Tunneling in Fractional Quantum Hall line junctions

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We study the tunneling current between two counterpropagating edge modes described by chiral Luttinger liquids when the tunneling takes place along an extended region. We compute this current perturbatively by using a tunnel Hamiltonian. Our results apply to the case of a pair of different two-dimensional electron gases in the fractional quantum Hall regime separated by a barrier, e. g. electron tunneling. We also discuss the case of strong interactions between the edges, leading to nonuniversal exponents even in the case of integer quantum Hall edges. In addition to the expected nonlinearities due to the Luttinger properties of the edges, there are additional interference patterns due to the finite length of the barrier.

I. INTRODUCTION

Edge states in the fractional quantum Hall effect (FQHE) are very interesting examples of one-dimensional strongly interacting quantum systems. The right and left moving edge modes of a quantum Hall bar are spatially separated by a macroscopic length and this leads to exponentially small backscattering. The main reason for localization is thus suppressed and the edge modes are a nearly ideal ballistic system. Many experiments have been devoted to the study of their unique characteristics [1]. An interesting geometry is a constriction of the electron gas, the so-called quantum point contact. By means of an electrostatic potential created by a gate, the two edges of a bar are brought in close proximity, allowing tunneling phenomena to take place at a single point of the fluid. Recently, progress in the technique of cleaved-edge overgrowth [2, 3] has led to the fabrication of samples in which the tunneling now occurs along a barrier of mesoscopic extent between two spatially separated two-dimensional electron gases. Kang et al. [2, 4] have performed detailed studies of the conductance of these new structures. Their samples consist of two-dimensional electron gases (2DEGs) separated by an atomically precise barrier of length 100 μm and of width 8.8 nm. They have studied the conductance of this structure as a function of the applied magnetic field and the voltage bias between the two gases. Many theoretical works have tried to explain their results [5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

In this paper, we give the results of a perturbative calculation of tunneling between two edge modes that are counterpropagating. Each of these edge modes are described by a chiral Luttinger liquid and we focus on the situation where they are characterized by the same anomalous exponent g. We consider the situation where tunneling takes place along an extended region with constant amplitude. The anomalous exponent g of the chiral Luttinger liquid is governed by the bulk FQHE fluid(s) and enter in the expression of the correlation function of the particle that tunnel. The extended tunneling geometry is potentially rich of new interference phenomena not found in single point-contact devices. Notably, T. L. Ho

pointed out the existence of oscillating currents without AC drive in the case of integer edge modes[15]. We find that in the FQHE regime there is a nontrivial interplay between the well-known nonlinearities of the Luttinger liquid and the interference pattern of the tunnel barrier. We also discuss the case of strong interactions between the left and right-moving modes (but still weak tunneling) which is amenable to the same theoretical treatment. The exponent g then takes nonuniversal values dictated by the details of the interaction potential, even for integer quantum Hall edges. The formulas we obtain are generalizations of previous results already available in the literature [16, 17]. This geometry may also give rise in the presence of disorder to a delocalization transition[18, 19]. The line junction geometry may also be relevant to a recent set of experiments using samples with an extended constriction [20]. A detailed modelling of such line junctions has been performed by Papa and McDonald [21, 22].

In section II, we introduce a model of edge states in the quantum Hall regime with an extended tunnel barrier. In section III, we present the perturbative treatment of the weak tunneling regime. Section IV gives the results of our study. Finally section V contains our conclusions.

II. THE GEOMETRY AND KINEMATICS OF TUNNELING

The geometry we consider is illustrated in fig. 1. It is that of an extended barrier whose height remains constant along some spatial extent we call L. The barrier separates spatially two 2DEGs. This is the geometry of the samples studied by Kang et al. [2]. In the quantum Hall regime there are edge modes of conduction that are counterpropagating. With the atomically precise barrier of ref. [2], there is tunneling only in the *integer* quantum Hall regime when the applied bias between the two 2DEGs is small. This geometry may also be realized by electrostatic gates, in which case the energetics may be different and extended tunneling may eventually be realized also in the fractional quantum Hall regime.

We first discuss the kinematics of tunneling by reasoning in the case of bulk filling factor $\nu = 1$. This situation

was first investigated by T. L. Ho [15]. The Landau levels are degenerate in the bulk and their energies raise when they approach a barrier. This scheme applies to both sides of the barrier. Ultimately the Landau levels of both sides will cross in the forbidden region inside the barrier and tunnel effect will lead to the opening of single-particle gaps. A simple effective theory clearly displays these phenomena. We first introduce the left and right moving chiral fermions ψ_R and ψ_L that are the relevant modes when the bulk filling factor is between $\nu = 1$ and $\nu = 2$. They are the counter-propagating edge states [16]. Their kinetic energy purely due to the confining potential is then :

$$\mathcal{H}_{kin} = -iv \int dx (\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L), \qquad (1)$$

where v is the drift velocity along the barrier. In momentum space the left modes have energy $\epsilon_L(k) = -vk$ and the right modes have $\epsilon_R(k) = vk$. These dispersion relations cross at $k = 0$ for zero energy. It is at this point that the tunnel effect is strongest. The tunneling through the extended barrier can be described by a Hamiltonian mixing these modes :

$$\mathcal{H}_{tunnel} = \mathcal{T} \int_{-L/2}^{+L/2} dx \ (\psi_R^\dagger(x)\psi_L(x) + \psi_L^\dagger(x)\psi_R(x)), \qquad (2)$$

where $\mathcal{T}$ is the tunnel amplitude, constant along the barrier by hypothesis. This is the tunneling Hamiltonian we treat in this paper. The coordinate x is defined as in fig. 1 so that the modes propagate for all values of x and tunneling is restricted to the range $(-L/2, +L/2)$.
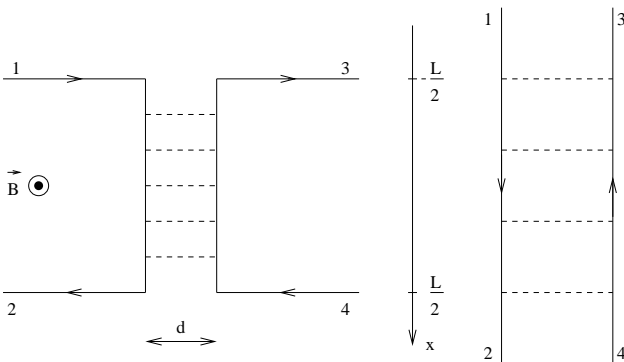

Figure 1: Two laterally separated 2DEGs under a magnetic field. A tunnel barrier causes mixing of the counterpropagating edge modes. Realistic configuration (a), topologically equivalent geometry (b)

This Hamiltonian can be seen a mass term of the fermion field for an infinite-length barrier. This mass leads to a gap in the free fermion excitation spectrum and this gap implies the existence of a range of energies for which there are no propagating solutions but only evanescent waves along the barrier. This phenomenon

repeats in a real sample also at the crossings of the other higher-lying Landau levels as discussed by Kang et al..

The problem defined by $\mathcal{H}_{kin} + \mathcal{H}_{tunnel}$ is a free fermion theory and has been studied exactly [15]. The scattering properties of the barrier can be deduced by a straightforward Landauer-type calculation. However this simplicity does not extend into the realm of the FQHE : here the electron operators are described by a chiral Luttinger liquid (χLL) theory and no longer by the free theory Eq.(1). We study the case with no interactions through the barrier so the χLLs are decoupled and all the properties of their correlation functions are known. We still use the tunneling Hamiltonian Eq.(2) and treat it in perturbation to obtain the current - voltage $I - V$ characteristic of this system. We consider the case of a single edge appropriate to a $\nu = 1/m$, m odd, FQHE fluid. A single chiral boson is then enough to describe the χLL theory. The bulk fluid determines the value of the anomalous exponent g characterizing the correlations at the edge. When the tunneling is between two distinct gases then the operators ψ_R and ψ_L should be taken as *electron* operators. Contrary to the case of the quantum point contact geometry, the bosonized version of the theory cannot be treated exactly since the tunneling term is no longer a boundary operator. We are thus left only with the perturbative calculation.

In real samples, there may be also noticeable Coulomb interactions along and across the barrier which can be modeled by the following kind of Hamiltonian :

$$\begin{aligned} H_{int} &= \int dx dy \ V_1(x-y) \left[\rho_R(x)\rho_R(y) + \rho_L(x)\rho_L(y)\right] \\ &+ 2 \int dx dy \ V_2(x-y) \, \rho_R(x)\rho_L(y), \qquad (3) \end{aligned}$$

where $\rho_R(x) =: \psi_R^\dagger(x)\psi_R(x) :$ is the right-moving edge electron density operator describing density fluctuation at point x ($\rho_L(x)$ is similarly defined). This may be potentially important in the structure of Kang et al where the width of the barrier is smaller than the magnetic length. In this case, Wen [16] has shown that the eigenmodes are a mixture of the uncoupled left and right moving edge modes. He showed also that the study of the tunnel effect may be performed along the standard lines with simply a redefinition of the Luttinger parameter g when the momentum dependence of the interactions may be neglected. So the results of our perturbative tunneling calculation also apply to interacting edges if we use an appropriate value of g. For example, Mitra and Girvin [6] have estimated the value of g to be $\approx 0.6 - 0.7$ in the structures studied in refs.([2, 4]). This value is an example of what can be expected in the integer quantum Hall regime when interactions destroy the perfect quantization of g by the bulk physics. Strictly speaking, this analysis applies when the interedge is translation invariant along the barrier, like in Eq.(3) and the tunnel

We note that such perturbative calculations have been done in various tunneling situations and are a valuable

tool to perform spectroscopy of reduced dimensionality samples[17, 23, 24, 25, 27] in the quantum Hall regime.

In a given sample under a magnetic field, the edge states will be populated up to a definite Fermi level. We will consider the symmetric situation where the Fermi level is the same on both sides of the sample. Application of a bias voltage induces a difference between these Fermi levels. The external magnetic field may be tuned so that the Fermi points meet at $k = 0$. This is the case first observed by Kang et al where there is a zero-bias peak in the conductance. The peculiar dispersion of the edge modes leads to special kinematic constraints in the case of an extended barrier. Indeed, when the length of the barrier goes to infinity, momentum along the barrier is a conserved quantity during tunneling (contrary to the case of point tunneling). Also energy conservation in the tunnel effect means that only states at the intersection of the dispersion relations of the left and right moving modes will tunnel. For a nonzero Fermi wavevector (defined with respect to the "vacuum" situation Eqs.(1,2)) this implies that one needs a finite bias eV to get a tunneling current. The voltage threshold will go to zero if the filling factor is tuned to the degeneracy point $k_f = 0$.

III. PERTURBATIVE TREATMENT

We now describe the application of the standard tunneling formalism to the line junction. Tunneling occurs from a non-equilibrium situation in which the chemical potential of the left edge μ_L is different from μ_R of the right side because of the applied voltage $eV = \mu_L - \mu_R$. The current is expressed as the rate of change of the number of particles $\widehat{N}_L = \int dx(\psi_L^\dagger(x)\psi_L(x))$, for example, on the left-hand side of the sample. Thus, the average tunneling current is given by:

$$I(t) = -e\langle \dot{N}_L(t)\rangle, \qquad (4)$$

where e is the electron charge. We set $\hbar = 1$ and $k_B = 1$ everywhere.

The total Hamiltonian is of the form :

$$\mathcal{H} = \mathcal{H}_L + \mathcal{H}_R + \mathcal{H}_{tunnel}, \qquad (5)$$

where $\mathcal{H}_L$ and $\mathcal{H}_R$ are Hamiltonians for the chiral Luttinger liquids on each side of the junction. We treat $\mathcal{H}_{tunnel}$ as a perturbation and keep only the leading term. Since we limit ourself to the tunneling at low voltage, low temperature and small tunneling amplitude $\mathcal{T}$, $\mathcal{H}_{tunnel}$ is the only term which does not commute with $\widehat{N}_L$ as $\mathcal{H}_L$ and $\mathcal{H}_R$ separately conserve $\widehat{N}_L$ and $\widehat{N}_R$. Standard first order expansion in the interaction representation yields [28] :

$$I(t) = ie \int_{-\infty}^t dt' \langle [\dot{N}_L(t), \mathcal{H}_{tunnel}(t')]\rangle. \qquad (6)$$

Following [28], this procedure leads to the following formula for the tunneling current :

$$I = e\mathcal{T}^2 \int \frac{dkdk'}{4\pi^2} \frac{\sin^2[(k-k')L/2]}{(k-k')^2} \times \qquad (7)$$

$$\times \int \frac{d\epsilon}{2\pi} A_R(k,\epsilon)A_L(k',\epsilon-eV)\{n_f(\epsilon-eV) - n_f(\epsilon)\}.$$

In this equation $n_f(\epsilon) = 1/(1 + \exp(\beta\epsilon))$ is the Fermi factor at temperature T ($\beta = 1/T$) and $A_R(k,\omega)$ (resp. $A_L(k,\omega)$) is the chiral spectral function for the right (resp. left) moving chiral Luttinger liquid.

The spectral function may be obtained from the imaginary part of the retarded Green's function :

$$A_{R,L}(k,\omega) = -2\,\mathbf{Im} \int\int \frac{dtdx}{4\pi^2} e^{i\omega t - ikx} \qquad (8)$$

$$\times \left[-i\theta(t)\langle\{\psi_{R,L}(x,t), \psi_{R,L}^\dagger(0,0)\}\rangle_\beta \right],$$

where we take the thermal average $\langle\rangle_\beta$ in a grand canonical ensemble including chemical potentials $\mu_{L,R}$. We first evaluate the Fourier transform in space and time of the forward Green function $G_{R,L}^>$ of right-movers and left-movers; then the spectral densities are obtained by use of the following identity which holds for fermions at finite temperature :

$$G_{R,L}^>(k,\omega) = (1 - n_f(\omega))A_{R,L}(k,\omega). \qquad (9)$$

The Green functions for a chiral Luttinger liquid are given by :

$$G_{R,L}^>(k,\omega) = \alpha^{g-1} \int\int \frac{dtdx}{4\pi^2} e^{i\omega t - ikx} e^{\pm ik_f x}$$

$$\times \frac{(\pi T/v)^g}{[i\,\sinh(\pi T(t\mp x/v))]^g}, \qquad (10)$$

where g is the Luttinger liquid parameter [16], k_f is the fermi wavevector and α is a microscopic cut-off. In the case of *electron* tunneling, the anomalous exponent is $g = 1/\nu = m$ when the two FQHE fluids have filling $\nu = 1/m$. This will describe the situation where a barrier separates two distinct electron gases. Here we consider only the simplest situation with the same filling factor on both sides of the barrier. If we consider tunneling with interactions through the barrier, then g is non-quantized, e. g. in an integer Hall system it may be slightly smaller than one (due to repulsive interactions).

The Fourier transforms of the spectral functions can be computed [29] :

$$G_{R,L}^>(k,\omega) = (\frac{2\pi\alpha}{v\beta})^{g-1} e^{\beta\omega/2} \delta(\omega + v(k_f \mp k))$$

$$\times B(\frac{g}{2} + i\frac{\beta\omega}{2\pi}, \frac{g}{2} - i\frac{\beta\omega}{2\pi}). \qquad (11)$$

This leads to the following closed formula :

$$A_{R,L}(k,\omega) = 2(\frac{2\pi\alpha}{v\beta})^{g-1}\cosh(\frac{1}{2}\beta\omega)\,\delta(\omega+v(k_f\mp k))$$

$$\times B(\frac{g}{2}+i\frac{\beta\omega}{2\pi},\frac{g}{2}-i\frac{\beta\omega}{2\pi})\,, \qquad (12)$$

where $B(x,y)$ is the Euler Beta function. In the zero temperature limit, formula (12) reduces to the standard zero-temperature chiral Luttinger liquid spectral function :

$$A_{R,L}(k,\omega) = \alpha^{g-1}\frac{2\pi}{\Gamma(g)}|k\mp k_f|^{g-1}\delta(\omega+v(k_f\mp k)). \quad (13)$$
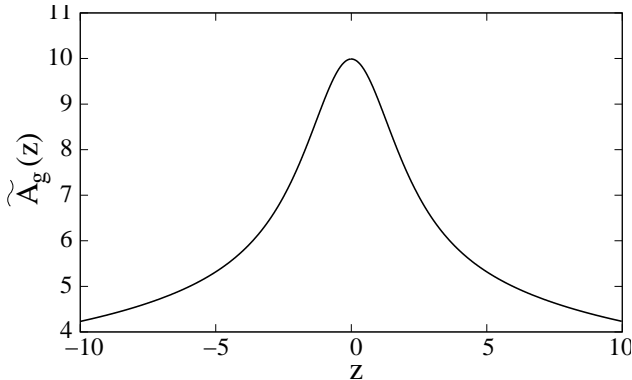

Figure 2: The scaling function $\widetilde{A}_g(z)$of a chiral Luttinger liquid as a function of the scaling variable $z = \omega/T$ in the case g=0.7.

The spectral function can reexpressed to show its scaling properties (we define the Fermi energy $\varepsilon_f = vk_f$) :

$$A_{R,L}(k,\omega) = 2(2\pi)^{g-1}\left(\frac{T}{\varepsilon_f}\right)^{g-1}\widetilde{A}_g(\omega/T)\,\delta(\omega+\varepsilon_f\mp vk),$$
$$(14)$$

with the scaling function :

$$\widetilde{A}_g(z) = (\alpha k_f)^{g-1}\cosh(z/2)\,B(\frac{g}{2}+i\frac{z}{2\pi},\frac{g}{2}-i\frac{z}{2\pi}). \quad (15)$$

We note that the ω/T scaling is destroyed when $g \neq 1$ by the overall factor $(T/\varepsilon_f)^{g-1}$. The scaling function $\widetilde{A}_g$ is plotted in Fig. 2.

IV.   RESULTS

Our formula Eq.(7) yields the value of the tunneling current for any barrier length L, interaction parameter g, finite applied voltage and temperature. It is convenient to define a dimensionless voltage $\overline{v} = eV/(2\varepsilon_f)$. We will concentrate on the zero temperature limit. The tunnel current can be written as :

$$I = I_0\,(k_fL)^2\int_{-\overline{v}}^{+\overline{v}}du\,\frac{\sin^2[(1+u)k_fL]}{(1+u)^2(k_fL)^2}(\overline{v}^2-u^2)^{g-1},$$
$$(16)$$

where we have defined an overall current scale $I_0 = e\mathcal{T}^2(\alpha k_f)^{2g-2}/(8\pi\varepsilon_f\Gamma(g)^2)$. It is not possible to compare directly currents with and without interactions because the current scale I_0 is a function of g. More precisely the microscopic cut-off appears through the dimensionless combination αk_f when $g \neq 1$. In the $g = 1$ case, one can perform the remaining integral in Eq.(16) in terms of the Sine integral [30] :

$$\frac{I}{I_0} = k_fL\,[Si(2k_fL(\overline{v}+1)) + Si(2k_fL(\overline{v}-1))] \quad (17)$$

$$-\frac{1-\cos(2k_fL(\overline{v}+1))}{2(\overline{v}+1)} - \frac{1-\cos(2k_fL(\overline{v}-1))}{2(\overline{v}-1)}.$$

A.   Non-resonant case

We first discuss the case when $k_f \neq 0$. This means that, in the absence of bias, either the two Fermi points are both below the maximum tunneling point $k = 0$ where the dispersion relations do cross or both are above the $k = 0$. So tunneling is suppressed till a finite bias. This can be seen in formula Eq.(16). If we set the barrier length to infinity then the sine function in the integrand of the tunnel current Eq.(16) peaks to a delta function. The current is then nonzero only beyond a bias threshold given by $|eV| > 2vk_f$. This is simply due to the conservation of momentum in the limit $L \to \infty$ which severely restricts tunneling. This threshold is smoothed out for finite L values and disappears in the point contact limit $L \to 0$. We will refer to the generic case $k_f \neq 0$ as being "non-resonant". On the contrary, for $k_f = 0$ tunneling is allowed for infinitesimal bias, the "resonant" case treated in the next section. In a realistic set-up one needs a special fine-tuning of the magnetic field to reach the resonant case. In the experiment of Kang et al, there is an extended region of k_f, i.e. of magnetic field where there is sizeable tunneling for infinitesimal bias as revealed by a zero-bias peak in the conductance. Such an extended range of tunneling is out of reach of the present model where there is a single point satisfying the resonance condition.

In fact, the tunnel current is $O(L^0)$ below the threshold and grows as $O(L)$ above. Below the threshold, the $L \to \infty$ limit leads to :

$$\frac{I}{I_0} = \left(\frac{eV}{2\varepsilon_f}\right)^{2g-1}\frac{\sqrt{\pi}\Gamma(g)}{2\Gamma(g+1/2)}\times{}_2F_1(1,\frac{3}{2},g+\frac{1}{2};\left(\frac{eV}{2\varepsilon_f}\right)^2),$$
$$(18)$$

where ${}_2F_1$ is the hypergeometric function [30]. Above threshold, we find the simple formula :

$$\frac{I}{I_0} = \pi\,k_fL\,\left(\left(\frac{eV}{2\varepsilon_f}\right)^2-1\right)^{g-1}. \qquad (19)$$

Some $I - V$ curves are displayed in fig. (3) for a very long barrier, $k_fL \gg 1$. In the FQHE case, we have taken
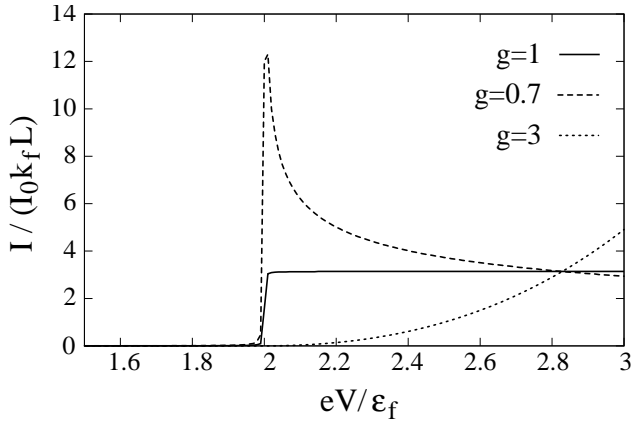
Figure 3: Tunneling current vs eV/ε_f for three values of g : $g = 3$, (dotted line) $g = 1$ (solid line), $g = 0.7$ (dashed line) for a barrier of length $k_f L = 1000$. Since the normalization I_0 of the currents is a function of g, no comparison can made between the absolute value of these $I - V$ curves.

the Luttinger parameter $g = 3$ for tunneling between two $\nu = 1/3$ liquids. The case $g = 1$ refers to free fermions, here there is a saturation of the tunnel current, i. e. I becomes O(L) independent of V. Finally, we have drawn the $I - V$ curve for a Luttinger parameter less than one, possibly relevant to the case of IQHE edge modes with interedge interactions. There is a sharp current spike at threshold. The current indeed behaves as $\sim (eV - 2\varepsilon_f)^{g-1}$ close to threshold in the large-L limit. For $V \to 0$, the current behaves as V^{2g-1}.

For a finite length barrier with $k_f L$ of order unity, the kinematic singularities are smoothed out. Typical curves are presented in Fig.(4). The behavior for V small is unchanged but there are additional oscillations due to the diffraction pattern generated by the barrier acting like a slit. We note that in the case with g less than one, the singular behavior for small voltage $\propto V^{2g-1}$ shows up only for small voltage. In fig.(5) is presented a close-up of the case $g = 0.7$ at very small bias : here one can observe the nonlinearity typical of a Luttinger liquid and the additional interference pattern. The period of the oscillations is $2\pi/(k_f L)$ in terms of the dimensionless voltage $eV/(2\varepsilon_f)$.

Fig. 6 shows the tunneling current vs the length of the barrier for a fixed voltage $eV/(2\varepsilon_f) = 0.1$ and several values of the interaction parameter g. There is a saturation for L larger than hundreds of the Fermi wavelength λ_f, as discussed above, because the voltage bias is below the threshold. There is also a beating pattern in both cases. The fast oscillations are due to the interference of L versus the Fermi wavelength while the slow oscillations are ruled by the voltage scale introduced in the problem by the bias voltage $eV/(2\varepsilon_f)$ set to 0.1. We have omitted the case $g = 3$ because it has a similar shape but with a very small current in units of I_0.
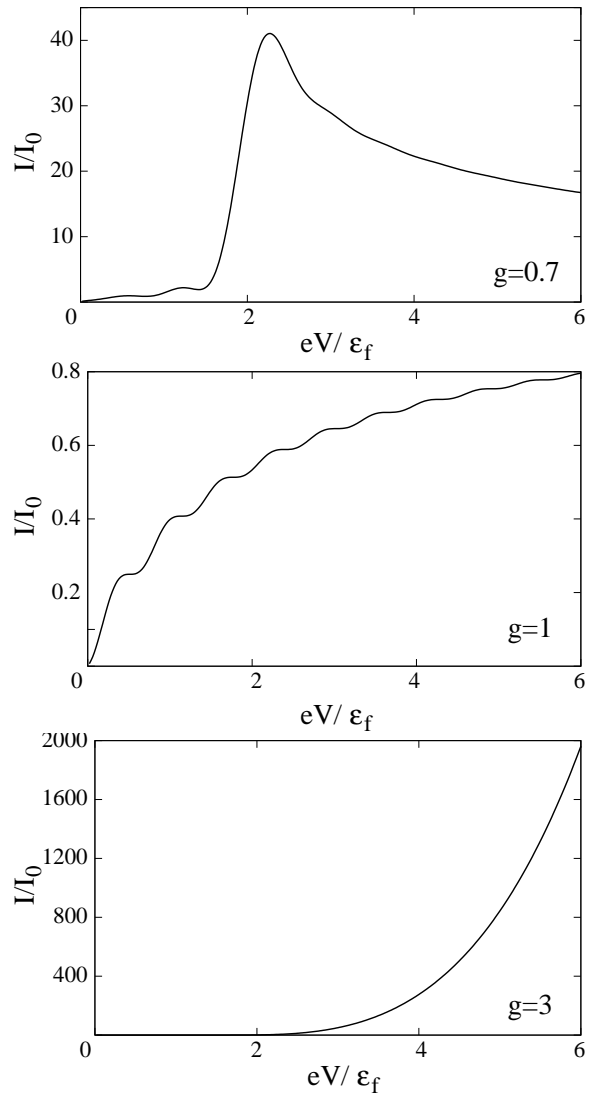

Figure 4: Tunneling current vs eV/ε_f for three values of g : $g = 0.7$, $g = 1$ and $g = 3$ from top to bottom for a barrier of length $k_f L = 10$.
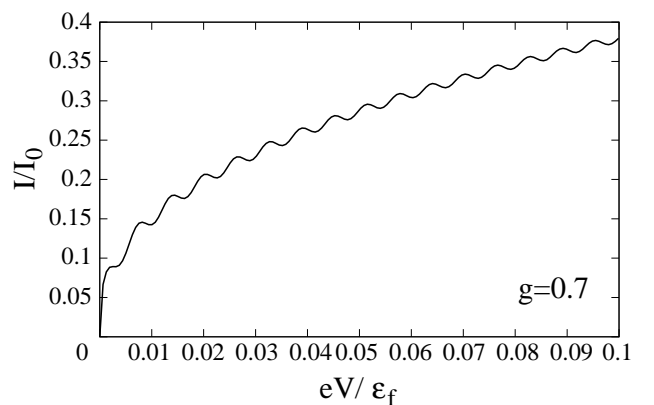

Figure 5: Tunneling current vs eV/ε_f for the interacting parameter $g = 0.7$ for a finite barrier length $k_f L = 1000$.
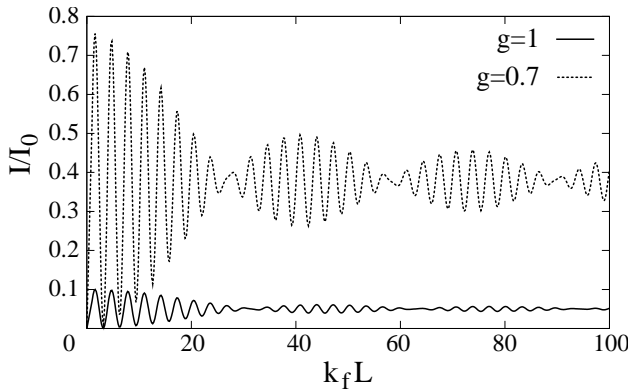
Figure 6: Tunneling current in unit of I_0 vs length of the barrier for the interacting parameter $g = 1$ (solid line) and $g = 0.7$ (dotted line). The bias is fixed at $eV/(2\varepsilon_f) = 0.1$.

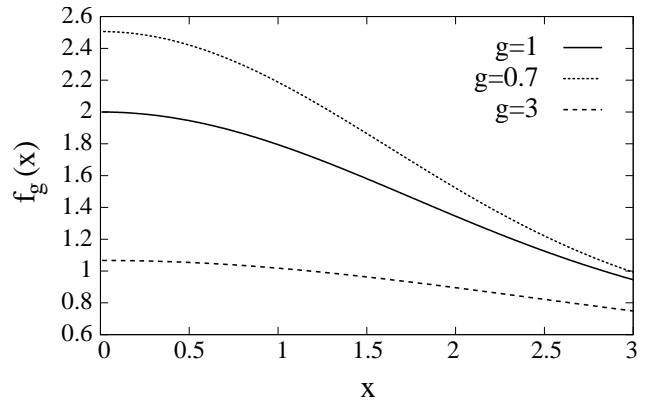

Figure 7: The auxiliary function governing the crossover between the regimes of long and short barriers for values of the parameter g.

B. Resonant case

We now turn to the case $k_f = 0$ which can be obtained by tuning the external applied magnetic field. This means that in the absence of bias the left and right Fermi levels exactly coincide at $k = 0$. The tunnel current from our perturbative calculation is now simpler because the Fermi energy disappears from the problem. As a consequence we find :

$$I = \frac{e\mathcal{T}^2}{\Gamma(g)^2} \frac{L^2}{8\pi\alpha v} \left(\frac{\alpha eV}{2v}\right)^{2g-1} f_g\left(\frac{eVL}{2v}\right), \qquad (20)$$

where we define the auxiliary function $f_g(x)$ by :

$$f_g(x) = \int_{-1}^{+1} dt \ (1-t^2)^{g-1} \frac{\sin^2(xt)}{(xt)^2}. \qquad (21)$$

This function decreases as π/x for large x and any value of g and has a nonzero value at the origin. This means that $I \propto V^{2g-1}$ if $eVL/v \ll 1$ while $I \propto V^{2g-2}$ if $eVL/v \gg 1$. For a typical $g = 0.7$ value this means that the current should first rise as $V^{0.4}$ before going down at larger voltage as $V^{-0.6}$. The function that governs the crossover is shown in Fig.(7). Some $I - V$ curves are displayed in Fig.(8). While the Luttinger liquid nonlinearities are still present, the oscillations formerly due to the presence of the scale k_f no longer exist. The $g = 0.7$ curve shows that strong interedge interactions in a situation of weak tunneling lead to a diverging conductance at zero bias.

Finally we note for completeness that our study may be extended at nonzero temperature by using the finite-temperature spectral functions. Using a dimensionless temperature $z = \beta\varepsilon_f$, we have :

$$I = \frac{e\mathcal{T}^2}{8\pi\varepsilon_f} \frac{(\alpha k_f)^{2g-2}}{\Gamma(g)^2} \left(\frac{2\pi}{z}\right)^{2g-2} \frac{\Gamma(g)^2}{4\pi^2} (k_f L)^2 \times \qquad (22)$$

$$\int du \ \frac{\sin^2[(1+u)k_f L]}{(1+u)^2(k_f L)^2} e^{zu}(1 + e^{-z(u+\overline{v})})(1 + e^{-z(u-\overline{v})})$$
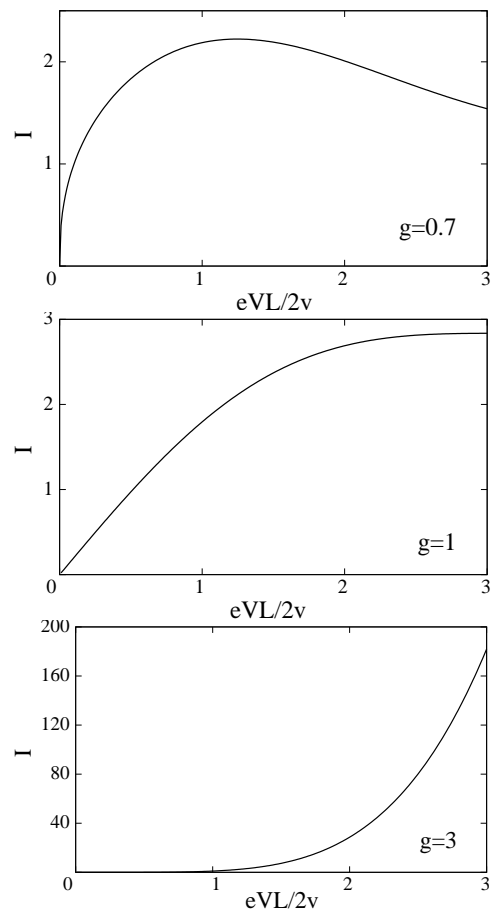

Figure 8: Tunneling current vs bias $eVL/(2v)$ at resonance for three values of g : $g = 0.7$, $g = 1$ and $g = 3$ from top to bottom.

$$\times B\left(\frac{g}{2} + i\frac{z}{2\pi}(u+\overline{v}), \frac{g}{2} - i\frac{z}{2\pi}(u+\overline{v})\right)$$
$$\times B\left(\frac{g}{2} + i\frac{z}{2\pi}(\overline{v}-u), \frac{g}{2} - i\frac{z}{2\pi}(\overline{v}-u)\right).$$

We expect thermal rounding of the oscillatory features as

well as the threshold behavior in the non-resonant case.

V. CONCLUSIONS

In this paper we have studied the tunneling between two counterpropagating edges modes pertaining to different electron gases by means of a perturbative expansion. We find the characteristic nonlinearities of the Luttinger liquid properties. There are additional oscillations due to the finite extent of the barrier. Our study applies to the case of tunneling between two FQHE fluids for $\nu = 1/m$ and also applies to the case of tunneling between edges when there is strong interactions throughout the barrier (but weak tunneling) in which case the characteristic ex-

ponent entering the spectral functions is no longer quantized by the bulk physics. It remains to be seen if samples with the correct properties can be artificially produced. We note that the present experiment by Kang et al is apparently in the regime of strong tunneling (even if the conductance throughout the barrier is far below the expected Landauer value).

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