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# Bose-Einstein Condensates in Optical Quasicrystal Lattices

L. Sanchez-Palencia<sup>1,2</sup> and L. Santos<sup>3</sup>

<sup>1</sup>*Laboratoire Charles Fabry, Institut d'Optique Théorique et Appliquée,  
Université Paris-Sud XI, F-91403 Orsay cedex, France*

<sup>2</sup>*Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany*

<sup>3</sup>*Institut für Theoretische Physik III, Universität Stuttgart, D-70569 Stuttgart, Germany*

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We analyze the physics of Bose-Einstein condensates confined in quasi-periodic optical lattices (optical quasicrystals), which offer an intermediate situation between ordered and disordered systems. First, we analyze the time-of-flight interference pattern that reveals quasi-periodic long-range order. Second, we demonstrate localization effects associated with quasi-disorder as well as Bloch-like oscillations associated with the extended nature of the wavefunction of a Bose-Einstein condensate in an optical quasicrystal. In addition, we discuss in detail the crossover between diffusive and localized regimes when the quasi-periodic potential is switched on, as well as the effects of interactions.

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The last few years have witnessed a fastly growing interest on ultracold atomic gases in periodic potentials generated by interfering laser beams (optical lattices). These potentials present neither defects nor phonons, offering a powerful tool for investigating the quantum behavior of periodic systems under unique control possibilities. In this sense, the physics of ultracold atoms in optical lattices presents fascinating resemblances with solid-state physics, which range from Bloch oscillations [1, 2] and Wannier-Stark ladders [3], to Josephson arrays of Bose-Einstein condensates (BECs) [4], or to the superfluid to Mott-insulator transition [5].

These remarkable experiments have been performed in regular cubic lattices. However, these lattices do not exhaust the rich possibilities offered by the current control of optical potentials. More sophisticated lattice geometries have been already proposed, including honey-comb [6] and Kagomé and triangular [7] lattices. In addition, although optical lattices present defectless periodicity, controlled defects may be introduced to generate a random or pseudorandom optical potential [8], allowing for the realization of Kondo-like physics [9], and for the observation of Anderson localization and Bose-Glass phases [10]. In this sense, laser speckle has been employed very recently to achieve a BEC in a random potential [11] opening new very exciting experimental possibilities.

As an intermediate situation between ordered and disordered structures, quasicrystals have attracted a large interest since their discovery in 1984 [12]. A quasicrystal is a long-range ordered material that does not present translational invariance, and therefore shares properties with both ordered crystals and amorphous solids [13]. In particular, quasicrystals show intriguing structure [14] as well as electronic conduction properties [15] at the border between ordered and disordered systems.

Surprisingly, up to now, only few works have been devoted to optical analogues of quasicrystals, despite the fact that optical lattices offer dramatic possibilities for

designing a wide range of geometrical configurations [16]. Optical quasicrystal lattices have been first studied experimentally in the context of atomic laser cooling [17]. In these works, the atomic gas, which was far from quantum degeneracy, was confined in a dissipative optical lattice where coherence was lost through spontaneous emission of photons. The temperature and spatial diffusion were found to behave similarly as in periodic optical lattices. The physics of one-dimensional quasiperiodic optical lattices has been also subject of recent research in the context of cold atomic gases, including a proposal for the atom-optical realization of Harper's model [18], and the analysis of Fibonacci potentials [19].

In this Letter, we investigate theoretically the BEC dynamics in an optical quasicrystal. First, we show that the BEC wavefunction displays quasi-periodic long-range order, which may be easily probed experimentally via matter-wave interferometry. Second, we show that macroscopic quantum coherence dramatically modifies the transport phenomena compared with the above-mentioned studies in dissipative lattices. In particular, effects usually associated with both order and disorder physics emerge. On one hand, due to quasi-disorder, spatial localization occurs, in contrast to ballistic expansion in periodic lattices. The crossover between ballistic expansion and localization is analyzed when the quasi-periodicity of the lattices is continuously increased. On the other hand, we show that due to extended character of the BEC wavefunction, Bloch oscillations take place. These oscillations are however quasi-periodic rather than periodic. Additionally, we briefly discuss the effects of the interatomic interactions in the BEC diffusion.

In the following we consider a dilute Bose gas trapped in the combination of a weak harmonic potential  $V_{\text{ho}}(\vec{r}) = \frac{M}{2}(\omega_{\perp}^2 \vec{r}_{\perp}^2 + \omega_z^2 z^2)$  and an optical lattice  $V_{\text{latt}}(\vec{r}_{\perp})$ , which can present periodic or quasi-periodic order. In the previous expression,  $M$  is the atomic mass,  $\omega_j$  are the harmonic trap frequencies, and  $\vec{r}_{\perp} = (x, y)$

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is the position vector on the lattice plane. We assume  $\omega_z$  to be sufficiently large to guarantee that the physics remains 2D on the  $xy$  plane. Following Ref. [17], we consider a laser configuration consisting on  $N_b$  laser beams arranged on the  $xy$  plane with  $N_b$ -fold symmetry rotation (Fig. 1). The polarization  $\vec{e}_j$  of laser  $j$  with wavevector  $\vec{k}_j$  is linear and makes an angle  $\alpha_j$  with the  $xy$  plane. The optical potential is thus [20]:

$$V_{\text{latt}}(\vec{r}_\perp) = \frac{V_0}{|\sum_j \mathcal{E}_j|^2} \left| \sum_{j=0}^{N_b} \mathcal{E}_j \vec{e}_j e^{-i(\vec{k}_j \cdot \vec{r}_\perp + \varphi_j)} \right|^2, \quad (1)$$

where  $0 \leq \mathcal{E}_j \leq 1$  stand for eventually different laser intensities and  $\varphi_j$  are the corresponding phases. In the following, we are mostly interested in the fivefold symmetric configuration ( $N_b = 5$ ,  $\mathcal{E}_j = 1$ ), similar to the Penrose tiling [21], which supports no translational invariance (see Fig. 1). The lattice displays potential wells which are clearly not periodically arranged. We also consider the configuration obtained by switching off lasers 1 and 4, which results in an anisotropic periodic lattice.

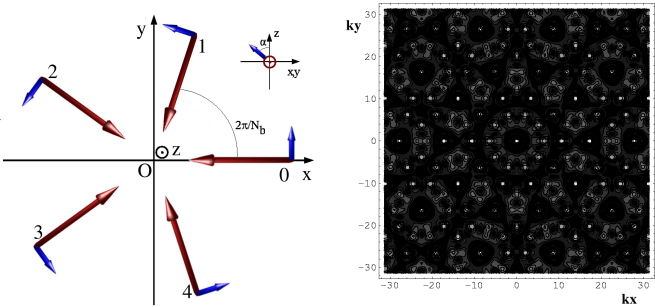


FIG. 1: Left: laser arrangement (see text for details). Right: Quasi-periodic lattice potential for  $N_b = 5$ ,  $V_0 < 0$  and  $\alpha_j = 0$ . White points correspond to potential minima.

1) *Equilibrium properties* - We first investigate the properties of a BEC at zero temperature in the combined optical lattice and harmonic trap. The static BEC wavefunction  $|\psi_{\text{BEC}}\rangle$  is obtained from evolution in imaginary time of the 2D-Gross-Pitaevskii equation (GPE):

$$i\hbar\partial_t\psi = \left[ -\hbar^2\vec{\nabla}^2/2M + V_{\text{ho}} + V_{\text{latt}} + g_{2D}|\psi|^2 \right] \psi, \quad (2)$$

where  $g_{2D} = \sqrt{8\pi\hbar^3\omega_z/M}a_{\text{sc}}$ , with  $a_{\text{sc}}$  the  $s$ -wave scattering length. In order to elucidate the long-range order properties of the BEC, we compute the momentum distribution of  $|\psi_{\text{BEC}}\rangle$ . The periodic and quasi-periodic cases are shown in Fig. 2a and Fig. 2b, respectively. In the periodic case, as expected, the momentum distribution displays discrete peaks corresponding to combinations of the elementary basis vectors of the reciprocal lattice,  $n_1\vec{k}_1 + n_2\vec{k}_2$  with integer coefficients  $n_1$  and  $n_2$ . As already obtained in previous experimental works [22, 23], this confirms the periodic long-range order of  $|\psi_{\text{BEC}}\rangle$ .

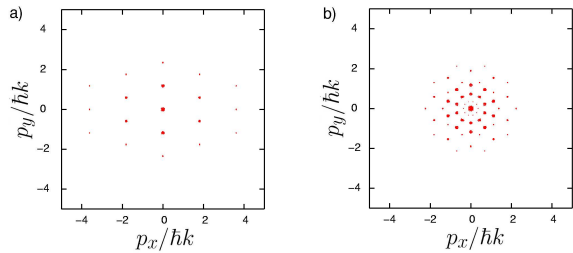


FIG. 2: Matterwave interference pattern of a BEC released from a combined optical lattice and harmonic trap: a) periodic case; b) quasi-periodic case. Both correspond to  $^{87}\text{Rb}$  and  $V_0 = -10E_R$ , where  $E_R = \hbar^2k^2/2M$  is the recoil energy.

The quasi-periodic case is more intriguing, resulting in a more complex structure. The momentum distribution also displays sharp peaks, being the signature of the quasi-periodic long-range order [24]. As in the periodic case, the positions of the peaks are linear combinations of integer numbers of  $N_b = 5$  wavevectors:  $\sum_j^{N_b-1} n_j \vec{k}_j$  where  $\vec{k}_0 = \vec{k}_1 - \vec{k}_0$  and  $\vec{k}_j = \mathcal{R}[2\pi j/N_b]\vec{k}_0$  is the wavevector obtained by a rotation of angle  $2\pi j/N_b$  of  $\vec{k}_0$ . The geometry of the reciprocal space thus shows a five-fold rotation symmetry incompatible with any translation invariance [25]. This situation is similar to the Penrose tiling [21] and to the first quasicrystal phases observed in solid state physics via Bragg diffraction [12].

The momentum distribution of a BEC initially trapped in a combined optical lattice plus harmonic confinement can be directly imaged via matter-wave interferometry after time-of-flight expansion. Indeed, although interatomic interactions are crucial for determining local populations of each potential well, they do not contribute significantly to the coherent expansion of the BEC in free space when released [22]. Such measurements have already been performed in periodic optical lattices [22, 23] and could easily be extended to quasi-periodic ones.

2) *Quantum transport* - One should not deduced that all physical properties of optical quasicrystals can be directly interpolated from the behavior of periodic lattices. Indeed, solid quasicrystals show intriguing dynamical properties that are not yet completely understood [13]. In the following, we investigate dynamical properties of quasi-periodic lattices. These are exemplified by the behavior of (i) coherent spatial diffusion and (ii) coherent oscillations in tilted lattices. Both effects can be easily investigated in current experiments on BECs.

*Coherent diffusion* - After equilibration of the Bose gas in the combined optical lattice plus harmonic confinement at  $t < 0$ , we switch off the harmonic trap at  $t = 0$  and let the BEC evolve in the optical lattice. Eventually, we simultaneously switch off the interactions at  $t = 0$ . The latter eases our analysis, but it is not strictly necessary. In experiments,  $s$ -wave scattering interactions can be rigorously turned off using a Feshbach resonance

[26]. The BEC expansion is calculated using a Crank-Nicholson algorithm for the time-dependent GPE (2).

Fig. 3 (inset) shows the time evolution of  $\langle x^2 \rangle$  and  $\langle y^2 \rangle$  of the BEC along  $x$  and  $y$ , in both periodic and quasiperiodic cases. In the periodic case, the condensate expands coherently as one expects from tunnel couplings between adjacent lattice sites. Let us recall that in free space,  $\langle r_j^2 \rangle$  increases quadratically in time at large times as  $\langle r_j^2(t) \rangle \propto v_j^2 t^2$  with  $v_j \propto 1/M$  [27]. In periodic lattices, we expect enhancement of inertia due to confinement in the lattice sites. The resulting low-excitation dynamics of the condensate is then the same as in free space provided substitution of the atomic mass  $M$  by an effective mass  $M^* > M$  [28]. We expect  $v_j \propto 1/M^{*2}$  and  $M/M^* \propto J$  (where  $J$  is the site-to-site tunneling rate) Numerical computations for various depths of the lattice potential  $V_0$  show that  $v_j^2$  decreases exponentially with  $V_0$  as expected from the well-known exponential decay of  $J$ . Anisotropic ballistic expansion of the BEC reflects the anisotropy of our periodic lattice.

The behavior of the BEC in the quasiperiodic lattice is dramatically different and can definitely not be understood in terms of an effective mass. After a short transient phase, the condensate spatially localizes so that its expansion vanishes as shown in the Inset of Fig. 3. Note that because of the five-fold symmetry of the lattice, the expansion is now isotropic. This behavior strongly contrasts with results obtained in the context of laser cooling where similar classical normal expansion ( $\langle r_j^2(t) \rangle \sim 2D_j t$ ) was found in both periodic and quasi-periodic optical lattices [17]. Here, spatial localization is a coherent effect induced by quasi-disorder due to the lack of periodicity. Indeed, the BEC populates localized (Wannier-like) states centered on each lattice site. In the periodic case, because all sites are equivalent, these states have the same energy and are strongly coupled through quantum tunneling. On the contrary, in the quasi-periodic lattice, all sites are different. In particular, the average difference of depths of adjacent sites  $\Delta$  can be of the order of magnitude of (but smaller than)  $V_0$ . Therefore, the tunneling is not resonant and the BEC remains localized.

The remarkable flexibility of optical lattices [16] offers the possibility to accurately study the competition between tunneling and quasi-disorder. Indeed, by ramping up gradually the intensity of lasers 1 and 4 while keeping constant the intensity of lasers 0, 2 and 3, one turns continuously from an anisotropic periodic lattice to a five-fold symmetric quasiperiodic lattice and thus correspondingly from ballistic expansion to spatial localization. In the considered range of parameters, all calculations fit well with  $\langle r_j^2 \rangle = \langle r_{j0}^2 \rangle + 2D_j t + v_j^2 t^2$ . We characterize the expansion through the ballistic velocity  $v_j$ . The behavior of  $v_x$  versus the quasi-disorder parameter  $\Delta$  is shown in Fig. 3 with and without interatomic interactions. In both cases, as expected, coherent diffusion dramatically decreases when quasi-disorder increases

and spatial localization occurs for  $\Delta \gtrsim J_x$ . This supports the interpretation that competition between coherent tunneling and inhomogeneities turns into localization as soon as tunneling becomes non-resonant. As shown in Fig. 3, interatomic interactions contribute to expansion for small inhomogeneities whereas they help localization for stronger inhomogeneities. This non-trivial effect will be described in more detail in a future publication.

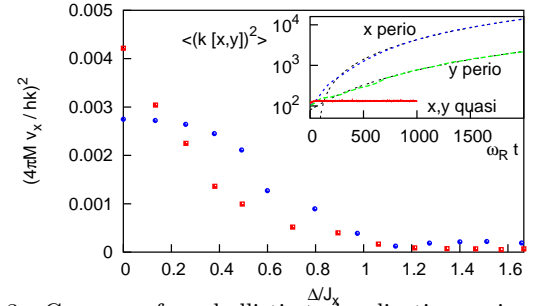


FIG. 3: Crossover from ballistic to localization regimes with (squares) and without (circles) interactions, for  $V_0 = -7.5E_R$ . Inset: Coherent diffusion in periodic and quasi-periodic lattices for  $V_0 = -5E_R$ . The time unit is  $\omega_R^{-1} = \hbar/E_R$ . Fits to  $\langle r_j^2(t) \rangle = r_{j0}^2 + 2D_j t + v_j^2 t^2$  in the periodic case are also shown.

*Quasiperiodic Bloch oscillations* - One of the most appealing predictions of the Bloch quantum theory of solids [25] is that homogeneous static forces induce oscillatory rather than constant motion in periodically ordered structures [29]. The corresponding Bloch oscillations have already been observed in superlattice superconductors [30] and in ultracold atoms in optical lattices [1, 2]. It is a fundamental question whether such a phenomenon also exists in less ordered systems like quasicrystals. Arguments based on general spectral properties of quasicrystals [31] and numerical simulations of 1D Fibonacci lattices [32] support the existence of Bloch oscillations in quasiperiodic lattices. However, to the best of our knowledge, this effect has never been observed experimentally. Using accelerated lattices [1, 2] or gravity [33], this question can be addressed experimentally in our system. After the equilibration of the interacting BEC in the combined harmonic trap and quasiperiodic lattice, we switch off the harmonic trap and the interactions at time  $t = 0$  and we tilt the quasiperiodic lattice in the  $x$  direction. The latter evolution of the quantum gas is shown in the inset of Fig. 4. We find noisy-like oscillatory motion in the (tilted)  $x$  direction and no motion in the (non-tilted)  $y$  direction. The oscillations in the  $x$  direction are clearly not periodic [34]. However, they definitely have an ordered structure that can be evidenced computing the time Fourier transform (FT) of the BEC mean position  $\langle x(t) \rangle$  as shown in Fig. 4. The FT of  $\langle x(t) \rangle$  is composed of discrete sharp peaks corresponding to a quasiperiodic motion [24]. The Bloch-like quasiperiodic oscillations can be interpreted as follows. As in periodic lattices, the BEC extends over several lattice wells. The

wavefunction can be decomposed into a sum of localized (Wannier-like) states, each one basically oscillating at a given frequency corresponding to the on-site energy. In periodic lattices, Bloch oscillations are due to the time-evolution of the non-vanishing overlap of the Wannier states. In quasiperiodic lattices, there is a discrete set of different (non commensurate) differences of on-site energies in adjacent wells and this leads to quasiperiodic oscillations.

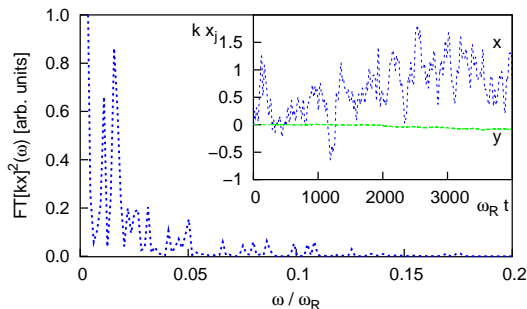


FIG. 4: Fourier transform of the mean position of a BEC in a periodic or quasiperiodic lattice. Inset: Time evolution of a BEC in a tilted quasiperiodic lattice. In the simulation,  $V_0 = -2E_R$  and  $V_{\text{tilt}} = 0.002E_R \times kx$ .

Summarizing, we have investigated the physics of BECs trapped in optical quasicrystals. We have shown that (i) the equilibrium BEC wavefunction displays long-range quasiperiodic order and that (ii) quantum transport shares properties with both ordered and disordered systems. On the one hand, because of quasi-disordered inhomogeneities, diffusion turns from ballistic to localization when quasi-periodicity is switched on. On the other hand, because of coherence extending over several lattice sites, quasiperiodic Bloch oscillations occur in quasiperiodic BECs. Finally, it is worth noticing that we have given evidences of these phenomena by considering quantities (momentum distribution, positions and widths of possibly expanding BECs) that can be accurately investigated in current experiments. Therefore, our system may be used to address experimentally some still unresolved issues on quasicrystals [13].

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- [1] M. Ben Dahan *et al.*, Phys. Rev. Lett. **76**, 4508 (1996); E. Peik *et al.*, Phys. Rev. A **55**, 2989 (1997).  
 [2] O. Morsch *et al.*, Phys. Rev. Lett. **87**, 140402 (2001); M. Cristiani *et al.*, Phys. Rev. A **65**, 063612 (2002).  
 [3] S. R. Wilkinson *et al.*, Phys. Rev. Lett. **76**, 4512 (1996).

- [4] B. P. Anderson and M. Kasevich, Science **282**, 1686 (1998); F. S. Cataliotti *et al.*, Science **293**, 843 (2001).  
 [5] D. Jaksch *et al.*, Phys. Rev. Lett. **81**, 3108 (1998); M. Greiner *et al.*, Nature (London) **415**, 39 (2002).  
 [6] L.-M. Duan, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **91**, 090402 (2003).  
 [7] L. Santos *et al.*, Phys. Rev. Lett. **93**, 030601 (2004).  
 [8] P. Horak, J.-Y. Courtois, and G. Grynberg, Phys. Rev. A **58**, 3953-3962 (1998).  
 [9] B. Paredes, C. Tejedor, and J. I. Cirac, condmat/0306497.  
 [10] B. Damski *et al.*, Phys. Rev. Lett. **91**, 080403 (2003); R. Roth and K. Burnett, J. Opt. B Quantum Semiclassical Opt. **5**, S50 (2003).  
 [11] J. E. Lye *et al.*, condmat/0412167.  
 [12] D. Shechtman *et al.*, Phys. Rev. Lett. **53**, 1951 (1984); D. Levine and P. J. Steinhardt, *ibid.* **53**, 2477 (1984).  
 [13] *The Physics of Quasicrystals*, eds. P. J. Steinhardt and S. Ostlund (World Scientific, Singapore, 1987); *Quasicrystals, the State of the Art*, eds. D. P. DiVincenzo and P. J. Steinhardt (World Scientific, Singapore, 1991); *Lectures on Quasicrystals*, eds. F. Hippert and D. Gratias (Les Editions de Physique, Paris, 1994).  
 [14] D. Levine and P. J. Steinhardt, Phys. Rev. B **34**, 596 (1986); M. J. Jarić, *ibid.* **34**, 4685 (1986).  
 [15] K. Ueda, and H. Tsunetsugu, Phys. Rev. Lett. **58**, 1272 (1987); S. Martin *et al.*, *ibid.* **67**, 719 (1991); C. Berger *et al.*, Phys. Scr. T **35**, 90 (1991); B. Passaro, C. Sire, and V. G. Benza, Phys. Rev. B **46**, 13751 (1992); D. Mayou *et al.*, Phys. Rev. Lett. **70**, 3915 (1993); F. S. Pierce, Q. Guo, and S. J. Poon, *ibid.* **73**, 2220 (1994).  
 [16] G. Grynberg and C. Robilliard, Phys. Rep. **355**, 335 (2000).  
 [17] L. Guidoni *et al.*, Phys. Rev. Lett. **79**, 3363 (1997); L. Guidoni *et al.*, Phys. Rev. A **60**, R4233 (1999).  
 [18] K. Drese and M. Holthaus, Phys. Rev. Lett. **78**, 2932 (1997).  
 [19] Y. Eksioğlu, P. Vignolo, and M. P. Tosi, condmat/0405440.  
 [20] R. Grimm, M. Weidemüller, and Yu. B. Ovchinnikov, Adv. At. Mol. Opt. Phys. **42**, 95 (2000).  
 [21] R. Penrose, Bull. Inst. Math. Appl. **10**, 266 (1974).  
 [22] P. Pedri *et al.*, Phys. Rev. Lett. **87**, 220401 (2001).  
 [23] M. Greiner *et al.*, Phys. Rev. Lett. **87**, 160405 (2001).  
 [24] A function is said to be quasi-periodic whenever its Fourier transform is composed of discrete (numerable) peaks [13].  
 [25] N. W. Ashcroft and N. D. Mermin, *Solid state physics*, (Holt, Rinehart and Winston, New York, 1976).  
 [26] S. Inouye *et al.*, Nature (London) **392**, 151 (1998); S. L. Cornish *et al.*, Phys. Rev. Lett. **85**, 1795 (2000).  
 [27] J.-L. Basdevant and J. Dalibard, *Quantum Mechanics* (Springer, Berlin, 2002).  
 [28] M. Krämer, L. Pitaevskii, and S. Stringari, Phys. Rev. Lett. **88**, 180404 (2002).  
 [29] F. Bloch, Z. Phys. **52**, 555 (1928); C. Zener, Proc. R. Soc. London, Ser. A. **145**, 523 (1934).  
 [30] C. Waschke *et al.*, Phys. Rev. Lett. **70**, 3319 (1993).  
 [31] C. Janot, L. Loreto, and R. Farinato, Phys. Lett. A **276**, 291 (2000).  
 [32] E. Diez *et al.*, Phys. Rev. A **54**, 16792 (1996).  
 [33] G. Roati *et al.*, Phys. Rev. Lett. **92**, 230402 (2004).  
 [34] The amplitude of the quasiperiodic oscillations may be enhanced by reducing the intensity of beams 1 and 4.