'}(\omega)=&-\frac{e^2v_{\rm F}^2\Gamma^2}{2(\pi a)^2}\sum_{\eta_1\eta_2 r\sigma}\eta_1\eta_2\\&\times\Big(\tilde{A}^{r\sigma}_{\eta_1\eta_2}(0)\tilde{B}^{r\sigma}_{-,\eta_1}(x,0,\omega)\tilde{B}^{r\sigma}_{+,\eta_1}(x',0,-\omega)\\&-\tilde{A}^{r\sigma}_{\eta_1\eta_2}(-\omega)\tilde{B}^{r\sigma}_{-,\eta_2}(x,0,\omega)\tilde{B}^{r\sigma}_{+,\eta_1}(x',0,-\omega)\\&-\tilde{A}^{r\sigma}_{\eta_1\eta_2}(\omega)\tilde{B}^{r\sigma}_{-,\eta_1}(x,0,\omega)\tilde{B}^{r\sigma}_{+,\eta_2}(x',0,-\omega)\\&+\tilde{A}^{r\sigma}_{\eta_1\eta_2}(0)\tilde{B}^{r\sigma}_{-,\eta_2}(x,0,\omega)\tilde{B}^{r\sigma}_{+,\eta_2}(x',0,-\omega)\Big),\end{aligned}\tag{17}$$

where $\tilde{A}(\omega)$, $\tilde{B}(x,0,\omega)$ are Fourier transforms, $\tilde{A}(\omega)=\int A(t)e^{i\omega t}\,dt$ and similarly for $\tilde{B}(x,0,\omega)$, of factors $A(t)$ and $B(x,0,t)$ defined in Eqs. (15) and (16).

IV. GREEN'S FUNCTIONS

Let us now concentrate on the calculation of the bosonic Green's functions in the nanotube. Going from the bosonic Hamiltonian of Eq. (3) to the Lagrangian in imaginary time $\tau=it+\delta$ which is in general dependent on both fields $\phi_{j\delta}$ and $\theta_{j\delta}$[9]. To derive the Green's functions for the two fields $\phi_{j\delta}(x,\tau)$ ($\theta_{j\delta}(x,\tau)$) it is convenient to integrate the partition function over the field $\theta_{j\delta}(x,\tau)$ ($\phi_{j\delta}(x,\tau)$). The corresponding actions follow:

$$S_{\phi_{j\delta}}=\frac{1}{2}\int dxd\tau\,K_{j\delta}\Big[v^{-1}_{j\delta}\,(\partial_\tau\phi_{j\delta})^2+v_{j\delta}(\partial_x\phi_{j\delta})^2\Big],\tag{18}$$

$$S_{\theta_{j\delta}} = \frac{1}{2}\int dxd\tau K_{j\delta}^{-1}\Big[v_{j\delta}^{-1}(\partial_\tau\theta_{j\delta})^2 + v_{j\delta}(\partial_x\theta_{j\delta})^2\Big], \tag{19}$$

where $K_{j\delta}(x)$ and $v_{j\delta}(x)$ are functions of the coordinate x. From these actions, one can conclude that the Green's function, $G_{j\delta}^{\phi\phi}(x,x',\bar\omega)$ for the field $\phi_{j\delta}(x,\tau)$ ($G_{j\delta}^{\phi\phi}(x,x',\bar\omega)$ is the Fourier transform of $G_{j\delta}^{\phi\phi}(x,x',\tau)$ in imaginary time), obeys the following differential equation:

$$\Big[\frac{K_{j\delta}(x)}{v_{j\delta}(x)}\,\bar\omega^2 - \partial_x v_{j\delta}(x)K_{j\delta}(x)\partial_x\Big]G_{j\delta}^{\phi\phi}(x,x',\bar\omega) = \delta(x-x'). \tag{20}$$

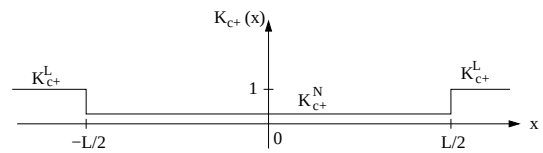

FIG. 3: 1D model for non interacting leads: the Luttinger liquid parameter varies over distance.

A similar equation holds for $G_{j\delta}^{\theta\theta}(x,x',\bar\omega)$, making the transformation $K_{j\delta}(x)\to K_{j\delta}^{-1}(x)$ as is obvious from Eqs. (18) and (19). It follows from Eq. (20) that the Green's function $G_{j\delta}^{\phi\phi}(x,x',\bar\omega)$ is a continuous function and its derivative $\partial_x G_{j\delta}^{\phi\phi}(x,x',\bar\omega)$ has a jump at $x=x'$:

$$-v_{j\delta}(x)K_{j\delta}(x)\partial_x G_{j\delta}^{\phi\phi}(x,x',\bar\omega)\Big|_{x=x'-\epsilon}^{x=x'+\epsilon} = 1. \tag{21}$$

To solve Eq. (20) with the condition given by Eq. (21), one chooses a model where the interaction parameter is a step like function: $K_{j\delta}(x) = K_{j\delta}^{\rm N}$ in the nanotube ($|x|<\frac{L}{2}$) and $K_{j\delta}(x) = K_{j\delta}^{\rm L}$ in the leads ($|x|>\frac{L}{2}$). Then looking for solution for $|x'|<\frac{L}{2}$ in the form:

$$G^{\phi\phi}(x,x',\bar\omega) = \begin{cases} Ae^{\frac{|\bar\omega|}{v_{\rm L}}x}, & x<-\frac{L}{2},\\ Be^{\frac{|\bar\omega|}{v_{\rm N}}x} + Ce^{-\frac{|\bar\omega|}{v_{\rm N}}x}, & -\frac{L}{2}<x<x',\\ De^{\frac{|\bar\omega|}{v_{\rm N}}x} + Ee^{-\frac{|\bar\omega|}{v_{\rm N}}x}, & x'<x<\frac{L}{2},\\ Fe^{-\frac{|\bar\omega|}{v_{\rm L}}x}, & x>\frac{L}{2}, \end{cases} \tag{22}$$

where the Green's function $G^{\phi\phi}(x,x',\bar\omega)$ vanishes at $x\to\pm\infty$, corresponding to outgoing boundary conditions (we have omitted subindex $j\delta$ to shorten the notations). Solving all matching conditions at $x=\pm\frac{L}{2}$ and $x=x'$, one specifies all coefficients $A,B,...$ in Eq. (22). In order to obtain the crossed Green's functions $G^{\phi\theta}(x,x',\bar\omega)$ and $G^{\theta\phi}(x,x',\bar\omega)$, one uses the equation of motion for the field $\phi(x,\tau)$ and $\theta(x,\tau)$ generated from Hamiltonian of Eq. (6). This leads to:

$$G^{\phi\theta}(x,x'\bar\omega) = -\frac{1}{\bar\omega}\frac{v(x)}{K(x)}\,\partial_x G^{\theta\theta}(x,x',\bar\omega), \tag{23}$$

$$G^{\theta\phi}(x,x'\bar\omega) = -\frac{v(x)K(x)}{\bar\omega}\,\partial_x G^{\phi\phi}(x,x',\bar\omega). \tag{24}$$

Performing the inverse Fourier transform one obtains the bosonic Green's functions in real time. From the real time Green's functions the Keldysh Green's functions matrix elements can be specified. Its two time arguments t and 0 are assigned to the lower/upper $(+/-)$ branch of the Keldysh contour. This procedure has been described in detail in Ref. 9, and shall therefore not be repeated. In order to find the noise correlation in the nanotube one needs only the real time Green's functions at the origin for factor $A_K^{r\sigma}(t)$ of Eq. (15) and one needs the Fourier transform of the coordinate derivative of the Green's functions in the leads for factors $B_K^{r\sigma}(x,x',t)$ of Eq. (16). Using the scheme of calculation depicted above one finds:

$$\tilde G_{j\delta}^{\phi\theta}(0,0,t) = \tilde G_{j\delta}^{\theta\phi}(0,0,t) = 0, \tag{25}$$

and

$$\tilde{G}^{\phi\phi}_{j\delta}(0,0,t) = -\frac{1}{2\pi K^{\rm N}_{j\delta}}\Bigg\{\ln\Big(1+\frac{iv_{\rm F}t}{a}\Big) + \sum_{r=\pm1}\sum_{n=1}^{\infty} b^n_{j\delta}\ln\Big[1+\frac{iv_{\rm F}t}{a+irnK^{\rm N}_{j\delta}L}\Big]\Bigg\}, \tag{26}$$

where $b_{j\delta} = (K^{\rm N}_{j\delta} - K^{\rm L}_{j\delta})/(K^{\rm N}_{j\delta} + K^{\rm L}_{j\delta})$. The Green's function $\tilde{G}^{\theta\theta}_{j\delta}(0,0,t)$ can be obtained by transformation $K^{\rm N}_{j\delta} \to (K^{\rm N}_{j\delta})^{-1}$ of the total factor of Eq. (26). Another necessary Green's functions has the form:

$$\begin{aligned}
\partial_x G^{\phi\phi}_{++}(x,0,\omega) &= \frac{i\,sgn(x)}{(K^{\rm N}+K^{\rm L})v_{\rm L}}\, f(|\omega|)e^{i\frac{|\omega|}{v_{\rm L}}(|x|-\frac{L}{2})},\\
\partial_x G^{\phi\phi}_{-+}(x,0,\omega) &= \frac{i\,sgn(x)\Theta(\omega)}{(K^{\rm N}+K^{\rm L})v_{\rm L}}\Big(f(\omega)e^{i\frac{\omega}{v_{\rm L}}(|x|-\frac{L}{2})} - C.c.\Big),\\
\partial_x G^{\phi\theta}_{++}(x,0,\omega) &= \frac{-i\,sgn(\omega)K^{\rm N}}{(K^{\rm N}+K^{\rm L})v_{\rm L}}\, f(|\omega|)e^{i\frac{|\omega|}{v_{\rm L}}(|x|-\frac{L}{2})},\\
\partial_x G^{\phi\theta}_{-+}(x,0,\omega) &= \frac{-i\Theta(\omega)K^{\rm N}}{(K^{\rm N}+K^{\rm L})v_{\rm L}}\Big(f(\omega)e^{i\frac{\omega}{v_{\rm L}}(|x|-\frac{L}{2})} + C.c.\Big),\\
f(\omega) &= \frac{e^{i\frac{\omega}{v_{\rm N}}\frac{L}{2}}}{1-b\,e^{i\frac{\omega}{v_{\rm N}}L}}\,,
\end{aligned} \tag{27}$$

where $\pm$ refers to the branch of the Keldysh contour and $\Theta(\omega)$ is the Heaviside function. To ease notations, the index $j\delta$ has been omitted from each coefficient $K^{\rm N(L)}$ and $v_{\rm N(L)}$. The remaining set of Keldysh Green's functions can be found from symmetry properties:

$$\partial_x G^{\phi\phi}_{--}(x,0,\omega) = [\partial_x G^{\phi\phi}_{++}(x,0,-\omega)]^*, \tag{28}$$

$$\partial_x G^{\phi\phi}_{+-}(x,0,\omega) = [\partial_x G^{\phi\phi}_{-+}(x,0,-\omega)]^*, \tag{29}$$

with the same relations for $\partial_x G^{\phi\theta}_{K}(x,0,\omega)$ Green's functions.

V. NOISE CORRELATIONS FOR AN INFINITE NANOTUBE

Let us first consider the case of infinite nanotube $L \to \infty$ where the noise correlations are measured in the Luttinger liquid. In this case, in all factors $\tilde{A}^{r\sigma}_{\eta\mu}(\omega)$ and $\tilde{B}^{r\sigma}_{\eta\mu}(x,x',\omega)$ calculated in previous sections one should insert interaction parameters to be equal $K^{\rm N}_{j\delta} = K^{\rm L}_{j\delta} = K_{j\delta}$. This corresponds to an homogeneous nanotube Hamiltonian (3) with constant velocity $v_{j\delta}$, without leads. Then, substituting $\tilde{A}^{r\sigma}_{\eta\mu}(\omega)$, $\tilde{B}^{r\sigma}_{\eta\mu}(x,x',\omega)$ into Eq. (17) one gets the following expression for the noise:

$$\begin{aligned}
S_{xx'}(\omega) &= \frac{e^2\Gamma^2}{(\pi a)^2}\,\frac{K^2_{c+} + sgn(x)sgn(x')}{2}\\
&\times\Theta(-\omega)\,e^{i\omega\tau_-}\Big(\tilde{A}_{+-}(|\omega|) + \tilde{A}_{-+}(-|\omega|)\Big),
\end{aligned} \tag{30}$$

where $\tau_- = (|x| - |x'|)/v_{c+}$ is defined to be the retardation time.

According to Ref. 14, only negative frequencies are entering in the expression for the spectral noise correlator $S_{xx'}(\omega)$ at zero temperature. Physically, this corresponds to the measurable part of our definition $S(\omega) = \int\langle\hat{I}(t)\hat{I}(0)\rangle\, e^{i\omega t}\,dt$ for the auto-correlation function.

As is explicit in Eq. (30), the current cross-correlator is a complex quantity, which reflects the fact that the product of two Hermitian operators in general is not a Hermitian operator itself. Yet the noise cross-correlator is a quantity which should in principle be measured experimentally. This is a similar issue which was encountered for the autocorrelation noise: what is the quantity which is physically measurable in experiments ? In general this correlator is a complex quantity for non symmetric positions $x \neq -x'$ so it cannot be directly measurable quantity even at positive frequencies.

A general theory of noise cross-correlation measurements goes beyond the scope of this paper. Here we only cite existing results. When considering the same model as in Ref.[14], for the cross correlator at two arbitrary positions x and x', with an LC circuit inductively coupled to our mesoscopic circuit at these two locations, one finds that[20] the noise correlator should be symmetrized with respect to the two positions where current are being measured:

$$S^{meas}_{xx'}(\omega) = \frac{1}{2}[S_{xx'}(\omega) + S_{x'x}(\omega)] \tag{31}$$

Note that these issues can be avoided if one considers the two positions where currents are being measured to be symmetrically located with respect to the injection point of the STM ($x = -x'$).

In the plots of Figs. 4 and 5 of this section and the following section the trivial phase $e^{i\omega\tau_-}$ will thus be omitted. Note that it is equal to 1 for a measurement geometry where $|x| = |x'|$ (equal length between the two current measurements on either side of the nanotube and the tip).

The factors $\tilde{A}_{+-}(\omega)$ and $\tilde{A}_{-+}(\omega)$ entering expression (30) are defined as:

$$\tilde{A}_{+-}(\omega) = \int_{-\infty}^{\infty} \frac{\cos\omega_0 t\, e^{i\omega t}}{(1 - iv_{\rm F}t/a)^{\nu+1}}\, dt, \tag{32}$$

and $\tilde{A}_{-+}(\omega) = \tilde{A}^*_{+-}(-\omega)$, where

$$\nu = \sum_{j\delta} \frac{1}{8}\Big(K_{j\delta} + \frac{1}{K_{j\delta}}\Big). \tag{33}$$

In the limit of a vanishing cutoff $a \to 0$, one can compute this integral explicitly and find the current fluctuations in the form:

$$\begin{aligned} S_{xx'}(\omega) &= \frac{K^2_{c+} + sgn(x)sgn(x')}{2}\, e^{i\omega\tau_-} \\ &\times \frac{2e^2\Gamma^2}{\pi v^2_{\rm F}} \left(\frac{a}{v_{\rm F}}\right)^{\nu-1} \Theta(|\omega_0| - |\omega|)\, \frac{(|\omega_0| - |\omega|)^\nu}{\mathbf{\Gamma}(\nu+1)}. \end{aligned} \tag{34}$$

According to Eq. (34), the noise cross correlator and the noise auto-correlations differ by a prefactor only. The convention for measuring the currents has been chosen as in Ref. 9. The positive direction is the same for $x > 0$ and for $x < 0$. Consequently the noise autocorrelation is positive while the noise cross-correlations are negative. Recall that in Ref. 9, it was stated that the usual convention for measuring noise correlations, i.e. in Hanbury-Brown type experiments[21], is to chose the positive direction to correspond to both currents flowing away from the point of injection[22]. This means, for instance that when considering $S_{x,-x}(\omega = 0) < 0$, quasi-particles are moving away from the injection point and give rise to positive Hanbury-Brown and Twiss correlations, as for photons.

Consider a nanotube with all interaction parameters $K_{j\delta} = 1$. This corresponds to the non-interacting case. Physically, this situation is reached when, for instance, the nanotube is put in close proximity with a metallic gate or with a heavily doped semiconductor substrate. According to Eq. (34) there is an autocorrelation signal corresponding to electrons being emitted either right or left of the emitter. However, there is no cross-correlation signal whatsoever, even at finite frequencies. In order to measure a cross-correlation signal, it would be necessary to go to higher order (fourth order) in the tunneling amplitude[22], where scattering theory predicts negative Hanbury-Brown and Twiss correlations, a result in agreement with experimental observations[23].

When considering the auto-correlation at finite frequencies, one recovers (Fig. 4) the known finite frequency spectrum of noise associated with non-interacting mesoscopic systems[24]: the noise decreases linearly with frequency, until the characteristic frequency $eV/\hbar$ is reached. The derivative of the noise bears a singularity at this point, which has been diagnosed experimentally in a diffusive mesoscopic conductor[25].

Next, one chooses $K_{c+} < 1$. All other interactions parameters are equal to 1: for the spin sector, this assumes that time reversal symmetry applies. For convenience, in the plots of Fig. 4, we have omitted the prefactor $(K^2_{c+} + sgn(x)sgn(x'))/2$ in order to plot the auto-correlation and the cross-correlation noise spectrum in a single plot. At $\omega = 0$, one recovers the result of Ref. 9: positive Hanbury-Brown and Twiss correlations due to entangled quasi-particles flowing at opposite ends of the nanotube. At frequencies larger than ω_0, the noise is equal to zero as in the non-interacting case.

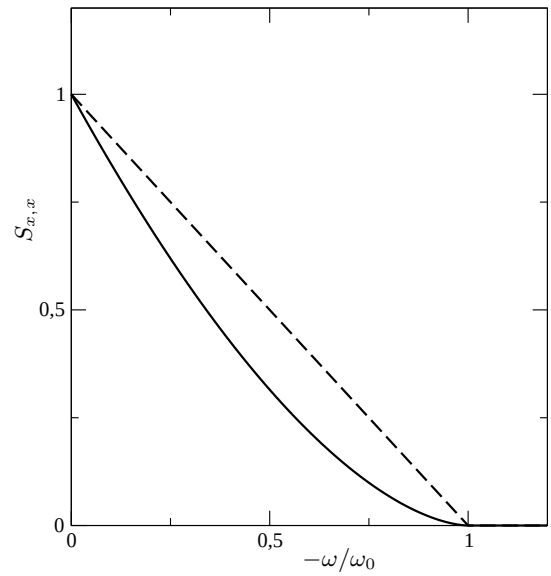

FIG. 4: Finite frequency noise auto-correlation for an infinite nanotube, in the non interacting case ($K_{c+} = 1$ dashed lines), as well as in the case of repulsive interactions ($K_{c+} = 1/3$ full line). The noise is normalized to the zero frequency value. In the case of $K_{c+} = 1/3$, the cross-correlations $-S_{x,-x}$ have the same dependence on ω.

.

For $\omega < \omega_0$, the noise differs from the non-interacting case. First of all, recall that in *chiral* Luttinger liquids as in the fractional quantum Hall effect[26], the finite frequency spectrum associated with quasiparticle tunneling gives rise to a singularity at $\omega = \nu eV/\hbar$ (ν is the filling factor) due to the tunneling density of states of Laughlin quasiparticles. Recall also that, when tunneling occurs between two quantum Hall fluids, the noise at $\omega < \nu eV/\hbar$ has a power law behavior, because of the tunneling density of states of electrons. The situation is quite similar when electrons are injected into a non-chiral Luttinger liquid: the spectrum for $\omega < \omega_0$ (Fig. 4) has a power law behavior with no singularity. The spectral density of noise is thus always lower than that of the non-interacting case.

From the point of view of the auto-correlation noise, there is therefore no qualitative difference between the spectrum for electron tunneling into a chiral and a non-chiral Luttinger liquid. From the point of view of the noise cross-correlation spectrum, one concludes that positive correlations persist at finite frequencies. In what follows, one should enquire whether such features remain robust in the presence of FL leads.

VI. NOISE CORRELATIONS FOR A NANOTUBE CONNECTED TO LEADS

Consider now the case where the nanotube has a finite length ($L \gg a$), and where it is connected to a FL leads with $K^{\rm L}_{j\delta} = 1$. Keeping the same conventions as in the preceding sections, the expression for the spectral density of noise – measured in the leads – reduces to:

$$S_{xx'}(\omega) = \frac{K_{c+}^2|\phi_+(\omega)|^2 + sgn(x)sgn(x')|\phi_-(\omega)|^2}{(1+K_{c+})^2} \times \frac{2e^2\Gamma^2}{(\pi a)^2}\,\Theta(-\omega)\,e^{i\omega\tau_-}\Big(\tilde{A}_{+-}(|\omega|) + \tilde{A}_{-+}(-|\omega|)\Big), \tag{35}$$

where $\phi_\pm(\omega) = (1 \pm (K_{c+}-1)/(K_{c+}+1)e^{i\omega L/v_{c+}})^{-1}$ reflects the presence of the leads.

In principle, the functions $\tilde{A}_{+-}(\omega)$ also contain information on the presence of the leads. In what follows, we will neglect the effect of the leads only in $\tilde{A}_{+-}(\omega)$ under specific assumptions. Consider the expression of A(t) in Eq. (15), which reflect the local tunneling contribution in the noise correlator. It can be decomposed as a product of three functions, when one considers the expression for the Green functions in the presence of the leads: a) the $\cos(\omega_0 t)$ prefactor; b) a function which is the Green's function in the absence of leads; this function only has a dependence on t and the cutoff v_F/a; c) a function which has all the information about the leads with a time scale $\tau_{\rm L} = v_{c+}/L$. Our approximation is to forget about this last term because when the Fourier transform is taken, if the voltage frequency ω_0 is large, the variation of the factor c) will not matter: the combination of the oscillation and the slowly decaying

part of the (infinite nanotube) contribution b) will provide the dominant contribution. Note that this approximation is justified if the Fourier frequency which is sampling $\tilde{A}_{+-}(\omega)$ is smaller than (or at least less than) ω_0. The validity limit of this approximation is that $\tau_{\rm V}$ is small compared to $\tau_{\rm L}$.

Here, one shall use the expression for $\tilde{A}_{+-}(\omega)$ of Eq. (32) corresponding to an infinite nanotube. Finally, one gets the following expression for current fluctuations:

$$S_{xx'}(\omega)=\frac{e^2\Gamma^2}{\pi v_{\rm F}^2}\left(\frac{a}{v_{\rm F}}\right)^{\nu-1}e^{i\omega\tau_-}\Theta(|\omega_0|-|\omega|)\frac{(|\omega_0|-|\omega|)^\nu}{\mathbf{\Gamma}(\nu+1)}$$
$$\times\left(\frac{1}{1-(1-K_{c+}^{-2})\sin^2\omega\tau_{\rm L}}+\frac{sgn(x)sgn(x')}{1-(1-K_{c+}^{2})\sin^2\omega\tau_{\rm L}}\right), \tag{36}$$

where $\tau_{\rm L}=L/2v_{c+}$ is the traveling time needed for Luttinger liquid excitations to reach the leads.

These results are illustrated in Fig. 5, for different parameters. In addition to the frequency scale imposed by the voltage bias, when considering the inhomogeneous Luttinger model for the FL leads, there appears a new frequency scale $\tau_{\rm L}^{-1}$ associated with the time of flight defined above. The first observation which can be made from the two plots of Figs. 5a and 5b is that at zero frequency, the noise cross-correlation vanish. The work of Ref. 9 had expressed suspicion that this may be the case, although a full computation of noise correlations was not provided there. The issue of the presence of FL leads for the two-terminal conductance of a Luttinger liquid leads was addressed several years ago[27].

At finite frequencies however, the noise correlations are always positive (with the "usual" convention for current signs), or zero. Positive noise correlations are reminiscent of systems where the two constituents (here pairs of quasi-particles) of a particle (here an electron) are separated into two different branches. Positive correlations where encountered previously in branched normal-superconducting junctions, when the two constituent electrons of a Cooper pair are split into two normal metal terminals[28].

The fact that the spectral power vanishes at zero frequency seems rather natural when using a second order approximation scheme in the tunneling amplitude Γ. Indeed, the second order calculation takes into account coherent transport associated with *only one* electron at a time injected into nanotube. It is a Poissonian result: there is no correlation between two successive electrons. At the same time, the zero frequency fluctuations are directly related to the cross-correlations of the *total* charge transmitted to the right FL lead and to the left FL lead. Let us inject an electron into the nanotube at time $t=0$. The cross correlations of the total charge transmitted to the FL leads $\hat{Q}_{\rm L}(t)=\int_0^t\hat{I}(x<0,t')dt'$ and $\hat{Q}_{\rm R}(t)=\int_0^t\hat{I}(x>0,t')dt'$ are defined as:

$$\langle\hat{Q}_{\rm L}(t)\hat{Q}_{\rm R}(t)\rangle=\int S_{xx'}(\omega)\,\frac{\sin^2(\omega t/2)}{(\omega/2)^2}\,\frac{d\omega}{2\pi}. \tag{37}$$

In the limit $t\to\infty$, one has $\langle\hat{Q}_{\rm L}(t)\hat{Q}_{\rm R}(t)\rangle=tS_{xx'}(0)$ (since $\sin^2(\omega t/2)/(\omega/2)^2\to 2\pi t\delta(\omega)$ in this limit). Since one has injected only one electron, it can be transmitted as whole either to the left or to the right FL lead and thus $\langle\hat{Q}_{\rm L}(t)\hat{Q}_{\rm R}(t)\rangle=0$. This simple observation accounts for the vanishing of the current fluctuations at zero frequency. Of course the next order in perturbation theory takes into account the coherence effect between two consecutively injected electrons, which may result in finite current fluctuations at zero frequency. This is for instance true for electron injection in a FL, where the cross-correlations at $\omega=0$ are negative.

The next observation coming from Eq. (36) is that there are two different limits for electron transport depending on the ratio of the voltage time $\tau_{\rm V}=\hbar/eV$ and the transport time $\tau_{\rm L}$. The situation for the case where $\tau_{\rm L}\gg\tau_{\rm V}$ is depicted on Fig. 5a where a sequence of resonances at $\omega\tau_{\rm L}=(2p+1)\pi/2$ appears in the current fluctuations. These resonances can be accounted by the analogy with multiple Andreev reflection on the boundaries between the nanotube and the FL leads[7].

The injection of an electron, say at $t=0$, results in the creation of two anomalously charged excitations propagating in opposite directions toward the boundaries. The characteristic width of the anomalously charged quasiparticle wave packets, $\delta x_{Q_\pm}$ is specified by the voltage time: $\delta x=v_{c+}\tau_{\rm V}$. Recall that these quasiparticle wave packets consist of collective electron-hole excitations. In the limit $\tau_{\rm L}\gg\tau_{\rm V}$, the width of these anomalously charged wave packets is much smaller then the length of the nanotube ($L\gg\delta x$). As a result, it takes a finite time $\tau_{\rm L}$ for these excitations to reach the boundary. At the boundary, these excitations can either be transmitted to the FL leads or can be reflected back to the nanotube, resulting in the splitting of these anomalously charged wave packets. The transmitted part (which consists of multiple electron-hole excitations, now in the FL lead) is then measured, resulting in the first peak of Fig. 5a. On the other hand, the reflected part then moves to the opposite boundary and reaches it after a time $t=3\tau_{\rm L}$. There it is again transmitted or reflected from this boundary. The transmitted parts in the FL leads on both side give rise to another signal (second maximum in Fig. 5a). Such multiple reflection processes therefore result in the

sequence of peaks in the spectral power curve at the corresponding frequencies. The amplitude of the peaks are lower, the higher the number of reflections inside the nanotube region, as expected. In all of the above, the cross-correlations signal is always positive in the Hanbury-Brown and Twiss sense.

In the opposite limit $\tau_{\rm L} \ll \tau_{\rm V}$ (see Fig. 5b), the size of the anomalously charged wave packets is much larger than the length of the nanotube and there is no room for resonances to occur in the cross-correlation spectral power curve. In the following one can argue that the positive noise correlation signal in either situations – many resonances or one resonance – can be used to identify anomalous charges without any ambiguity. However, note that given our assumptions on the computation of $\tilde{A}_{+-}(\omega)$, results are to be trusted mostly for multiple bounces: in the case of a single bounce, a correction of the density of states due to the presence of the leads should in principle be included.

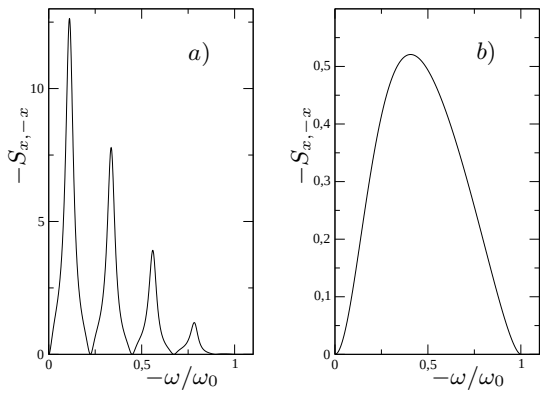

FIG. 5: Finite frequency noise correlations for a nanotube connected to leads. a) case where $\tau_{\rm L} = 14\tau_{\rm V}$; b)case where $\tau_{\rm L} = \tau_{\rm V}$. The noise is normalized according to the frequency independent prefactor of Eq. (36).

VII. CONCLUSION

The purpose of the present paper has been to further analyze the effect of electron injection in a one-dimensional correlated electron system, chosen to be a metallic carbon nanotube. A crucial question was to ask whether the positive noise correlations survive at zero frequencies, when leads are included in the model. The negative answer to this question motivated us to analyze the full finite frequency spectrum of auto-correlation and cross-correlation noise.

The presence of positive correlations in the finite frequency cross-correlations is a manifestation of non-local behavior in an interacting one-dimensional system, which was stressed in Ref. 9 in the case of vanishing frequencies. Here, we have shown that a finite frequency analysis allows to recover such information. In addition, in is particularly interesting to uncover the role of the different time scales which govern the dynamics of electron injection in a Luttinger liquid connected to leads. In one extreme, one obtains a simple vanishing of zero frequency noise at the cutoff frequency, while in the other extreme, the noise spectrum shows a diffraction pattern due to the interference of quasi-particles traveling in the central region.

In all of the above, we conclude that the finite frequency analysis of noise allows to clearly identify the anomalous charges which are associated with the chiral modes traveling in this non-chiral Luttinger liquid. In an experiment, these could be detected by choosing a frequency where the noise autocorrelation and the noise cross-correlations give a maximum as a function of frequency. A suggestion for identifying the interaction parameter – and thus the anomalous charges – is to tune the frequency so that $\omega\tau_{\rm L} = (2p+1)\pi/2$ (p integer), so that the sine function which enters in Eq. (36) is equal to one. Then, one can measure experimentally – and compute – the ratio $|S_{x,-x}/S_{x,x}| = (1-K_{c+}^4)/(1+K_{c+}^4)$. to extract the chiral charges $Q_\pm = (1 \pm K_{c+})/2$. Note that the fact that such chiral charges enter the Fabry-Perot interpretation[7] for recombination into electron charges at the Luttinger liquid interfaces gives a motivation for their identification as elementary charges of the non-chiral Luttinger liquid.

While this was in progress, we noticed a work[19] dealing with two terminal measurements in a Luttinger liquid containing an impurity, connected to FL leads. There is a claim that a charge $e^* = K_{c+}e$ (the translation to the nanotube interaction parameter in ours) can be identified in a periodic structure of the noise. It is likely that their results are connected to the present results. In some sense, the injection of an electrons may play the same role as

the presence of an impurity. However, our approach has the advantage that it merely relies on the comparison of the spectrum for autocorrelation and for cross-correlation noise. Note that the present results apply just as well to a one-dimensional quantum wire constructed from semiconductor cleaved-edge heterostructures such as the ones used to measure spin-charge separation[29].

The present work, like many others, has described the contacts as one-dimensional leads which are adiabatically connected to the Luttinger liquid. Extensions of the present work could include a more general description of the connection between the Fermi liquid leads and the nanotube. This has been achieved using both a coherent and an incoherent approach for the fractional quantum Hall effect[30]. Also, as mentioned in our previous work, more complications could arise due to screening[31] of the interactions by the FL leads or even by the tip. The former should not play any role as long as the size of the nanotube is larger than the screening length, which should be comparable to the lattice constant or cutoff parameter a. Concerning screening effect due to the tip, however, the fact that a tunneling geometry is chosen should minimize its effects. A final remark is that this finite frequency diagnosis could be also simulated by a zero-frequency noise measurement operated on a nanotube which is subject to the superposition of a DC and an AC bias from the tip, as was recently illustrated for the fractional quantum Hall effect[32].

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