HAL open science

Electron injection in a nanotube: noise correlations and entanglement

Adeline Crépieux, Rodolphe Guyon, Pierre Devillard, Thierry Martin

To cite this version:

Adeline Crépieux, Rodolphe Guyon, Pierre Devillard, Thierry Martin. Electron injection in a nanotube: noise correlations and entanglement. Physical Review B: Condensed Matter and Materials Physics (1998-2015), 2003, 67 (20), pp.205408. 10.1103/PhysRevB.67.205408 . hal-00004468

HAL Id: hal-00004468

https://hal.science/hal-00004468v1

Submitted on 4 Sep 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

arXiv:cond-mat/0209291v3 [cond-mat.mes-hall] 20 Mar 2003

Electron injection in a nanotube: noise correlations and entanglement

A. Crépieux[1,2], R. Guyon[1,2], P. Devillard[1,3] and T. Martin[1,2]
[1] *Centre de Physique Théorique, Case 907 Luminy, 13288 Marseille Cedex 9, France*
[2] *Université de la Méditerranée, 13288 Marseille Cedex 9, France and*
[3] *Université de Provence, 13331 Marseille Cedex 03, France*

Transport through a metallic carbon nanotube is considered, where electrons are injected in the bulk by a scanning tunneling microscope tip. The charge current and noise are computed both in the absence and in the presence of one dimensional Fermi liquid leads. For an infinite homogeneous nanotube, the shot noise exhibits effective charges different from the electron charge. Noise correlations between both ends of the nanotube are positive, and occur to second order only in the tunneling amplitude. The positive correlations are symptomatic of an entanglement phenomenon between quasiparticles moving right and left from the tip. This entanglement involves many body states of the boson operators which describe the collective excitations of the Luttinger liquid.

I. INTRODUCTION

Over the years, the study of current noise and noise correlations has become a respected and useful diagnosis for transport measurements on mesoscopic conductors. Theoretically, noise was first computed mostly for non–interacting systems[1]. However, it soon became clear that low frequency noise could be used to isolate the quasiparticle charge[2,3] and to study the statistical correlations[4,5] in specific quasi one–dimensional correlated electron systems, such as the edge waves in the quantum Hall effect. In these chiral Luttinger liquids, the charge of the collective excitations along the edges corresponds to the electron charge multiplied by the filling factor.

Attention is now turning towards conductors – individual nano-objects – which occur naturally, and which can be connected to current/voltage probes in order to perform a transport experiment. The crucial advantage of such nano–objects is that they are essentially free of defects and in some circumstances they have an inherent one dimensional character. Carbon nanotubes constitute the archetype of such 1D nano-objects: single wall armchair nanotubes have metallic behavior, with two propagating modes at the Fermi level. Incidentally, electronic correlations are known to play an important role in such systems. Carbon nanotubes seem to constitute good candidates to study Luttinger liquid behavior. In particular, their tunneling density of states – and thus the tunneling $I(V)$ characteristics is known to have a power law behavior[6,7,8] in accordance with Luttinger liquid theory.

Luttinger models for nanotubes differ significantly from their quantum Hall effect counterpart, because of their non-chiral character. Forward and backward fields describing collective excitations effectively mix, because the interactions between electrons are spread along the whole length of the nanotube. For this reason, a straightforward transposition of the results obtained for chiral edge system proves difficult. Nevertheless, non–chiral Luttinger liquids can be described with chiral fields[9,10]. Such chiral fields correspond to excitations with anomalous (non-integer) charge, which has eluded detection so far.

In the present work, we propose an experimental geometry which allows to probe directly the underlying charges of the collective excitations. The setup consists of a nanotube whose bulk is contacted by a scanning tunneling microscope (STM) tip which injects electrons, while both extremities of the nanotube collect the current (Fig. 1). The current, the noise and the noise correlations are computed, and the effective charges are determined by comparison with the Schottky formula[11] for an "infinite" nanotube, the striking result is that noise correlations contribute to second order in the electron tunneling, in sharp contrast with a fermionic system which requires fourth order. The noise correlations are then positive, because the tunneling electron wave function is split in two counter propagating

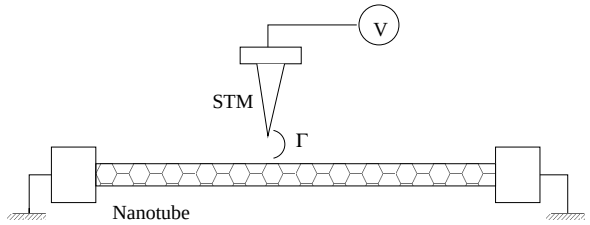

FIG. 1: Schematic configuration of the nanotube–STM device: electrons are injected from the tip at $x = 0$: current is measured at both nanotube ends, which are set to the ground.

modes of the collective excitations in the nanotube. We conjecture that in the presence of 1D Fermi liquid leads, modeled as in Ref.[12], the absence of renormalization/interaction effects of the nanotube is recovered.

A recent two terminal experiment studied the current–current fluctuations in ropes of nanotubes[13]. There, it is pointed out that the strong reduction of the low frequency noise cannot be understood within the context of scattering theory[14]. Naive comparison with existing non-chiral Luttinger liquid models[10] would imply an interaction parameter much inferior to the free electron case. Also, we mention that other multi-terminal geometries where a nanotube or a one–dimensional wire is attached to more than two leads, have been considered[15,16,17,18,19]. Our proposal deals with the same geometry as Ref[16], where a renormalization analysis identified the exponents of the current voltage characteristics. However, here the emphasis is put on the low frequency current fluctuation spectrum, both for the autocorrelation and the cross correlations between the two ends of the nanotube.

The paper is organized as follows: the Hamiltonian of our setup is specified in the next section, followed by a general non-equilibrium scheme based on the Keldysh formalism to study transport in this device, which is independent of the type of leads chosen (Sect. 3). Results for a nanotube connected to leads are then presented in Sect. 4. A connection with the effective charges of Refs.[9,10,16] is established in Sect. 5.

II. MODEL HAMILTONIAN

The transport geometry (Fig. 1) implies tunneling from the tip (normal or ferromagnetic metal) to the nanotube, and subsequent propagation of collective excitations along the nanotube. In the absence of tunneling, the Hamiltonian is thus simply the sum of the nanotube Hamiltonian, described by a two mode Luttinger liquid, together with the tip Hamiltonian. Using the standard conventions[20], the operator describing an electron with spin σ moving along the direction r, from mode α is specified in terms of a bosonic field:

$$\Psi_{r\alpha\sigma}(x,t) = \frac{1}{\sqrt{2\pi a}} e^{i\alpha k_F x + i r q_F x + i\varphi_{r\alpha\sigma}(x,t)} \ , \tag{1}$$

with a a short distance cutoff, k_F the Fermi momentum, q_F the momentum mismatch associated with the two modes, and the convention $r = \pm$, $\alpha = \pm$ and $\sigma = \pm$ are chosen for the direction of propagation, for the nanotube branch, and for the spin orientation. It is convenient to express this bosonic phase in terms of the conventional non-chiral Luttinger liquid fields $\theta_{j\delta}$ and $\phi_{j\delta}$, with $j\delta \in \{c+, c-, s+, s-\}$ identifying the charge/spin and total/relative fields:

$$\varphi_{r\alpha\sigma}(x,t) = \sqrt{\frac{\pi}{2}} \sum_{j\delta} h_{\alpha\sigma j\delta} (\phi_{j\delta}(x,t) + r\theta_{j\delta}(x,t)) \ , \tag{2}$$

with $h_{\alpha\sigma c+} = 1$, $h_{\alpha\sigma c-} = \alpha$, $h_{\alpha\sigma s+} = \sigma$ et $h_{\alpha\sigma s-} = \alpha\sigma$. $\theta_{j\delta}$ and $\phi_{j\delta}$ are dual non-chiral fields. A plausible alternative would have been to express $\varphi_{r\alpha\sigma}$ in terms of the chiral Luttinger liquid fields. However, the present choice will be simpler later on when dealing with inhomogeneous Luttinger liquids (in order to include the leads), as the Green's functions for $\theta_{j\delta}$, $\phi_{j\delta}$ are known. The Hamiltonian which describes the collective excitations in the nanotube has the standard form:

$$H = \frac{1}{2} \sum_{j\delta} \int_{-\infty}^{\infty} dx \left(v_{j\delta} K_{j\delta} (\partial_x \phi_{j\delta}(x,t))^2 + \frac{v_{j\delta}}{K_{j\delta}} (\partial_x \theta_{j\delta}(x,t))^2 \right) \ , \tag{3}$$

with an interaction parameter $K_{j\delta}$ and velocity $v_{j\delta}$.

For the STM tip, one assumes for simplicity that only one electronic mode couples to the nanotube. The tip can thus be described by a semi-infinite Luttinger liquid, as in Kondo type problems. This turns out to be convenient in this problem where both bosonized nanotube fermions operators and tip fermions operators intervene. For the sake of generality, we allow the two spin components of the tip fields to have different Fermi velocities u_F^σ, which allows to treat the case of a ferromagnetic metal. The fermion operator at the tip location $x = 0$ is then:

$$c_\sigma(t) = \frac{1}{\sqrt{2\pi a}} e^{i\tilde{\varphi}_\sigma(t)} \ . \tag{4}$$

Here, $\tilde{\varphi}_\sigma$ is the chiral Luttinger liquid field, whose Keldysh Green's function at $x = 0$ is given by[21]:

$$\begin{aligned} g_{\sigma(\eta_1\eta_2)}(t_1,t_2) &\equiv \langle T_K \{ \tilde{\varphi}_\sigma(t_1^{\eta_1}) \tilde{\varphi}_\sigma(t_2^{\eta_2}) \} \rangle \\ &= -\frac{1}{2\pi} \ln \{ 1 + i[(\eta_1+\eta_2) sgn(t_1-t_2) - (\eta_1-\eta_2)] u_F^\sigma (t_1-t_2)/2a \} \ , \end{aligned} \tag{5}$$

where $\eta_{1,2} = \pm$ refer to the upper or lower branch of the Keldysh contour.

The tunneling Hamiltonian is a standard hopping term:

$$H_T(t) = \sum_{\varepsilon r\alpha\sigma} \Gamma^{(\varepsilon)}_{r\alpha\sigma}(t)[\Psi^\dagger_{r\alpha\sigma}(0,t)c_\sigma(t)]^{(\varepsilon)} \ . \tag{6}$$

Here the superscript (ε) leaves either the operators in bracket unchanged ($\varepsilon = +$), or transforms them into their Hermitian conjugate ($\varepsilon = -$). The voltage bias between the tip and the nanotube is included using the Peierls substitution: the hopping amplitude $\Gamma^{(\varepsilon)}_{r\alpha\sigma}$ acquires a time dependent phase $\exp(i\varepsilon\omega_0 t)$, with the bias voltage identified as $V = \hbar\omega_0/e$. We will use the convention $\hbar \to 1$. Similarly, the tunneling current is defined as:

$$I_T(t) = ie \sum_{\varepsilon r\alpha\sigma} \varepsilon\Gamma^{(\varepsilon)}_{r\alpha\sigma}(t)[\Psi^\dagger_{r\alpha\sigma}(0,t)c_\sigma(t)]^{(\varepsilon)} \ . \tag{7}$$

In Eqs. (1) and (4), we have omitted the Klein factors which guarantee the anti-commutation of the 3 types of fermions operators – written in terms of bosonic fields – for this problem: the two nanotube branches and the STM single mode. It has been established[5,22] that Klein factors are in principle necessary to treat multi-Luttinger system, as illustrated in the computation of noise correlations between three edge states in the FQHE. In the present work, Klein factors can be dropped because we intend to work with lowest order perturbation theory. To order Γ^2, statistical correlations between the three Luttinger systems do not occur. However, they should show up when calculating higher order corrections (Γ^4).

For this problem which implies propagation along the nanotube, it is also necessary to compute the (total) charge and (total) spin currents using the bosonized fields of Eq. (1):

$$\begin{aligned} I_\rho(x,t) &= ev_F \sum_{r\alpha\sigma} r\Psi^\dagger_{r\alpha\sigma}(x,t)\Psi_{r\alpha\sigma}(x,t) \\ &= 2ev_F\sqrt{\frac{2}{\pi}}\partial_x\phi_{c+}(x,t) \ . \end{aligned} \tag{8}$$

Similarly, we consider the spin current in the $\hat{z}$ direction:

$$\begin{aligned} I_{\sigma_z}(x,t) &= ev_F \sum_{r\alpha\sigma} r\Psi^\dagger_{r\alpha\sigma}(x,t)\sigma_z\Psi_{r\alpha\sigma}(x,t) \\ &= 2ev_F\sqrt{\frac{2}{\pi}}\partial_x\phi_{s+}(x,t) \ . \end{aligned} \tag{9}$$

Note that the contribution from terms containing $2k_F$ oscillations has been dropped. This is equivalent to requiring that the current measurement along the nanotube is effectively a spatial average over a length scale larger than λ_F. In practice, $2k_F$ terms are necessary in order to establish a connection between current fluctuations and density fluctuations.

III. NON EQUILIBRIUM TRANSPORT FORMALISM

In this section, the general approach used to calculate the tunneling current and noise, as well as the current and noise in the nanotube is described. All quantities are computed at zero temperature for simplicity. The calculation of the tunneling current and noise is quite similar to the perturbative results in Ref.[2] for the FQHE. Here it is summarized in order to compare with the nanotube transport quantities.

A. Tunneling current and noise

The Keldysh technique is used to compute the average tunneling current and noise. We adopt the convention that the coefficients $\eta, \eta_{1,2} = \pm$ identify the upper/lower branch of the Keldysh contour:

$$\langle I_T(t)\rangle = \frac{1}{2}\sum_\eta \langle T_K\{I_T(t^\eta)e^{-i\int_K dt_1 H_T(t_1)}\}\rangle \ , \tag{10}$$

$$S_T(t,t') = \frac{1}{2}\sum_\eta \langle T_K\{I_T(t^\eta)I_T(t'^{-\eta})e^{-i\int_K dt_1 H_T(t_1)}\}\rangle \ , \tag{11}$$

which applies in typical tunneling situations where the product of the current averages is of order Γ^4. In order to collect the lowest order contribution in the tunneling amplitude, the exponential is expanded to first order for the current, and to zeroth order for the noise:

$$\langle I_T(t)\rangle = \frac{e\Gamma^2}{2}\sum_{r\alpha\sigma\varepsilon\eta\eta_1}\eta\varepsilon\int_{-\infty}^{+\infty}dt_1 e^{-i\varepsilon\omega_0(t-t_1)}\langle T_K\{\Psi_{r\alpha\sigma}(0,t^\eta)^{(\varepsilon)}\Psi_{r\alpha\sigma}(0,t_1^{\eta_1})^{(-\varepsilon)}\}\rangle\langle T_K\{c_\sigma(t^\eta)^{(-\varepsilon)}c_{\sigma_1}(t_1^{\eta_1})^{(\varepsilon)}\}\rangle \quad (12)$$

$$S_T(t,t') = \frac{e^2\Gamma^2}{2}\sum_{r\alpha\sigma\varepsilon\eta} e^{-i\varepsilon\omega_0(t-t')}\langle T_K\{\Psi^+_{r\alpha\sigma}(0,t^\eta)^{(\varepsilon)}\Psi^+_{r\alpha\sigma}(0,t'^{-\eta})^{(-\varepsilon)}\}\rangle\langle T_K\{c_\sigma(t^\eta)^{(\varepsilon)}c_\sigma(t'^{-\eta})^{(-\varepsilon)}\}\rangle\ , \quad (13)$$

where the last factor in Eqs. (12) and (13) is the tip fermion Green's function. Next the nanotube and tip fields are specified in terms of the bosonized fields (nonchiral and chiral), and the two Keldysh ordered exponential products are computed:

$$\langle I_T(t)\rangle = \frac{e\Gamma^2}{2(2\pi a)^2}\sum_{r\alpha\sigma\varepsilon\eta\eta_1}\eta\varepsilon\int_{-\infty}^{+\infty}dt_1 e^{-i\varepsilon\omega_0(t-t_1)}e^{2\pi g_{\sigma(\eta\eta_1)}(t-t_1)}$$
$$\times e^{\frac{\pi}{2}\sum_{j\delta}(G^{\phi\phi}_{j\delta(\eta\eta_1)}(0,0,t-t_1)+rG^{\phi\theta}_{j\delta(\eta\eta_1)}(0,0,t-t_1)+rG^{\theta\phi}_{j\delta(\eta\eta_1)}(0,0,t-t_1)+G^{\theta\theta}_{j\delta(\eta\eta_1)}(0,0,t-t_1))}\ , \quad (14)$$

$$S_T(t,t') = \frac{e^2\Gamma^2}{(2\pi a)^2}\sum_{r\alpha\sigma\varepsilon\eta} e^{-i\varepsilon\omega_0(t-t')}e^{2\pi g_{\sigma(\eta-\eta)}(t-t')}$$
$$\times e^{\frac{\pi}{2}\sum_{j\delta}(G^{\phi\phi}_{j\delta(\eta-\eta)}(0,0,t-t')+rG^{\phi\theta}_{j\delta(\eta-\eta)}(0,0,t-t')+rG^{\theta\phi}_{j\delta(\eta-\eta)}(0,0,t-t')+G^{\theta\theta}_{j\delta(\eta-\eta)}(0,0,t-t'))}\ . \quad (15)$$

As expected, the stationary current and the real time current correlator call for the time differences $t-t_1$, $t-t'$ only. Integrating over time, the zero frequency noise is introduced. Further using the symmetry properties of the Green's functions $g_{\sigma(\eta\eta)}(\tau)=g_{\sigma(\eta\eta)}(|\tau|)$ and $G^{\phi\phi}_{j\delta(\eta\eta)}(0,0,\tau)=G^{\phi\phi}_{j\delta(\eta\eta)}(0,0,|\tau|)$ (similarly for $\theta\theta$, $\theta\phi$ and $\phi\phi$), only $\eta=-\eta_1$ is retained for the current:

$$\langle I_T\rangle = -\frac{2ie\Gamma^2}{(2\pi a)^2}\sum_{r\sigma\eta}\eta\int_{-\infty}^{+\infty}d\tau\sin(\omega_0\tau)e^{2\pi g_{\sigma(\eta-\eta)}(\tau)}e^{\frac{\pi}{2}\sum_{j\delta}(G^{\phi\phi}_{j\delta(\eta-\eta)}(0,0,\tau)+rG^{\phi\theta}_{j\delta(\eta-\eta)}(0,0,\tau)+rG^{\theta\phi}_{j\delta(\eta-\eta)}(0,0,\tau)+G^{\theta\theta}_{j\delta(\eta-\eta)}(0,0,\tau))}\ , \quad (16)$$

$$S_T(\omega=0) = -\frac{e^2\Gamma^2}{(\pi a)^2}\sum_{r\sigma\eta}\int_{-\infty}^{+\infty}d\tau\cos(\omega_0\tau)e^{2\pi g_{\sigma(\eta-\eta)}(\tau)}e^{\frac{\pi}{2}\sum_{j\delta}(G^{\phi\phi}_{j\delta(\eta-\eta)}(0,0,\tau)+rG^{\phi\theta}_{j\delta(\eta-\eta)}(0,0,\tau)+rG^{\theta\phi}_{j\delta(\eta-\eta)}(0,0,\tau)+G^{\theta\theta}_{j\delta(\eta-\eta)}(0,0,\tau))}\ . \quad (17)$$

The tunneling current and noise imply the knowledge of the Green's functions at the tunneling location only.

B. Nanotube current and noise

The operator averages along the nanotube require a perturbative calculation up to second order in the tunneling Hamiltonian for the tunneling current and for the noise. Tunneling of an electron from the STM tip is followed by propagation of the collective excitations of the Luttinger liquid towards both ends of the nanotube.

$$\langle I_\rho(x,t)\rangle = -\frac{1}{4}\sum_{\eta\eta_1\eta_2}\eta_1\eta_2\langle T_K\{I_\rho(x,t^\eta)\int\int dt_1dt_2H_T(0,t_1^{\eta_1})H_T(0,t_2^{\eta_2})\}\rangle\ , \quad (18)$$

$$S_\rho(x,t;x',t') = -\frac{1}{4}\sum_{\eta\eta_1\eta_2}\eta_1\eta_2\langle T_K\{I_\rho(x,t^\eta)I_\rho(x',t'^{-\eta})\int\int dt_1dt_2H_T(0,t_1^{\eta_1})H_T(0,t_2^{\eta_2})\}\rangle\ , \quad (19)$$

where the contribution to the noise coming from $\langle I_\rho(x,t)\rangle\langle I_\rho(x',t')\rangle$ has been dropped because it contributes to order Γ^4. Expressing the Hamiltonian in terms of the fields, the limit $\lim_{\gamma\to 0}(i\gamma)^{-1}\partial_x\exp[i\gamma\phi_{c+}]=\partial_x\phi_{c+}$ is used in order to cast the time ordered averages into correlators of exponentials only:

$$\langle I_\rho(x,t)\rangle = -\frac{ev_F\Gamma^2}{4\pi a}\sqrt{\frac{2}{\pi}}\sum_{\eta\eta_1\eta_2\varepsilon_1r_1\alpha_1\sigma_1}\eta_1\eta_2\int\int dt_1dt_2e^{-i\varepsilon_1\omega_0(t_1-t_2)}\langle T_K\{c^{(-\varepsilon_1)}_{\sigma_1}(t_1^{\eta_1})c^{(\varepsilon_1)}_{\sigma_1}(t_2^{\eta_2})\}\rangle$$

$$\times \lim_{\gamma\to 0}\frac{1}{i\gamma}\partial_x\langle T_K\{e^{i\gamma\phi_{c+}(x,t^\eta)}e^{-i\varepsilon_1\varphi_{r_1\alpha_1\sigma_1}(0,t_1^{\eta_1})}e^{i\varepsilon_1\varphi_{r_1\alpha_1\sigma_1}(0,t_2^{\eta_2})}\}\rangle \ , \tag{20}$$

$$S_\rho(x,t;x',t') = -\frac{e^2v_F^2\Gamma^2}{\pi^2 a}\sum_{\eta\eta_1\eta_2\varepsilon_1 r_1\alpha_1\sigma_1}\eta_1\eta_2\int\int dt_1dt_2e^{-i\varepsilon_1\omega_0(t_1-t_2)}\langle T_K\{c_{\sigma_1}^{(-\varepsilon_1)}(t_1^{\eta_1})c_{\sigma_1}^{(\varepsilon_1)}(t_2^{\eta_2})\}\rangle$$

$$\times \lim_{\gamma\to 0}\frac{1}{\gamma^2}\partial_x\partial_{x'}\langle T_K\{e^{i\gamma\phi_{c+}(x,t^\eta)}e^{-i\gamma\phi_{c+}(x',t'^{-\eta})}e^{-i\varepsilon_1\varphi_{r_1\alpha_1\sigma_1}(0,t_1^{\eta_1})}e^{i\varepsilon_1\varphi_{r_1\alpha_1\sigma_1}(0,t_2^{\eta_2})}\}\rangle \ , \tag{21}$$

where the contribution from the STM tip is the same as before. The two time ordered products (one for the tip and one for the nanotube) are expressed in terms of Luttinger liquid Green's functions. Taking the spatial derivative, one obtains an expression with Green's functions as prefactors – implying propagation – as well as exponentiated Green's functions at the tunneling location. Operating variable changes in the integrals and noticing that only $\eta_1 = -\eta_2$ contributes, the current and noise become:

$$\langle I_\rho(x)\rangle = -\frac{ev_F\Gamma^2}{2\pi^2a^2}\sum_{\eta\eta_1 r_1\sigma_1}\int_{-\infty}^{+\infty}d\tau'\partial_x\left(G^{\phi\phi}_{c+(\eta\eta_1)}(x,0,\tau') - G^{\phi\phi}_{c+(\eta-\eta_1)}(x,0,\tau') + r_1G^{\phi\theta}_{c+(\eta\eta_1)}(x,0,\tau') - r_1G^{\phi\theta}_{c+(\eta-\eta_1)}(x,0,\tau')\right)$$

$$\times\int_{-\infty}^{+\infty}d\tau sin(\omega_0\tau)e^{2\pi g_{\sigma_1(\eta_1-\eta_1)}(\tau)}e^{\frac{\pi}{2}\sum_{j\delta}(G^{\phi\phi}_{j\delta(\eta_1-\eta_1)}(0,0,\tau)+G^{\theta\theta}_{j\delta(\eta_1-\eta_1)}(0,0,\tau)+r_1G^{\phi\theta}_{j\delta(\eta_1-\eta_1)}(0,0,\tau)+r_1G^{\theta\phi}_{j\delta(\eta_1-\eta_1)}(0,0,\tau))} \ , \tag{22}$$

$$S_\rho(x,x',\omega=0) = -\frac{e^2v_F^2\Gamma^2}{(\pi a)^2}\sum_{\eta\eta_1 r_1\sigma_1}$$

$$\times\int_{-\infty}^{+\infty}d\tau cos(\omega_0\tau)e^{2\pi g_{\sigma_1(\eta_1-\eta_1)}(\tau)}e^{\frac{\pi}{2}\sum_{j\delta}(G^{\phi\phi}_{j\delta(\eta_1-\eta_1)}(0,0,\tau)+r_1G^{\phi\theta}_{j\delta(\eta_1-\eta_1)}(0,0,\tau)+r_1G^{\theta\phi}_{j\delta(\eta_1-\eta_1)}(0,0,\tau)+G^{\theta\theta}_{j\delta(\eta_1-\eta_1)}(0,0,\tau))}$$

$$\times\int_{-\infty}^{+\infty}d\tau_1\partial_x\left(G^{\phi\phi}_{c+(\eta\eta_1)}(x,0,\tau_1) - G^{\phi\phi}_{c+(\eta-\eta_1)}(x,0,\tau_1) + r_1G^{\phi\theta}_{c+(\eta\eta_1)}(x,0,\tau_1) - r_1G^{\phi\theta}_{c+(\eta-\eta_1)}(x,0,\tau_1)\right)$$

$$\times\int_{-\infty}^{+\infty}d\tau_2\partial_{x'}\left(G^{\phi\phi}_{c+(-\eta\eta_1)}(x',0,\tau_2) - G^{\phi\phi}_{c+(-\eta-\eta_1)}(x',0,\tau_2) + r_1G^{\phi\theta}_{c+(-\eta\eta_1)}(x',0,\tau_2) - r_1G^{\phi\theta}_{c+(-\eta-\eta_1)}(x',0,\tau_2)\right) \ . \tag{23}$$

Note the temporal decoupling (which occurs after operating variable changes) in these expressions. The integral over τ contains information on electron tunneling at $x = 0$, while the remaining integrals involve propagation, thus the spatial dependence in the Green's functions arguments.

IV. CURRENT AND NOISE FOR AN INFINITE NANOTUBE

In the previous section, general expressions were derived for the current and noise, which are independent of the form of the Green's functions $G^{\phi\phi}_{j\delta}$, $G^{\phi\theta}_{j\delta}$, $G^{\theta\phi}_{j\delta}$ and $G^{\theta\theta}_{j\delta}$. The Green's functions are described in Appendix A and are used to compute the tunneling noise and current as well as the nanotube noise and current.

A. Tunneling current and noise

After substitution of the Green's function of a nanotube, the tunneling current and noise read:

$$\langle I_T\rangle = -\frac{2ie\Gamma^2}{(2\pi a)^2}\sum_{r\sigma\eta}\eta\int_{-\infty}^{+\infty}\frac{\sin(\omega_0\tau)d\tau}{(1-i\eta\frac{u_F^\sigma\tau}{a})(1-i\eta\frac{v_F\tau}{a})^\nu} \ , \tag{24}$$

$$S_T(\omega=0) = \frac{e^2\Gamma^2}{(\pi a)^2}\sum_{r\sigma\eta}\int_{-\infty}^{+\infty}\frac{\cos(\omega_0\tau)d\tau}{(1-i\eta\frac{u_F^\sigma}{a}\tau)(1-i\eta\frac{v_F}{a}\tau)^\nu} \ , \tag{25}$$

with the exponent:

$$\nu = \frac{1}{8}\sum_{j\delta}\left(K_{j\delta}+\frac{1}{K_{j\delta}}\right) \ . \tag{26}$$

ν is the bulk tunneling exponent of the current–voltage characteristics $\langle I_T(\omega_0)\rangle$.[6] The integrals are computed in Appendix B, we obtain :

$$\langle I_T\rangle \;=\; \frac{2e\Gamma^2}{\pi a}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right)\left(\frac{a}{v_F}\right)^\nu \frac{\mathrm{sgn}(\omega_0)|\omega_0|^\nu}{\mathbf{\Gamma}(\nu+1)}\ , \tag{27}$$

where we used the definition of the Gamma function $\mathbf{\Gamma}$. Only electrons can tunnel from the tip to the nanotube, so one can check that the classical Schottky formula holds always:

$$S_T(\omega=0) = e|\langle I_T\rangle|\ . \tag{28}$$

B. Nanotube current and noise

Some of the time integrals in Eq. (22) has already been encountered when computing the tunneling current and noise. The current and noise thus become:

$$\langle I_\rho(x)\rangle = -\frac{eiv_F\Gamma^2}{\pi a}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right)\left(\frac{a}{v_F}\right)^\nu \mathrm{sgn}(\omega_0)\frac{|\omega_0|^\nu}{\mathbf{\Gamma}(\nu+1)}\sum_{\eta\eta_1}\eta_1\int_{-\infty}^{+\infty} d\tau'\partial_x\left(G^{\phi\phi}_{c+(\eta\eta_1)}(x,0,\tau') - G^{\phi\phi}_{c+(\eta-\eta_1)}(x,0,\tau')\right)\ , \tag{29}$$

$$\begin{aligned}
S_\rho(x,x',\omega=0) &= -\frac{2e^2v_F^2\Gamma^2}{\pi a}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right)\left(\frac{a}{v_F}\right)^\nu \frac{|\omega_0|^\nu}{\mathbf{\Gamma}(\nu+1)}\\
&\times\Bigg[\sum_{\eta\eta_1}\int_{-\infty}^{+\infty} d\tau_1\partial_x\left(G^{\phi\phi}_{c+(\eta\eta_1)}(x,0,\tau_1) - G^{\phi\phi}_{c+(\eta-\eta_1)}(x,0,\tau_1)\right)\int_{-\infty}^{+\infty} d\tau_2\partial_{x'}\left(G^{\phi\phi}_{c+(-\eta\eta_1)}(x',0,\tau_2) - G^{\phi\phi}_{c+(-\eta-\eta_1)}(x',0,\tau_2)\right)\\
&+\sum_{\eta\eta_1}\int_{-\infty}^{+\infty} d\tau_1\partial_x\left(G^{\phi\theta}_{c+(\eta\eta_1)}(x,0,\tau_1) - G^{\phi\theta}_{c+(\eta-\eta_1)}(x,0,\tau_1)\right)\int_{-\infty}^{+\infty} d\tau_2\partial_{x'}\left(G^{\phi\theta}_{c+(-\eta\eta_1)}(x',0,\tau_2) - G^{\phi\theta}_{c+(-\eta-\eta_1)}(x',0,\tau_2)\right)\Bigg]\\
&= -\frac{2e^2v_F^2\Gamma^2}{\pi a}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right)\left(\frac{a}{v_F}\right)^\nu \frac{|\omega_0|^\nu}{\mathbf{\Gamma}(\nu+1)}(I^{\phi\phi}(x,x') + I^{\phi\theta}(x,x'))\ ,
\end{aligned} \tag{30}$$

where the last factors are computed in Appendix B. The standard assumptions of the calculation of the tunneling current and noise are recalled, as the same expressions appear in both results. We obtain:

$$\langle I_\rho(x)\rangle \;=\; \frac{e\Gamma^2}{\pi a}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right)\left(\frac{a}{v_F}\right)^\nu \frac{|\omega_0|^\nu \mathrm{sgn}(\omega_0)}{\mathbf{\Gamma}(\nu+1)}\mathrm{sgn}(x)\ , \tag{31}$$

$$S_\rho(x,x',\omega=0) \;=\; \frac{(K_{c+})^2 + \mathrm{sgn}(x)\mathrm{sgn}(x')}{2}e|\langle I_\rho(x)\rangle|\ . \tag{32}$$

Current conservation $|\langle I_\rho(x)\rangle| = |\langle I_T\rangle|/2$ is shown to hold. Results are valid for arbitrary voltages, with the expected power law behavior.

V. DISCUSSION

A. Local current correlations

One accepted diagnosis to detect effective or anomalous charges is to compare the noise with the associated current with the Schottky formula in mind. A striking result is that despite the fact that electrons are tunneling from the STM tip to the bulk of the nanotube, the zero frequency current fluctuations are proportional to the current for $x' = x >> a$:

$$S_\rho(x,x,\omega=0) = \frac{1+(K_{c+})^2}{2}e|\langle I_\rho(x)\rangle|\ , \tag{33}$$

with an anomalous effective charge for an infinite nanotube.

B. Positive cross-correlations

More can be learned from a measurement of the noise correlations. Noise correlations have been proposed to detect statistical correlations in quantum transport[5,14]. Indeed, our geometry can be considered as a Hanbury-Brown and Twiss correlation device. Such experiments have now been completed for photons and more recently for electrons in quantum waveguides. Here the novelty is that electronic excitations do not represent the right eigenmodes of the nanotube. For $x' = -x >> a$ the noise correlations read:

$$S_\rho(x, -x, \omega = 0) = -\frac{1-(K_{c+})^2}{2} e|\langle I_\rho(x)\rangle| \ . \tag{34}$$

This is a priori negative. However, if the current direction is chosen to be positive from the tip to the extremities of the nanotube, the sign of the cross–correlations is positive. Recall that the fermionic version of the Hanbury-Brown and Twiss experiment yields negative noise correlations[14,23]. So far, positive noise correlations have been attributed in priority to bosonic systems[24]. Nevertheless, there are at least two other situations where they are encountered. First, when the source of particle is a superconductor, noise correlations can also be positive depending on the junction configuration[25,26,27,28]. Second, they also occur in systems with floating voltage probes[29]. In the case of a superconductor, the emission of electron pairs through separate quantum dots guarantee that the noise correlations are always positive: a (singlet) entangled electron pair is generated outside the superconductor[30,31].

Note that the prefactors in Eq. (34) can readily be interpreted using the language of Ref.[9,10,16]. A tunneling event to the bulk of a nanotube is accompanied by the propagation of two counter-propagating charges $Q_\pm = (1 \pm K_{c+})/2$. Recall that the subscript $c+$ identifies the charge (as opposed to spin) excitation given by the total (rather than relative) contribution of the two modes propagating in the nanotube. Each charge is as likely to go right or left. According to Ref.[10] electron injection in a Luttinger liquid is characterized by chiral charges $Q_\pm$ and chiral spin charges $S_\pm$ which describe the elementary excitations of the nanotube.

$$\begin{pmatrix} Q_+ \\ Q_- \\ S_+ \\ S_- \end{pmatrix} = \sum_\sigma \left[n_\sigma \begin{pmatrix} 1 \\ 1 \\ \sigma/2 \\ \sigma/2 \end{pmatrix} + J_\sigma \begin{pmatrix} (1+K_{c+})/2 \\ (1-K_{c+})/2 \\ \sigma(1+K_{s+})/4 \\ \sigma(1-K_{s+})/4 \end{pmatrix} \right] \ , \tag{35}$$

with integers $n_\sigma, J_\sigma = 0, 1, 2, ...$ ($\sigma = \uparrow, \downarrow$). In particular, the addition of an electron with spin σ corresponds to the choice $n_\sigma = 0$ and $J_\sigma = 1$.

The current noise and noise correlations can be interpreted as an average over the two types of excitations:

$$S_\rho(x, x) \ \sim \ \frac{(Q_+^2 + Q_-^2)}{2} = \frac{1+(K_{c+})^2}{4} \ , \tag{36}$$

$$S_\rho(x, -x) \ \sim \ -Q_+ Q_- = -\frac{1-(K_{c+})^2}{4} \ . \tag{37}$$

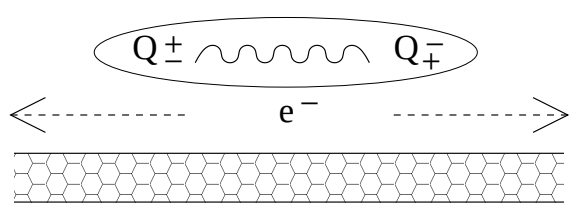

FIG. 2: Schematic description of entangled quasiparticles $Q_\pm$ being emitted right and left of the tunnel junction.

A drawing where the two types of charges "flow away" from the tip while propagating along the nanotube is depicted in the lower part of Fig. 2. Both charges $Q_\pm$ are equally likely to go right or left, and they are emitted as a pair with opposite labels. The noise correlations of Eq.(37) are rendered positive if one adopts the standard convention for measuring the current in multi-terminal conductors[1]. Here these "positive" noise correlations resulting from charges moving toward both extremities of the nanotube have the added particularity that they occur to second order in a perturbative tunneling calculation. In superconducting-normal systems, the two electrons which emanate from the same Cooper pair and which propagate in the two Luttinger liquids provide a manifestation of the non local character

of quantum mechanics. In the present case, only one electron is injected, but it is split into left and right excitations, unless one imposes one dimensional Fermi liquid leads. Here, we are dealing with entanglement between collective excitations of the Luttinger liquid. Written in terms of the chiral quasiparticle fields, the addition of an electron with given spin σ on a nanotube in the ground state $|O_{LL}\rangle$ gives:

$$\sum_{r\alpha}\Psi^{\dagger}_{r\alpha\sigma}(x=0)|O_{LL}\rangle = \frac{1}{\sqrt{2\pi a}}\sum_{r\alpha}\exp\left[-i\sum_{j\delta}\sqrt{\frac{\pi}{2K_{j\delta}}}h_{\alpha\sigma j\delta}\left(\frac{1+rK_{j\delta}}{2}\tilde{\varphi}^{+}_{j\delta}(x)+\frac{1-rK_{j\delta}}{2}\tilde{\varphi}^{-}_{j\delta}(x)\right)\right]|O_{LL}\rangle \ , \quad (38)$$

with $\tilde{\varphi}^{r}_{j\delta}$ the chiral bosonic fields of the (nonchiral) Luttinger liquid. This wave function is characterized by right and left movers $r=\pm$ whose fields appear explicitly in the phase operator of this many-particle wave function. These fields are independent of each other, therefore the exponential can be written as a product of fields:

$$\sum_{r\alpha}\Psi^{\dagger}_{r\alpha\sigma}(x=0)|O_{LL}\rangle = \frac{1}{\sqrt{2\pi a}}\sum_{\alpha}\prod_{j\delta}\left[(\tilde{\psi}^{\dagger}_{j\delta+})^{Q_{j\delta+}}(\tilde{\psi}^{\dagger}_{j\delta-})^{Q_{j\delta-}}+(\tilde{\psi}^{\dagger}_{j\delta+})^{Q_{j\delta-}}(\tilde{\psi}^{\dagger}_{j\delta-})^{Q_{j\delta+}}\right]|O_{LL}\rangle \ , \quad (39)$$

where for each sector (charge/spin, total/relative mode) the charges $Q_{j\delta\pm}=(1\pm K_{j\delta})/2$ have been introduced, and chiral fractional operators are defined as:

$$\tilde{\psi}_{j\delta\pm}(x) = \exp\left[i\sqrt{\frac{\pi}{2K_{j\delta}}}h_{\alpha\sigma j\delta}\tilde{\varphi}^{\pm}_{j\delta}(x)\right] \ . \quad (40)$$

The wave function described by Eq. (39) has all the characteristics of an entangled state. Because the two types of excitations travel towards opposite ends of the nanotube, the time evolution of this "injected electron " state is simply obtained with the substitution $\tilde{\varphi}^{r}_{j\delta}(x)\to\tilde{\varphi}^{r}_{j\delta}(x-rv_{j\delta}t)$. Consequently, quantum mechanical non-locality is quite explicit here. The detection of a charge $Q_{\pm}$ in one arm is necessarily accompanied by the simultaneous detection of a charge $Q_{\mp}$ in the other extremity of the nanotube.

This entanglement is the direct consequence of the correlated state of the Luttinger liquid. When additional electrons are injected, these break up into the specific modes which can propagate in either direction in the nanotube. It therefore differs significantly from its analogs which use superconductors as electron injectors, where two electrons from the same Cooper pair are dissociated[30,31,32,33,34].

When considering only one sector, such as $j\delta=c+$, it is interesting to note that the wave function has the same structure of say, a triplet spin state (a symmetric combination of "up" and "down" states, or "plus" and "minus" charges) for electrons, with the electrons being replaced by chiral quasiparticle operators. Indeed, one has to recognize that each chiral field $\tilde{\varphi}^{r}_{j\delta}$ can be written as a superposition of boson operators:

$$\tilde{\varphi}^{r}_{j\delta}(x) = \frac{1}{4\sqrt{K_{j\delta}}}\sum_{r'\alpha\sigma}h_{\alpha\sigma j\delta}(r+r'K_{j\delta})\sum_{(r'k)>0}\sqrt{\frac{1}{|k|L}}\left(d^{\dagger}_{\alpha\sigma}(k)e^{-ikx}+d_{\alpha\sigma}(k)e^{ikx}\right)e^{-a|k|/2} \ , \quad (41)$$

where $d^{\dagger}_{\alpha\sigma}(k)$ creates a boson with nanotube mode α, spin σ and momentum k, and characterizes the collective modes of the one dimensional liquid. According to the state written in Eq. (40), this linear superposition of boson operators appears in an exponential. This expresses that non-local "many–boson" correlations are created when an electron is injected in a nanotube, and these many-body states are entangled in the present geometry.

C. Spin current

Effects similar to the detection of effective charges show up in the spin sector when time reversal symmetry ($K_{s+}\neq 1$) does not hold. The spin current and spin noise are obtained in a similar manner:

$$\langle I_{\sigma_z}(x)\rangle \ = \ \frac{e\Gamma^2}{\pi a}\left(\sum_{\sigma}\frac{\sigma}{u^{\sigma}_F}\right)\frac{sgn(\omega_0)|\omega_0|^{\nu}}{\mathbf{\Gamma}(\nu+1)}\left(\frac{a}{v_F}\right)^{\nu}sgn(x) \ . \quad (42)$$

So that at large distances:

$$S_{\sigma_z}(x,-x,\omega=0) \ = \ -\frac{1-(K_{s+})^2}{2}e|\langle I_{\sigma_z}(x)\rangle| \ , \quad (43)$$

$$S_{\sigma_z}(x,x,\omega=0) \ = \ \frac{1+(K_{s+})^2}{2}e|\langle I_{\sigma_z}(x)\rangle| \ . \quad (44)$$

In practice, when time reversal symmetry holds ($K_{s+} = 1$), spin noise correlations vanish to order Γ^2 independently from the presence or the nature of the leads. In the case where the tip is non magnetized, the spin current and spin noise correlations also vanish.

D. 1D Fermi liquid leads

In the presence of one-dimensional Fermi liquid leads, where the leads are considered to be Luttinger liquids whose interaction parameters are set to $K^L_{j\delta} \equiv 1$, quasiparticles suffer Andreev type reflections[9] at both exterminites of the nanotubes. Multiple reflections of quasiparticles in the Fabry-Perot geometry – Fermi liquid/Nanotube/Fermi liquid – are expected to lead to a concellation of the interaction effects in the nanotube, as in the two terminal calculations of conductance and noise[12]. Although the detailed calculation is not presented here, dimensional analysis of the time integrals suggest that, the nanotube current and noise read:

$$\langle I_\rho(x)\rangle = \frac{e\Gamma^2\omega_0}{\pi v_F}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right) sgn(x) \ , \tag{45}$$

$$S_\rho(x,x',\omega=0) = \frac{1+sgn(x)sgn(x')}{2} e|\langle I_\rho(x)\rangle| \ . \tag{46}$$

For $x = x'$, this whould give the classical Schottky formula, in the very same spirit as in Ref.[12]. For x and x' on opposite ends of the nanotube, this noise correlator should vanish, to this order: the scattering theory result has a lowest non vanishing contribution of order Γ^4.

This low voltage result is modified by a higher power law behavior at higher voltage, with a threshold voltage specified by the size of the system $\hbar v_F/L$ as in Ref.[12].

VI. CONCLUSION

In summary, a diagnosis for detecting the chiral excitations of a Luttinger liquid nanotube has been presented, which is based on the knowledge of low frequency current fluctuation spectrum in the nanotube. Typical transport calculations either address the propagation in a nanotube, or compute tunneling I(V) characteristics. Here, both are addressed because they constitute the key for obtaining the quasiparticle charges. Both the noise (autocorrelation) and the noise correlations (cross-correlations) are needed to identify the charges $Q_\pm$. Independently, note that this measurement could also be confronted to other diagnoses of the nanotube interaction parameter using tunneling current voltage characteristics.

This result relies on the assumption that one dimensional Fermi liquid leads are avoided. Such leads have been treated in different approaches[35,36], and are also labelled radiative contacts. Radiative contacts imply equilibration with the electrons. In Ref.[16], both radiative contacts and equilibration with dressed eigenmodes were studied, with the obvious result that Luttinger liquid renormalization shows up in the conductance in the latter case. In special circumstances such as the case of Ref.[13] the nanotube is embedded in the metallic contacts, and it is suspended by its ends. Here, the absence of a screening gate is explicit. Electron transport between these two entities likely occurs in multiple electron scattering processes as studied in Ref.[37]. In these, or other contacts fabricated by growing techniques[34], quantities such as current and noise may not be affected by the presence of the contacts.

Standard fermion results should be recovered when the system is connected on one dimensional Fermi liquid leads. The auto-correlation noise in one end of the nanotube should be related to the charge current with the standard Schottky formula. The noise correlation signal should also vanish as expected and the next order correction $O(\Gamma^4)$ then needs to be computed.

A crucial test of the contacts is in order. It should be possible in practical situations to analyze the type of contacts which one has between the nanotube and its connections. If the ratio of the cross-correlations to the current $S_\rho(x,-x,\omega_0)/\langle I_\rho(x)\rangle$ does not depend on the tunneling distance ($\log\Gamma$), both contributions are of order Γ^2 and this constitutes an indication that the contacts do not affect this quasiparticle entanglement. If we are dealing with a Fermi liquid behavior, the noise correlation–current ratio should behave like Γ^2, rather than a constant.

Finally, we have remarked that the many-body wave function which describes a Luttinger liquid with an added electron has necessarily EPR[38] entangled degrees of freedom. Both electrons chiralities contribute to the emission of quasiparticle pairs moving in opposite direction. This entanglement involves many particle states, unlike its electron counterpart. A suggestion for detection of such Luttinger liquid entanglement without perturbing the system with leads is nevertheless needed. The issue – how to detect this many-body entanglement – should be addressed while

taking into account different models for the leads, possibly involving multiple reflections within one contact[37]. Multiple reflections of the quasiparticles from one contact to the other kill this entanglement in one dimensional Fermi liquid leads, which is implicit in the vanishing of the noise correlations to this order. At any rate, this is the first time that collective excitations entanglement is discussed in a condensed matter setting.

Acknowledgments

Discussions with A. Lebedev, I. Safi on the Keldysh Green's function and with M. Büttiker are gratefully acknowledged.

APPENDIX A: GREEN'S FUNCTIONS IN THE PRESENCE OF CONTACTS

In this appendix, the Green's functions are computed, assuming a Luttinger liquid with an homogeneous interaction parameter $K_{j\delta}$ and velocity $v_{j\delta}$. The product $v_{j\delta}K_{j\delta}$ corresponds to the Fermi velocity v_F.

The finite temperature action associated with this problem has the general form:

$$S = \frac{1}{2}\sum_{j\delta}\int_0^\beta d\tau \int_{-\infty}^{\infty} dx \Big(v_{j\delta}K_{j\delta}(\partial_x\phi_{j\delta}(x,\tau))^2 + \frac{v_{j\delta}}{K_{j\delta}}(\partial_x\theta_{j\delta}(x,\tau))^2 + 2i(\partial_x\phi_{j\delta}(x,\tau))(\partial_\tau\theta_{j\delta}(x,\tau))\Big) \ . \tag{A1}$$

Which implies that the Fourier transform of the time ordered Green's functions $G^{\theta\theta}_{j\delta(T)}$ and $G^{\phi\phi}_{j\delta(T)}$, define as:

$$G^{\theta\theta}_{j\delta(T)}(x,x',t) = \langle T\theta_{j\delta}(x,t)\theta_{j\delta}(x',0)\rangle - \langle T\theta^2_{j\delta}(x,t)\rangle \ , \tag{A2}$$

$$G^{\phi\phi}_{j\delta(T)}(x,x',t) = \langle T\phi_{j\delta}(x,t)\phi_{j\delta}(x',0)\rangle - \langle T\phi^2_{j\delta}(x,t)\rangle \ , \tag{A3}$$

where T is the time ordered operator, satisfies the differential equations[12]:

$$\Big(\frac{\omega^2}{v_{j\delta}K_{j\delta}} - \partial_x\frac{v_{j\delta}}{K_{j\delta}}\partial_x\Big)G^{\theta\theta}_{j\delta(T)}(x,x',\omega) = 4\pi\delta(x-x') \ , \tag{A4}$$

$$\Big(-\frac{K_{j\delta}\omega^2}{v_{j\delta}} + \partial_x v_{j\delta}K_{j\delta}\partial_x\Big)G^{\phi\phi}_{j\delta(T)}(x,x',\omega) = 4\pi\delta(x-x') \ . \tag{A5}$$

The Green's function $G^{\theta\theta}_{j\delta(T)}$ is continuous everywhere, and $v_{j\delta}[\partial_x G^{\theta\theta}_{j\delta(T)}]/K_{j\delta}$ has a discontinuity at $x = x'$. The similarity between Eq. (A4) and (A5) results from the duality properties of the underlying fields. All information on $G^{\phi\phi}_{j\delta(T)}$ is obtained by dividing $G^{\theta\theta}_{j\delta(T)}$ by $K^2_{j\delta}$.

According to Eqs. (14) and (15), there are additional Green's functions in our problem which involve the fields θ and ϕ. For instance:

$$G^{\phi\theta}_{j\delta(T)}(x,x',t) = \langle T\phi_{j\delta}(x,t)\theta_{j\delta}(x',0)\rangle - \langle T\phi_{j\delta}(x,t)\theta_{j\delta}(x,t)\rangle \ . \tag{A6}$$

Using the action (A1) one can show that (t is a real time variable):

$$\langle\partial_x\phi_{j\delta}(x,t)\theta_{j\delta}(x',0)\rangle = \frac{1}{v_{j\delta}K_{j\delta}}\langle\partial_t\theta_{j\delta}(x,t)\theta_{j\delta}(x',0)\rangle \ , \tag{A7}$$

and similarly for $G^{\theta\phi}_{j\delta(T)}$.

From these real time Green's functions, we further specify the Keldysh matrix elements which two times $t, 0$ are assigned to the upper/lower branch $(++, +-, -+, --)$. Given an arbitrary real time Green's function $G(x,x',t) = \langle A(x,t)B(x',0)\rangle - \langle A(x,t)B(x,t)\rangle$ a general procedure[39] for obtaining these elements is as follows:

$$G^{\theta\theta}_{j\delta(K)}(x,x',t) = \begin{pmatrix} G^{\theta\theta}_{j\delta}(x,x',|t|) & G^{\theta\theta}_{j\delta}(x',x,-t) \\ G^{\theta\theta}_{j\delta}(x,x',t) & G^{\theta\theta}_{j\delta}(x',x,-|t|) \end{pmatrix} \ , \tag{A8}$$

where:

$$G^{\theta\theta}_{j\delta}(x,x',t) = -\frac{K_{j\delta}}{8\pi}\sum_r \ln\Big(1 + i\frac{v_F t}{a} + ir\frac{K_{j\delta}(x-x')}{a}\Big) \ . \tag{A9}$$

The same applies to $G^{\phi\phi}_{j\delta(K)}$ for which we have:

$$G^{\phi\phi}_{j\delta}(x,x',t) = \; -\frac{1}{8\pi K_{j\delta}}\sum_r \ln\left(1+i\frac{v_F t}{a}+ir\frac{K_{j\delta}(x-x')}{a}\right) \; . \tag{A10}$$

The mixed correlators read:

$$G^{\phi\theta}_{j\delta(K)}(x,x',t) = \left(\begin{array}{cc} \begin{array}{c} t>0: G^{\phi\theta}_{j\delta}(x,x',t) \\ t<0: G^{\theta\phi}_{j\delta}(x',x,-t) \end{array} & G^{\theta\phi}_{j\delta}(x',x,-t) \\ G^{\phi\theta}_{j\delta}(x,x',t) & \begin{array}{c} t>0: G^{\theta\phi}_{j\delta}(x',x,-t) \\ t<0: G^{\phi\theta}_{j\delta}(x,x',t) \end{array} \end{array}\right) \; . \tag{A11}$$

where:

$$G^{\theta\phi}_{j\delta}(x,x',t) = \; -\frac{1}{8\pi}\sum_r r\ln\left(1+i\frac{v_F t}{a}+ir\frac{K_{j\delta}(x-x')}{a}\right) \; . \tag{A12}$$

The same applies to $G^{\theta\phi}_{j\delta(K)}$ for which we have:

$$G^{\phi\theta}_{j\delta}(x,x',t) = \; -\frac{1}{8\pi}\sum_r r\ln\left(1+i\frac{v_F t}{a}+ir\frac{K_{j\delta}(x-x')}{a}\right) \; . \tag{A13}$$

APPENDIX B: INTEGRALS

We now compute the integrals involved in the tunneling current and noise. The general integrals which will be required to compute the current and noise read:

$$\int_{-\infty}^{+\infty}\frac{\sin(\omega_0\tau)d\tau}{\left(\frac{a}{u_F^\sigma}-i\eta\tau\right)\left(\frac{a}{v_F}-i\eta\tau\right)^\nu} \approx i\pi\eta\,\mathrm{sgn}(\omega_0)\frac{|\omega_0|^\nu}{\mathbf{\Gamma}(\nu+1)} \; , \tag{B1}$$

$$\int_{-\infty}^{+\infty}\frac{\cos(\omega_0\tau)d\tau}{\left(\frac{a}{u_F^\sigma}-i\eta\tau\right)\left(\frac{a}{v_F}-i\eta\tau\right)^\nu} \approx \pi\,\mathrm{sgn}(\omega_0)\frac{|\omega_0|^\nu}{\mathbf{\Gamma}(\nu+1)} \; . \tag{B2}$$

We now write the integral which appears in the nanotube current, which refer to propagation along the nanotube:

$$I_2 \;=\; \int_{-\infty}^{+\infty} d\tau'\partial_x\left(G^{\phi\phi}_{c+(++)}(x,0,\tau')-G^{\phi\phi}_{c+(--)}(x,0,\tau')+G^{\phi\phi}_{c+(-+)}(x,0,\tau')-G^{\phi\phi}_{c+(+-)}(x,0,\tau')\right) \; . \tag{B3}$$

Using the expressions for the Green's functions (Appendix A):

$$I_2 \;=\; \frac{i}{\pi v_F}\arctan\left(\frac{K_{j\delta}x}{a}\right) \approx i\frac{\mathrm{sgn}(x)}{2v_F} \; , \tag{B4}$$

where the approximate sign holds at large distances.

The integrals which are involved for the computation of the noise read:

$$I^{\phi\phi}(x,x') \;=\; 4I_3(x)I_3(x') \; , \tag{B5}$$

$$I^{\phi\theta}(x,x') \;=\; 4I_4(x)I_4(x') \; , \tag{B6}$$

with

$$I_3(x) \;=\; \int_{-\infty}^{+\infty} d\tau\partial_x\left(G^{\phi\phi}_{c+(++)}(x,0,\tau)-G^{\phi\phi}_{c+(+-)}(x,0,t)\right) \approx i\;\mathrm{sgn}(x)/4v_F \; , \tag{B7}$$

$$I_4(x) \;=\; \int_{-\infty}^{+\infty} d\tau\partial_x\left(G^{\phi\theta}_{c+(++)}(x,0,\tau)-G^{\phi\theta}_{c+(+-)}(x,0,t)\right) \approx -iK_{c+}/4v_F \; . \tag{B8}$$

[1] Ya. M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).

[2] C. Kane and M.P.A. Fisher, Phys. Rev. Lett. **72**, 724 (1994); C. de C. Chamon, D.E. Freed, and X.G. Wen, Phys. Rev. B **51**, 2363 (1995-II); P. Fendley, A.W.W. Ludwig, and H. Saleur, Phys. Rev. Lett. **75**, 2196 (1995).
[3] L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. **79**, 2526 (1997); R. de-Picciotto *et al.*, Nature **389**, 162 (1997).
[4] H. Saleur and U. Weiss, Phys. Rev. B **63**, 201302 (2001).
[5] I. Safi, P. Devillard, and T. Martin, Phys. Rev. Lett. **86**, 4628 (2001).
[6] C. Kane, L. Balents, and M. P. A. Fisher, Phys. Rev. Lett. **79**, 5086 (1997).
[7] M. Brockrath, D.H. Cobden, J. Lu, A.G. Rinzler, R.E. Smalley, L. Balents, and P. McEuen, Nature **397**, 598 (1999).
[8] R. Egger, A. Bachtold, M.S. Fuhrer, M. Bockrath, D. Cobden, and P. McEuen, in *Interacting Electrons in Nanostructures*, edited by R. Haug, and H. Schoeller (Springer, 2001).
[9] I. Safi, Ann. Phys. Fr. **22**, 463 (1997).
[10] K.-V. Pham, M. Gabay, and P. Lederer, Phys. Rev. B **61**, 16397 (2000).
[11] W. Schottky, Ann. Phys. (Leipzig) **57**, 541 (1918).
[12] I. Safi and H. Schulz, Phys. Rev. B **52**, 17040 (1995); D. Maslov and M. Stone, Phys. Rev. B **52**, 5539 (1995); V.V. Ponomarenko, Phys. Rev. B **52**, 8666 (1995).
[13] P.-E. Roche, M. Kociak, S. Guéron, A. Kasumov, B. Reulet, and H. Bouchiat, Eur. Phys. J. B **28**, 217 (2002).
[14] T. Martin and R. Landauer, Phys. Rev. B **45**, 1742 (1992); M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990); Phys. Rev. B **46**, 12485 (1992).
[15] Ya. M. Blanter, F. Hekking, and M. Büttiker, Phys. Rev. Lett. **81**, 1925 (1998).
[16] K.-I. Imura, K.-V. Pham, P. Lederer, and F. Piéchon, Phys. Rev. B **66**, 035313 (2002).
[17] B. Trauzettel, R. Egger, and H. Grabert, Phys. Rev. Lett. **88**, 116401 (2002).
[18] S. Chen, B. Trauzettel, and R. Egger, Phys. Rev. Lett. **89**, 226404 (2002).
[19] K.-V. Pham, P. Piéchon, K.-I. Imura, and P. Lederer, cond-mat/0207294.
[20] R. Egger and A. Gogolin, Eur. Phys. J. B **3**, 781 (1998).
[21] C. Chamon and D.E. Freed, Phys. Rev. B **60**, 1842-1853 (1999).
[22] R. Guyon, P. Devillard, T. Martin, and I. Safi, Phys. Rev. B **65**, 153304 (2002).
[23] M. Henny *et al.*, Science **296**, 284 (1999); W. Oliver *et al.*, Science **296**, 299 (1999).
[24] R. Hanbury-Brown and Q.R. Twiss, Nature **177**, 27 (1956).
[25] J. Torrès and T. Martin, Eur. Phys. J. B **12**, 319 (1999).
[26] T. Gramespacher and M. Büttiker Phys. Rev. B **61**, 8125 (2000).
[27] J. Börlin, W. Belzig, and C. Bruder Phys. Rev. Lett. **88**, 197001 (2002).
[28] M. Schechter, Y. Imry, and Y. Levinson Phys. Rev. B **64**, 224513 (2001); F. Taddei and R. Fazio, Phys. Rev. B **65**, 134522 (2002).
[29] C. Texier and M. Büttiker Phys. Rev. B **62**, 7454 (2000).
[30] P. Recher, E.V. Sukhorukov, and D. Loss, Phys. Rev. B **63**, 165314 (2001).
[31] G.B. Lesovik, T. Martin, and G. Blatter, Eur. Phys. J. B **24**, 287 (2001).
[32] P. Recher and D. Loss, Phys. Rev. B **65**, 165327 (2002).
[33] C. Bena *et al.*, Phys. Rev. Lett. **89**, 037901 (2002).
[34] V. Bouchiat, N. Chtchelkatchev, D. Feinberg, G.B. Lesovik, T. Martin, and J. Torrès, Nanotechnology **14**, 77 (2003).
[35] I. Safi, Eur. Phys. J. B **12**, 451 (1999).
[36] R. Egger and H. Grabert, Phys. Rev. Lett. **79**, 3463 (1997); Phys. Rev. B **58**, 10761 (1998).
[37] C. de C. Chamon and E. Fradkin, Phys. Rev. B **56**, 2012 (1997); V. Ponomarenko and D. Averin, JETP Letters, **74**, 87 (2001).
[38] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[39] I. Safi, private communication.