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Local search for the minimum label spanning tree problem with bounded color classes

TOBIAS BRÜGGEMANN ^{*} JÉRÔME MONNOT [†] GERHARD J. WOEGINGER [‡]

Abstract

In the Minimum Label Spanning Tree problem, the input consists of an edge-colored undirected graph, and the goal is to find a spanning tree with the minimum number of different colors. We investigate the special case where every color appears at most r times in the input graph. This special case is polynomially solvable for $r = 2$, and NP-complete and APX-complete for any fixed $r \geq 3$.

We analyze local search algorithms that are allowed to switch up to k of the colors used in a feasible solution. We show that for $k = 2$ any local optimum yields an $(r + 1)/2$ -approximation of the global optimum, and that this bound is tight. For every $k \geq 3$, there exist instances for which some local optima are a factor of $r/2$ away from the global optimum.

Keywords: Graph algorithms; approximation algorithms; combinatorial optimization; local search; complexity; APX-completeness.

1 Introduction

In the *Minimum Label Spanning Tree problem* (MINLST, for short), we are given a simple, connected, undirected graph $G = (V, E)$ without loops on n vertices. The edges in E are colored (or labeled) with the colors c_1, c_2, \dots, c_q . For $i = 1, \dots, q$ we denote by $E(c_i) \subseteq E$ the set of edges with color c_i . The goal in MINLST is to find a spanning tree in G that uses the minimum number of colors. An equivalent formulation of MINLST asks to find a smallest cardinality subset $C \subseteq \{c_1, c_2, \dots, c_q\}$ of the colors, such that the subgraph induced by the edge sets $E(c_i)$ with $c_i \in C$ is connected and touches all vertices in V .

Motivated by certain applications in communication network design, Chang & Leu [4] introduced problem MINLST in 1997 and proved that it is NP-complete. Krumke & Wirth [9] formulated a greedy algorithm for MINLST, and showed that its worst case performance ratio is at most $2 \ln n + 1$. Moreover, [9] proved that no polynomial time approximation algorithm

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for MINLST can have a worst case performance ratio $(1 - \varepsilon) \ln n$, for any $\varepsilon > 0$. Wan, Chen & Xu [16] provided a better analysis of the greedy algorithm in [9]; they showed that its worst case performance ratio is at most $\ln(n - 1) + 1$.

Results of this paper. In this paper, we study the special case MINLST_r of MINLST in which every color occurs at most r times ($r \geq 2$) on the edges of G . For $r = 2$, this special case is equivalent to the Graphic Matroid Parity problem, and therefore can be solved in polynomial time (see Observation 5.1 in Section 5). For every $r \geq 3$, this special case MINLST_r is NP-complete and APX-complete; hence, for $r \geq 3$ this special case does not possess a polynomial time approximation scheme unless $\text{P}=\text{NP}$ (see Theorem 5.2 in Section 5).

In Section 2 we introduce a family of local search algorithms that are based on the so-called *k-switch* neighborhoods, where $k \geq 1$ is an integer. Sloppily speaking, a *k-switch* replaces up to k of the colors used in a feasible solution by other colors. Local optima for the *k-switch* neighborhoods can be computed in polynomial time. In Sections 3 and 4 we then discuss how well local optima for *k-switch* perform in comparison to global optima: For $k = 2$, any local optimum yields an $(r + 1)/2$ -approximation of the global optimum, and this bound of $(r + 1)/2$ is best possible. For every $k \geq 3$, there exist instances for which some local optimum is a factor of roughly $r/2$ away from the global optimum. Hence, from the worst case point of view there is almost no profit in moving from the (small) 2-switch neighborhood to the (much bigger) *k-switch* neighborhoods with $k \geq 3$.

In studying the worst case quality of local optima of local search algorithms for combinatorial problems, we follow the line of research of Finn & Horowitz [6] (for multiprocessor scheduling), Hurkens & Schrijver [8] (for set packing), Lu & Ravi [11] (for maximum-leaf spanning trees), Ausiello & Protasi [3] (for complexity aspects), Arkin & Hassin [2] (for weighted set packing), and Schuurman & Vredeveld [14] (for multiprocessor scheduling). For more information on this area, we refer the reader to the Ph.D. thesis [15] of Vredeveld.

2 The *k-switch* neighborhoods

Any spanning tree T for problem MINLST can be represented by the set $C(T) \subseteq \{c_1, \dots, c_q\}$ of colors used in T . In this section, we prefer to work with color sets. A color set C is *feasible* if and only if the corresponding set of edges is connected and touches all vertices in the graph.

Definition 2.1 *Let $k \geq 1$ be an integer, and let C_1 and C_2 be two feasible color sets for some instance of MINLST. Then the set C_2 is in the *k-switch* neighborhood $k\text{-SWITCH}(C_1)$ of the set C_1 , if and only if*

$$|C_1 - C_2| \leq k \quad \text{and} \quad |C_2 - C_1| \leq k. \tag{1}$$

In other words, we can get the color set C_2 from the color set C_1 by first removing up to k colors from C_1 , and then adding up to k colors to it.

As usual with neighborhood structures, we may build a local search algorithm around the *k-switch* neighborhood:

Start with an arbitrary feasible color set C . As long as there exists a feasible color set C' in k -SWITCH(C) with $|C'| < |C|$, replace the old set C by the better set C' .

Eventually, the local search algorithm will terminate in a *local optimum* C : For this local optimum C , any set C' in k -SWITCH(C) will satisfy $|C'| \geq |C|$. In a slight abuse of notation, we will say that a spanning tree is a local optimum for the k -switch neighborhood if and only if its associated color set $C(T)$ is a local optimum for the k -switch neighborhood.

The following observation shows that for every fixed value of k , a local optimum for the k -switch neighborhood can be determined in polynomial time.

Observation 2.2 *For any $k \geq 1$, a local optimum with respect to the k -switch neighborhood can be computed in $O(n^{3k+3})$ time.*

Proof. Without loss of generality, we assume that the starting point of the local search algorithm contains at most $n - 1$ colors. By equation (1) any neighborhood set k -SWITCH(C) contains at most $O(|C|^k q^k)$ feasible sets. Since $|C| \leq n - 1$ and since $q \leq |E| \leq n^2$, we conclude that $|k\text{-SWITCH}(C)| = O(n^{3k})$. Within $O(n^2)$ time, we can determine whether a color set in the neighborhood is feasible and we can determine its objective value. Hence, one replacement step in the local search takes only $O(n^{3k+2})$ time.

Since the possible objective values are integers in the range from 1 up to $n - 1$, the local search terminates after at most $n - 2$ replacement steps. ■

In the following two sections, we will analyze the quality of local optima with respect to k -switch neighborhoods for $k \geq 2$. The case $k = 1$ is trivial.

Observation 2.3 *Let $r \geq 2$ be an integer. For any instance of MINLST_r , a local optimum with respect to the 1-switch neighborhood gives an r -approximation of the global optimum. This bound is tight.* ■

3 Local optima for the 2-switch neighborhood

In this section, we provide a complete worst case analysis of local optima with respect to the 2-switch neighborhood: Every local optimum yields an $(r + 1)/2$ -approximation of the global optimum (Theorem 3.1), and this bound is best possible (Theorem 3.2).

Theorem 3.1 *For any integer $r \geq 2$ and for any instance G of MINLST_r , the objective value of any local optimum with respect to the 2-switch neighborhood is at most a factor of $(r + 1)/2$ above the optimal objective value.*

Proof. Suppose for the sake of contradiction that the statement is false, and consider a counterexample $G = (V, E)$ with the smallest number of edges. Let $T^* = (V, E^*)$ be an optimal spanning tree for G , and let $T^+ = (V, E^+)$ be a locally optimal tree with respect to the 2-switch neighborhood. Let $C^* = C(T^*)$ and $C^+ = C(T^+)$ denote the corresponding color sets with

$$|C^+| > \frac{r+1}{2} |C^*|. \tag{2}$$

We observe that in a smallest counterexample, $C^* \cap C^+ = \emptyset$ must hold: If there is a color $i \in C^* \cap C^+$, then we can contract all edges with this color i in G , and get a smaller instance where the global and local optimum both use one color less. Since this smaller instance still satisfies the inequality (2), we would have found a smaller counterexample. Hence, $C^* \cap C^+ = \emptyset$. Moreover, a smallest counterexample satisfies $E^* \cup E^+ = E$.

Let n denote the number of vertices in G . A color is called *singleton* if it shows up on exactly one edge of G . Let ℓ denote the number of singleton colors in C^+ , and let e_1, \dots, e_ℓ be an enumeration of the corresponding edges in T^+ . Consider the $\ell + 1$ subtrees $T_1^+, \dots, T_{\ell+1}^+$ that result from removing the ℓ edges e_1, \dots, e_ℓ from T^+ .

Suppose that there exists some color i , such that the edges with color i connect more than two of these subtrees $T_1^+, \dots, T_{\ell+1}^+$ to each other. Then one could add color i to C^+ , remove an appropriate pair of singleton colors from C^+ , and get another feasible color set C^- with strictly better objective value. Since the set C^- is in the 2-switch neighborhood of the local optimum C^+ , we arrive at a contradiction. Therefore, every color connects at most two of these $\ell + 1$ subtrees to each other. But this implies that also the global optimum must spend at least ℓ colors on connecting the corresponding $\ell + 1$ vertex sets to each other, and we get

$$|C^*| \geq \ell. \quad (3)$$

Since a spanning tree has $n - 1$ edges, and since every color occurs at most r times, we furthermore have that

$$|C^*| \geq \frac{n - 1}{r}. \quad (4)$$

Now let us estimate the number of colors in the local optimum T^+ : There are ℓ edges in T^+ with the ℓ singleton colors. Every non-singleton color i in T^+ occurs at least twice on the edges of G . Since $C^+ \cap C^* = \emptyset$, the color i cannot show up in T^* , and since $E^* \cup E^+ = E$, all edges with color i are contained in T^+ . This yields that there are at most $(n - 1 - \ell)/2$ non-singleton colors in C^+ . Hence,

$$|C^+| \leq \ell + \frac{1}{2}(n - 1 - \ell) = \frac{1}{2}(n - 1) + \frac{1}{2}\ell \leq \frac{r}{2}|C^*| + \frac{1}{2}|C^*| = \frac{r + 1}{2}|C^*|. \quad (5)$$

Here we used (3) and (4). The inequality (5) blatantly contradicts our initial assumption (2). This contradiction completes the proof of the theorem. ■

Theorem 3.2 *For any integer $r \geq 2$, there exist an instance G of MINLST_r and a spanning tree T for G that is a local optimum with respect to the 2-switch neighborhood, such that the objective value of T is $(r + 1)/2$ above the optimal objective value.*

Proof. Consider the graph G with vertices v_0, x_0, \dots, x_{r-1} , and y_0, \dots, y_{r-1} . There is an edge from v_0 to every other vertex. Moreover, the vertices x_0, \dots, x_{r-1} (in this ordering) induce a cycle and the vertices y_0, \dots, y_{r-1} (in this ordering) induce a cycle. There are $r + 3$ colors: For $i = 1, \dots, r - 1$ the two edges $[x_{i-1}, x_i]$ and $[y_{i-1}, y_i]$ have color i . The edge $[v_0, x_0]$ has color r , and the edge $[v_0, y_0]$ has color $r + 1$. The edge $[x_0, x_{r-1}]$ and all edges from v_0 to x_1, \dots, x_{r-1} have color $r + 2$; the edge $[y_0, y_{r-1}]$ and all edges from v_0 to y_1, \dots, y_{r-1} have color $r + 3$.

Then the edges with colors $r + 2$ and $r + 3$ form a spanning tree with 2 colors. The edges with colors $1, 2, \dots, r + 1$ form a spanning tree with $r + 1$ colors that is a local optimum with respect to the 2-switch neighborhood. See Figure 1 for an illustration. ■

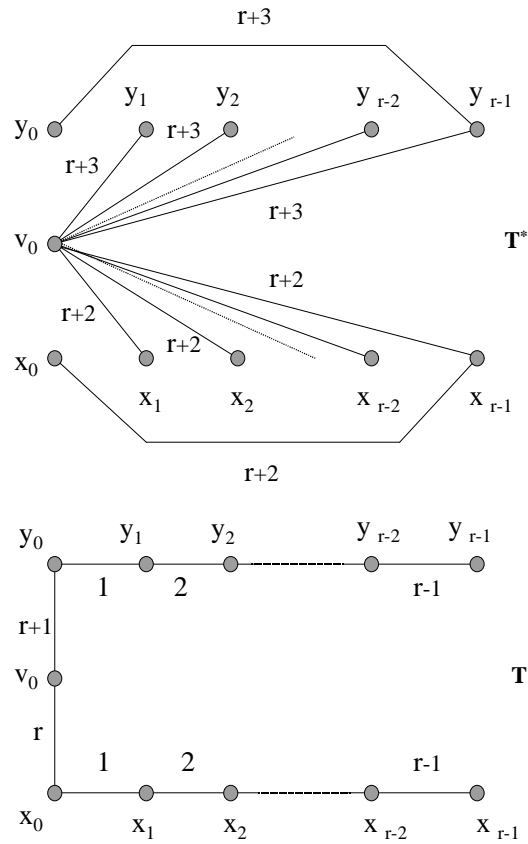


Figure 1: A global optimum and a local optimum for 2-switch in the proof of Theorem 3.2.

4 Local optima for the k -switch neighborhood

In this section we will show that from the worst case point of view, it will not help a lot if we move from the 2-switch neighborhood to the bigger k -switch neighborhoods with $k \geq 3$: There always will be instances for which a local optimum for a k -switch neighborhood is a factor of $r/2$ away from the global optimum.

Lemma 4.1 *For any $k \geq 2$ and for any $r \geq 3$, there exist arbitrarily large undirected, simple graphs $H = (V_H, E_H)$ that satisfy the following three properties.*

- H is r -regular (i.e., every vertex in H has degree exactly r)
- H has girth at least k (i.e., the shortest cycle in H has length at least k)
- H contains a perfect matching \mathcal{M} .

Proof. By applying a result of Erdős & Sachs [5], Hurkens & Schrijver [8] construct bipartite r -regular graphs of girth at least k . It is well-known that every regular bipartite has a perfect matching. By taking many disjoint copies of the graph in [8], we get arbitrarily large graphs with the desired three properties. ■

Now consider a graph $H = (V_H, E_H)$ as described in Lemma 4.1. Denote $|V_H| = 2h$, and let w_1, w_2, \dots, w_{2h} be an enumeration of the vertices in V_H such that for $i = 1, \dots, h$ the vertices w_i and w_{h+i} form an edge in the perfect matching \mathcal{M} . From H we will construct an instance graph $G = (V_G, E_G)$ for MINLST_r . The vertex set V_G consists of $2rh + 2$ vertices. There are two special vertices u_1 and u_2 , and for $i = 1, \dots, 2h$ there is a group G_i of r vertices $v_{i,j}$ with $1 \leq j \leq r$. The edges in G are defined as follows.

- There is an edge between the two special vertices u_1 and u_2 .
- The special vertex u_1 is connected to all vertices $v_{i,j}$ with $1 \leq i \leq 2h$ and $1 \leq j \leq r$.
- The special vertex u_2 is connected to all vertices $v_{i,1}$ with $1 \leq i \leq 2h$.
- Every group G_i induces a path through the vertices $v_{i,1}, v_{i,2}, \dots, v_{i,r}$ in exactly this ordering.

The edge colors are defined as follows.

- (C1) The edge $[u_1, u_2]$ has color c^* .
- (C2) For $i = 1, \dots, 2h$ every edge between the special vertex u_1 and the group G_i has color $c(i)$. We say that color $c(i)$ corresponds to the vertex w_i in H .
- (C3) For $i = 1, \dots, h$ the two edges $[u_2, v_{i,1}]$ and $[u_2, v_{h+i,1}]$ have color $\bar{c}(i)$. We say that color $\bar{c}(i)$ corresponds to the edge $[w_i, w_{h+i}]$ in \mathcal{M} .
- (C4) For every edge $[w_a, w_b] \in E_H - \mathcal{M}$, there is a corresponding color $c(a, b)$. This color $c(a, b)$ shows up exactly once on the path induced by group G_a and exactly once on the path induced by group G_b . Since w_a is incident to $r - 1$ edges in $E_H - \mathcal{M}$, this yields exactly $r - 1$ colors for the $r - 1$ edges in the path induced by G_a . The exact assignment of colors $c(a, b)$ to edges in G_a is irrelevant for our arguments; an arbitrary assignment will work.

We say that color $c(a, b)$ corresponds to the edge $[w_a, w_b]$ in $E_H - \mathcal{M}$.

Note that the color c^* in (C1) occurs once, that every color $c(i)$ in (C2) occurs exactly r times, and that every color $\bar{c}(j)$ in (C3) and every color $c(a, b)$ in (C4) occurs exactly twice. Hence, we have indeed constructed an instance of MINLST_r .

Lemma 4.2 *The optimal objective value of instance G is at most $2h + 1$.*

Proof. The edge $[u_1, u_2]$ of color c^* in (C1) together with the edges with colors $c(i)$ with $1 \leq i \leq 2h$ in (C2) form a spanning tree for G . ■

Lemma 4.3 *There exists a spanning tree T for G*

- (a) *that has objective value $rh + 1$, and*
- (b) *that is a local optimum with respect to the k -switch neighborhood.*

Proof. We let T consist of all color classes in (C1), (C3), and (C4). This yields a spanning tree with $rh + 1$ colors that satisfies property (a). It remains to prove that T also satisfies the property in (b). Suppose for the sake of contradiction that there is an improving k -switch for T . This k -switch removes $x \leq k$ colors from T , and it adds $y \leq x - 1$ colors to T . We make two observations:

- If the switch removes the color $\bar{c}(i)$ (that corresponds to the edge $[w_i, w_{h+i}]$ in H), then it must simultaneously add the two colors $c(i)$ and $c(h+i)$ (that correspond to the vertices w_i and w_{h+i} in H). Otherwise, one of the groups G_i and G_{h+i} will be separated from the rest of the graph.
- If the switch removes the color $c(a, b)$ (that corresponds to the edge $[w_a, w_b]$ in E_H) then it must simultaneously add the two colors $c(a)$ and $c(b)$ (that correspond to the vertices w_a and w_b in H). Otherwise, some vertices in group G_a or G_b will be isolated from the rest of the graph.

To summarize, whenever the switch removes a color in (C3) or (C4) that corresponds to an edge in H , then it must simultaneously add the two colors in (C2) that correspond to the vertices of this edge in H .

Let $Y \subset V_H$ denote the vertices in H that correspond to the $|Y| \leq k - 1$ colors from (C2) that the switch adds to T . Then the switch can remove the single color c^* , and it can remove the colors in (C3) and (C4) that correspond to edges induced by vertices in Y . Since H has girth k , the subgraph of H induced by Y is cycle-free. Hence it is a forest, and induces at most $|Y| - 1$ edges in H . But this means that the k -switch adds $|Y|$ colors, while it removes at most $|Y|$ colors; hence, it is not an improving k -switch. This contradiction completes the proof. ■

Theorem 4.4 *For any integer $k \geq 2$, for any integer $r \geq 2$, and for any real $\varepsilon > 0$, there exist an instance G of MINLST_r and a spanning tree T for G that is a local optimum with respect to the k -switch neighborhood, such that the objective value of T is at least $r/2 + \varepsilon$ above the optimal objective value.*

Proof. Lemmas 4.2 and 4.3 yield a ratio $(rh + 1)/(2h + 1)$ between the objective values of the local and of the global optimum. As h tends to infinity, this ratio tends to $r/2$. ■

5 Complexity and in-approximability

In this section, we first explain why problem MINLST_2 is easy, and then prove that problem MINLST_3 is difficult. MINLST_3 is APX-complete, which implies that it does not have a polynomial time approximation scheme unless $P=NP$.

Observation 5.1 For $r = 2$, the problem MINLST_r is polynomially solvable.

Proof. The problem MINLST_2 is essentially equivalent to the *Graphic Matroid Parity problem*; see for instance Lovász & Plummer [10] and Gabow & Stallman [7]: In the Graphic Matroid Parity problem, we are given a graph $G' = (V', E')$ and a partition of the edge set E' into disjoint pairs of edges $\{f, f'\}$. The goal is to find a forest F with the maximum number of edges, such that $f \in F$ holds if and only if $f' \in F$ for all pairs $\{f, f'\}$ in the partition.

In problem MINLST_2 , the edge pairs $\{f, f'\}$ are the pairs of edges with the same color. The goal is to use as many colors twice as possible, and then to connect the resulting forest to a tree by adding color classes of cardinality one. ■

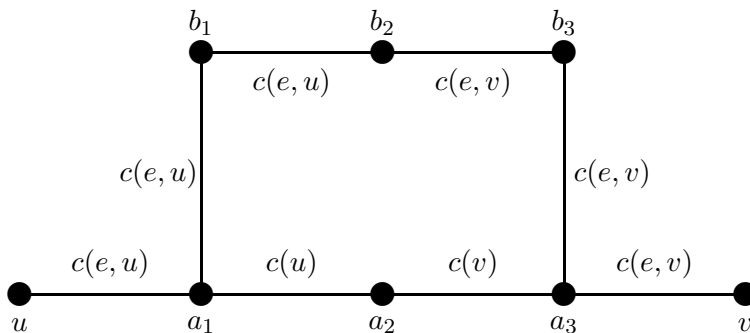


Figure 2: The gadget $Z(u, v)$ as used in the proof of Theorem 5.2.

Theorem 5.2 For $r \geq 3$ the problem MINLST_r is APX-complete even if the input graph G is restricted to be bipartite and of maximum degree 3.

Proof. The proof will be done via an approximation preserving L -reduction (cf. Papadimitriou & Yannakakis [13]) from the vertex cover problem in 3-regular connected graphs, VC3 for short: An instance of VC3 consists of a connected 3-regular graph $H = (V_H, E_H)$, and the goal is to find a minimum cardinality vertex cover W for H , that is, a subset $W \subseteq V_H$ that intersects every edge in E_H . Alimonti & Kann [1] proved that problem VC3 is APX-hard. This implies that there is some small $\varepsilon > 0$ such that the existence of a polynomial time approximation algorithm with performance guarantee $1 + \varepsilon$ would imply $P=NP$.

We consider an arbitrary instance $H = (V_H, E_H)$ of problem VC3, with $|V_H| = 2h$ and $|E_H| = 3h$. We construct a corresponding instance $G = (V_G, E_G)$ of problem MINLST_3 from it: For every vertex $v \in V_H$, there is a corresponding color $c(v)$. For every edge $e = [u, v] \in E_H$, there are two corresponding colors $c(e, u)$ and $c(e, v)$. G results from H by replacing every edge $e = [u, v] \in E_H$ by a copy of the gadget $Z(u, v)$ depicted in Figure 2. This gadget $Z(u, v)$ has six new vertices a_1, a_2, a_3 and b_1, b_2, b_3 . The edges and their colors are defined as follows:

- The edges $[u, a_1]$, $[a_1, b_1]$, $[b_1, b_2]$ are of color $c(e, u)$.

- The edges $[b_2, b_3]$, $[b_3, a_3]$, $[a_3, v]$ are of color $c(e, v)$.
- The edge $[a_1, a_2]$ has color $c(u)$.
- The edge $[a_2, a_3]$ has color $c(v)$.

This completes the description of the graph G . Note that the colors $c(e, u)$ and $c(e, v)$ only show up within the gadget $Z(u, v)$, and there they are used three times. Any color $c(v)$ shows up once in the three gadgets that correspond to the three edges incident to v in H . Hence, we have indeed constructed an instance of MINLST_3 . Moreover, the graph G clearly is bipartite and of maximum degree 3.

Since every vertex in H is incident to exactly three edges, the optimal vertex cover W^* for H must contain at least $|E_H|/3 = h$ vertices. Since there are altogether $|V_H| + 2|E_H| = 8h$ colors in G , the optimal spanning tree T^* for G uses at most $8h$ colors. Therefore,

$$|C(T^*)| \leq 8|W^*|. \quad (6)$$

Since in every gadget $Z(u, v)$ the vertex b_1 (respectively, the vertex b_3) is only adjacent to edges of color $c(e, u)$ (respectively, to edges of color $c(e, v)$), all these colors $c(e, u)$ and $c(e, v)$ must be used in any spanning tree of G . Moreover, in order to connect the vertex a_2 to the rest of the tree, any spanning tree must use at least one of the two colors $c(u)$ and $c(v)$. Based on these observations, it is easy to translate a spanning tree T for G into a corresponding vertex cover W_T for H : W_T consists of the vertices $v \in V_H$ for which the color $c(v)$ shows up in the tree T . Consequently, $|W_T| = |C(T)| - 6h$. By similar reasoning, we get that the optimal spanning tree T^* of G and the optimal vertex cover W^* of H satisfy $|W^*| = |C(T^*)| - 6h$. This implies that for any spanning tree T , $|W_T| - |W^*| = |C(T)| - |C(T^*)|$. Combining this fact with (6) yields

$$|W_T| - |W^*| \leq |C(T)| - |C(T^*)| \cdot \frac{8|W^*|}{|C(T^*)|}. \quad (7)$$

Now, if $|C(T)| \leq (1+\varepsilon)|C(T^*)|$ holds, then the inequality (7) yields $|W_T| \leq (1+8\varepsilon)|W^*|$. Hence, the existence of a polynomial time approximation scheme for problem MINLST_3 would imply the existence of a polynomial time approximation scheme for problem VC_3 . This establishes APX-hardness of MINLST_3 . Since MINLST_3 clearly is contained in APX, the proof of the theorem is complete. ■

Mohar [12] has shown that the vertex cover problem is NP-complete for planar 3-regular graphs. With this, the reduction in Theorem 5.2 yields that MINLST_3 is NP-complete even in planar, bipartite graphs of maximum degree 3. The approximability of MINLST and MINLST_r in planar graphs remain open.

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