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Abstract

We analyze the advantages of using ultra-cold coherent sources of atoms for matter-wave interferometry in space. We present a proof-of-principle experiment that is based on an analysis of the results previously published in [1] from which we extract the ratio h/m for ^{87}Rb. This measurement shows that a limitation in accuracy arises due to atomic interactions within the Bose-Einstein condensate. Finally we discuss the promising role of coherent-matter-wave sensors, in particular inertial sensors, in future fundamental physics missions in space.

Key words: Matter Waves, Optical Cooling and Trapping, Bose-Einstein Condensation, Atom Interferometry, Metrology

Atom interferometry [2,3,4,5,6] has long been one of the most promising candidates for ultra-precise and ultra-accurate measurement of gravito-inertial forces [7,8,9,10,11,12,13] or for precision measurements of fundamental constants [14]. The realization of Bose-Einstein condensation (BEC) of a dilute gas of trapped atoms in a single quantum state [15,16,17] has produced the matter-wave analog of a laser in optics [18,19,20,21]. As lasers have revolutionized optical interferometry [22,23,24], so it is expected that the use of Bose-Einstein condensed atoms will bring the science of atom optics, and in particular atom interferometry, to an unprecedented level of accuracy [25,26]. In addition, BEC-based coherent atom interferometry would reach its full potential in space-based applications where micro-gravity will allow the atomic interferometers to reach their best performance [27].

In this document, we discuss the prospects of using atom-lasers in future space missions to study fundamental physics. We point out that atomic ensembles at sub-microKelvin temperatures will be required, if the sensitivity of space-based atom-interferometers is to reach its full potential. In addition, we show

that interactions within a Bose-Einstein condensate (or atom laser) will limit the measurement accuracy of such devices and must be controlled to a very high level. We demonstrate the latter in a ground-based measurement, by re-analyzing the Bragg-spectroscopy data of [1] to extract the ratio h/m of the Planck constant h to the atomic mass m of ^{87}Rb. This ratio h/m is related to the fine structure constant α [28], of which precise knowledge is essential for testing the validity of measurements related to different branches of physics (QED, solid state physics, . . .) [29,30,31,32,33,34]. A measurement of h/m has been proposed as a candidate microgravity mission in the HYPER [27] and ICE [35] programs, as a follow-up to the state-of-the art measurements on earth using cold atoms [36,37].

Generally, atom interferometry is performed by applying two successive *coherent* beam-splitting processes separated by a time T to an ensemble of particles (see Figure 1) [38,39], followed by detection of the particles in each of the two output channels. The interpretation in terms of matter waves follows from the analogy with optical interferometry. The incoming matter wave is separated into two different paths by the first beam-splitter. The accumulation of phases along the two paths leads to interference at the second beam-splitter, producing complementary probability amplitudes in the two output channels [40,41,42]. The detection probability in each channel is then a sine function of the accumulated phase difference, $\Delta\phi$.

Atomic clocks [43,44,45] can be considered the most advanced application of atom interferometry. In this "interferometer", the two different paths of Figure 1 consist of the free evolution of atoms in different internal states with an energy separation $\hbar\Delta\omega$. An absolute standard of time is then obtained by servo-locking a local oscillator to the output signal of the interferometer, which varies as $\cos(\Delta\omega \times T)$. Atom interferometers can also be used as a probe for gravito-inertial fields. In such applications, the beam-splitters normally consist of pulsed near-resonance light fields which interact with the atoms to create a coherent superposition of two different *external* degrees of freedom, by coherent transfer of momentum from the light field to the atoms [2,38]. Consequently, the two interferometer paths are separated in space, and a change in the gravito-inertial field in either path will result in a modification of the accumulated phase difference. Effects of acceleration and rotation can thus be measured with very high accuracy. To date, ground-based experiments using atomic gravimeters (measuring acceleration) [7,46], gravity gradiometers (measuring acceleration gradients) [11,47] and gyroscopes [8,9] have been realized and proved to be competitive with existing optical [48] or artifact-based devices [49].

The ultimate phase-sensitivity of an atom interferometer is, aside from technical difficulties, limited by the finite number of detected particles N and scales as $\Delta\phi_{\min} = 2\pi/\sqrt{N}$ (quantum projection noise limit [50,51]). Of course, the
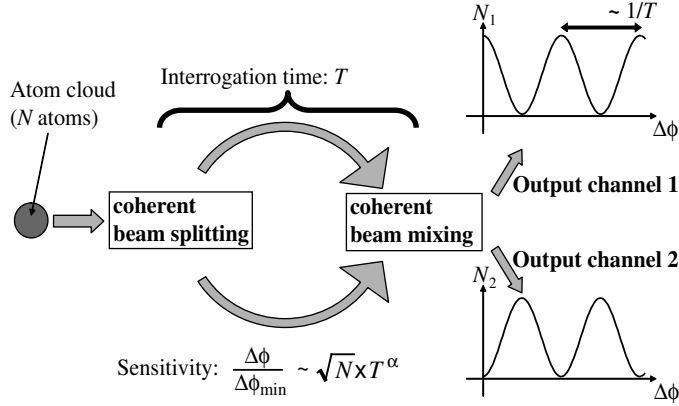
Fig. 1. Principle of an atom-interferometer. An initial atomic wavepacket is split into two parts by the first beam splitter. The wavepackets then propagate freely along the two different paths for an "interrogation time" T, during which the two wavepackets can accumulate different phases. A second pulse is then applied to the wavepackets so that the number of atoms at each output is modulated with respect to this phase difference. The maximum sensitivity achievable for such an apparatus can be defined by comparing the variation of the number of atoms ΔN due to the phase difference $\Delta\phi$ at the output ($\Delta N \sim N\Delta\phi/2\pi \propto NT^\alpha$) with the quantum projection noise arising from atom counting $\sqrt{N}$. It scales as $\sqrt{N} \times T^\alpha$.

relation between the relative phases accumulated along the two different paths and the actual physical property to be measured is a function of to the "interrogation" time T spent by the particles between the two beam-splitters. Thus, the ideal sensitivity of an atom interferometer is expected to scale[1] as $\sqrt{N} \times T^\alpha$ with $\alpha > 0$, and it is obviously of strong interest to increase these two factors.

Nevertheless, in practice, the *absolute accuracy* of an atom-interferometer is limited by uncontrolled, environmental phase shifts in the interferometer, for example, due to stray electromagnetic fields or mechanical vibrations. These residual phase shifts must therefore be controlled and measured to better than the desired accuracy. This is usually best achieved by keeping these shifts as small as possible, using passive isolation and active feedback [52]. Such constraints forbid in general the use of external fields as a means of controlling the position of the atoms during the phase accumulation period. In addition, any *inhomogeneity* in an external potential applied to the atoms would usually result in a loss of coherence, decreasing the sensitivity and dynamics of the atom interferometer. As a consequence, most high-precision atom interferometers require that the atoms are in free-fall between the two beam-splitting processes.

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[1] An atomic clock or an atomic gyrometer, for example, has a sensitivity proportional to T and on-ground gravimeter has a sensitivity proportional to T^2 due to the quadratic nature of free-fall trajectory in a constant gravitational field.
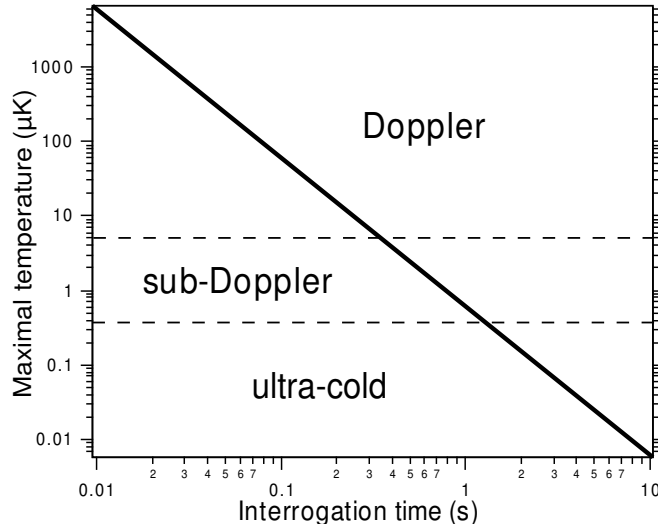
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Fig. 2. Maximum temperature of atom source for a given interrogation time. The maximum interrogation time for a given initial temperature has been calculated for a detection area of $10\,\mathrm{cm}^2$ and defined as the time at which half of the atoms are no longer detected. The dashed lines indicate the limits of Doppler and sub-Doppler cooling. Interrogation times of several seconds are compatible only with clouds of atoms at ultra-cold temperatures, close to the quantum degenerate regime.

Seeking to increase the sensitivity of on-ground atom interferometers by increasing the interrogation time T, one soon reaches a limit imposed by gravity. With the stringent requirements of ultra-high vacuum and a very well controlled environment, current state-of-the-art experimental apparatus does not allow more than a few meters of free-fall, with corresponding interrogation times of the order of $T \sim 200\,\mathrm{ms}$. Space-based applications will enable much longer interrogation times to be used, thereby increasing dramatically the sensitivity and accuracy of atom interferometers [27].

Even in space, atom interferometry with a *classical* atomic source will not outperform the highest-precision ground-based atom interferometers that use samples of cold atoms prepared using standard techniques of Doppler and sub-Doppler laser cooling [53]. Indeed, the temperature of such sub-Doppler laser-cooled atom cloud is typically $\sim 1\,\mu\mathrm{K}$ ($v_{\mathrm{rms}} \sim 1\,\mathrm{cm/s}$). In the absence of gravity, the time evolution of cold samples of atoms will be dominated by the effect of finite temperature: in free-space, a cloud of atoms follows a ballistic expansion until the atoms reach the walls of the apparatus where they are lost. Therefore the maximum interrogation time reasonably available for space-based atom interferometers will strongly depend on the initial temperature of the atomic source. As shown in Figure 2, the 200 ms limit imposed by gravity for a 30 cm free fall is still compatible with typical sub-Doppler temperatures, whereas an interrogation time of several seconds is only accessible by using an "ultra-cold" source of atoms (far below the limit of laser cooling) with a temperature of the order of a few hundred nanoKelvin.
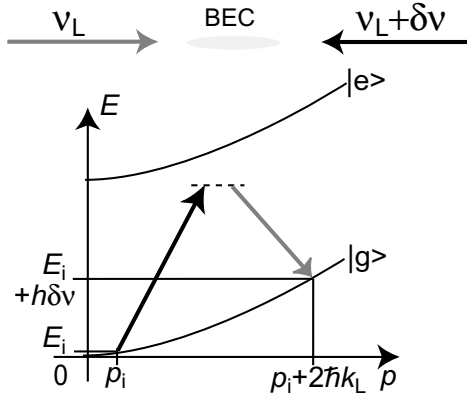
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Fig. 3. Principle of Bragg scattering: a moving standing wave, formed from two counter-propagating laser beams with a small relative detuning $\delta\nu$, can coherently transfer a fraction of the atoms to a state of higher momentum when the resonance condition is fulfilled. A 2-photon Bragg scattering event imparts a momentum $2\hbar k_{\mathrm{L}}$, and an energy of $h\delta\nu$ to the atoms: thus, the first-order (2-photon) Bragg resonance for atoms with zero initial velocity occurs at a detuning of $h\delta\nu = 4\hbar^2 k_{\mathrm{L}}^2/2m$. This resonance condition depends on the initial velocity of the atoms relative to the optical standing wave.

These dense, *ultra-cold* samples of atoms are now routinely produced in laboratories all around the world. Using evaporative cooling techniques [15,16,17], one can cool a cloud of a few 10^6 atoms to temperatures below $100\,\mathrm{nk}$ [54]. At a sufficiently low temperature and high density, a cloud of atoms undergoes a phase transition to quantum degeneracy. For a cloud of bosonic (integer spin) atoms, this is known as Bose-Einstein condensation, in which all the atoms accumulate in the same quantum state (the atom-optical analogy of the laser effect in optics). A BEC exhibits long range correlation [55,56,57] and can therefore be described as a coherent "matter wave": an ideal candidate for the future of atom interferometry in space. The extremely low temperature associated with a BEC results in a very slow ballistic expansion, which in turn leads to interrogation times of the order of several tens of seconds in a space-based atom interferometer. In addition, the use of such a coherent source for atom optics could give rise to novel types of atom interferometry [25,26,58,59].

In our laboratory we have realized a coherent matter-wave interferometer based on Bragg scattering [59]. The principle of Bragg scattering is the following [60,61]: two counter-propagating laser beams of wavevector $\pm\mathbf{k}_{\mathrm{L}}$ and frequencies ν_{L} and $\nu_{\mathrm{L}} + \delta\nu$ form a moving light-grating. The common frequency ν_{L} is chosen to be in the optics domain but far detuned from atomic resonances to avoid spontaneous emission. A two-photon transition, involving absorption of a photon from one beam and stimulated re-emission into the other beam, results in a coherent transfer of momentum $\mathbf{p}_{\mathrm{f}} - \mathbf{p}_{\mathrm{i}} = 2\hbar\mathbf{k}_{\mathrm{L}}$ from the light field to the atoms, where $\mathbf{p}_{\mathrm{i}}$ and $\mathbf{p}_{\mathrm{f}}$ are the initial and final momenta of the atoms. Conservation of energy and momentum leads to the resonance

conditions $E_{\mathrm{f}} = E_{\mathrm{i}} + h\delta\nu$, where (in free space) the initial and final energies of the atoms are given by $E_{\mathrm{i}} = p_{\mathrm{i}}^2/2m$ and $E_{\mathrm{f}} = p_{\mathrm{f}}^2/2m$ respectively. Bragg scattering can be used for different types of matter-wave manipulation, depending on the pulse length τ. Using a short pulse ($\tau < 100\,\mu\mathrm{s}$), the Bragg beams are sufficiently frequency broadened that the Bragg process is insensitive to the momentum distribution within the condensate: the resonance condition is then satisfied simultaneously for the entire condensate. If the Bragg laser power and pulse duration are then selected to correspond to the $\pi/2$ condition, the probability of momentum transfer to the atoms is 50 percent: this is a 50/50 beam splitter for the condensate, between two different momentum states. When using longer pulses (for example $\tau = 2\,\mathrm{ms}$ in [1]), the Bragg process is velocity selective, and one can apply this technique to momentum spectroscopy [57,1].

By carefully re-analyzing the Bragg-spectroscopy data of [1], we have extracted a measurement of the ratio h/m of the Planck constant h to the atomic mass m of ^{87}Rb. The experimental sequence proceeds as follows: a laser-cooled sample of ^{87}Rb atoms is magnetically trapped in the $5\mathrm{S}^{1/2}\,|F = 1, m_F = -1\rangle$ state [62] and then evaporatively cooled to quantum degeneracy. The magnetic trapping fields are switched off and the atoms fall for 25 ms. During this free-fall period, the coherent Bragg-scattering "velocimeter" pulse is applied. In this experiment, the implementation of Bragg scattering is as follows: two orthogonally polarised, co-propagating laser beams of frequencies ν_L and $\nu_L + \delta\nu$ and wave vector $\mathbf{k}_{\mathrm{L}}$ are retro-reflected by a highly stable mirror, with 90° polarisation rotation (see Figure 4). With this scheme, the atoms are subject to two standing waves moving in opposite directions and with orthogonal polarisations. In addition, the relative detuning $\delta\nu$ is chosen so as to fulfill the second-order (4-photon) resonance condition. This four laser Bragg-scattering scheme produces a coherent transfer of momentum of $+4\hbar\mathbf{k}_{\mathrm{L}}$ and $-4\hbar\mathbf{k}_{\mathrm{L}}$. This scheme enables us to reject the effect of a non-zero initial velocity, which can arise from imperfections in the magnetic trap switch-off. For an initial velocity p_{i}/m, the 4-photon resonance conditions for the two oppositely moving standing-waves are $\delta\nu_+ = \delta\nu_0(1 + p_{\mathrm{i}}/2\hbar k_{\mathrm{L}})$ and $\delta\nu_- = \delta\nu_0(1 - p_{\mathrm{i}}/2\hbar k_{\mathrm{L}})$ where $\delta\nu_0$ is the Doppler-free value, $\delta\nu_0 = (8/2\pi)(\hbar k_{\mathrm{L}}^2/2m)$ (see Figure 4). Scanning the Bragg scattering efficiency in the two directions as a function of $\delta\nu$ yields two peaks with widths corresponding to the condensate momentum width, centred at each of the resonance frequencies, $\delta\nu_+$ and $\delta\nu_-$ (Figure 4). After fitting each individual spectrum with a gaussian distribution, we can extract the two center frequencies $\delta\nu_{\pm}$. To correct the data for the non-zero initial velocity, we then re-center both spectra around the average value $\delta\nu_0 = (\delta\nu_+ + \delta\nu_-)/2$.

Using this method and averaging over 350 spectra (Figure 5), we find $\delta\nu_0 = 30.189(4)$ kHz where the figure in parentheses is the 68% confidence interval of the fit. We then deduce a value $h/m \equiv \lambda^2 \times \delta\nu_0/4 = 4.5946(7) \times 10^{-9}\,\mathrm{J.s.kg}^{-1}$ where the wavelength $\lambda = 780.246291(2) \times 10^{-9}$ of the Bragg beams, slightly
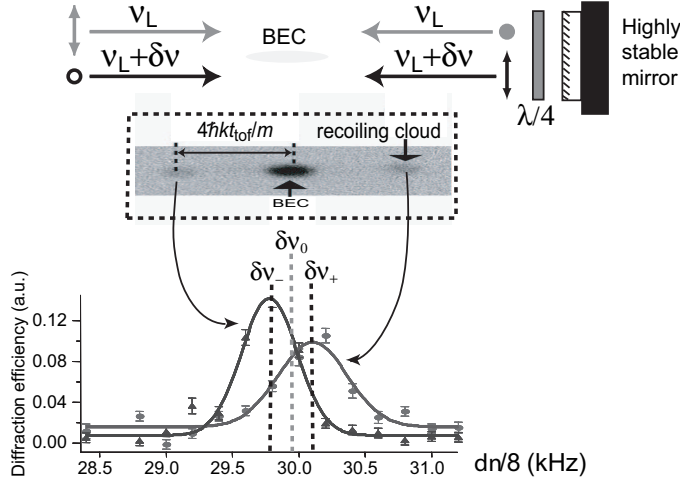
Fig. 4. Principle of our four photon, dual direction Bragg scattering scheme. Top: schematic of the experimental apparatus. Two retro-reflected laser beams form two standing waves of orthogonal polarisations, moving in opposite directions. Middle: normalized number of atoms diffracted into each of the two output channels as a function of Bragg detuning $\delta\nu$. (Inset: typical absorption image after Bragg diffraction and free evolution during a time t_{tof}.) Bottom: schematic of the 4-photon Bragg resonance condition. For zero initial momentum, the resonance condition is fulfilled by both standing waves for a detuning $\delta\nu_0$. For non-zero initial momentum p_i, the resonance frequency is equally and oppositely shifted for each of the two channels.

detuned from the $\left(5^2S_{1/2}, F = 2\right) \rightarrow \left(5^2P_{3/2}, F = 3\right)$ optical transition, is very accurately known from [63,64]. The offset between our measurement and the CODATA value of h/m (4.59136×10^{-9} J.s.kg^{-1}) can be explained by two major systematic effects. First, as described in [1], the frequencies ν_L and $\nu_L + \delta\nu$ of the Bragg scattering beams were obtained by using two independently driven acousto-optical modulators (AOM) of center frequency 80 MHz. The frequency difference $\delta\nu$ was then deduced from the measurement of the frequency of each AOM driver with a high precision frequency meter. The reference oscillator used in the frequency meter was later characterized to have an accuracy of about 4×10^{-7}, giving a ± 16 Hz inaccuracy in the actual frequency difference $\delta\nu$. The resulting systematic error in our measurement then gives $h/m = 4.5946(20)(7) \times 10^{-9}$ J.s.kg^{-1}. The second systematic effect is a collisional shift due to interactions in the high density atomic cloud. In the

following, we will show that this accounts for the remaining offset.

Indeed, ultra-cold ^{87}Rb atoms have repulsive interactions which modify the Bragg-scattering resonance condition. The energy of an atom in the condensate is $E_i = p_i^2/2m + Un(\mathbf{r})$. The second term is the condensate interaction energy: $n(\mathbf{r})$ is the local atomic density of the condensate and $U = 5.147(5) \times 10^{-51}$ J.m^3 is the interaction parameter. Immediately after Bragg scattering into a different momentum state, an atom experiences an effective potential $2Un(\mathbf{r})$ due to the surrounding condensate, and its energy is then $E_f = p_f^2/2m + 2Un(\mathbf{r})$ [57]. We therefore replace the Bragg resonance condition (for zero initial momentum) with a *local* resonance condition which takes into account the effect of interactions:

$$2h\delta\nu_0(\mathbf{r}) = 16\frac{\hbar^2 k_L^2}{2m} + Un(\mathbf{r}) \tag{1}$$

The parabolic density distribution of our Bose-Einstein condensate, at the moment when the Bragg diffraction occurs, is

$$n(x,y,z) = n_0 \cdot \max\left[0\,;\, 1 - (x^2 + y^2)/R_\perp^2 - z^2/R_z^2\right]$$

with peak density $n_0 \simeq 3.6(4) \times 10^{18}$ m^{-3} and half-lengths $R_\perp \simeq 9.8\,\mu$m and $R_z \simeq 126\,\mu$m, where z is the direction of the Bragg-scattering. Since our measurement of the diffraction efficiency averages over the whole cloud, the resulting spectrum is then shifted by $U\langle n\rangle/2h \sim 4Un_0/7$ and broadened. Taking this interaction shift into account, we correct our measured value of h/m:

$$\begin{aligned} h/m &= \lambda^2/4 \cdot \left[\langle\delta\nu_0\rangle - U\langle n\rangle/2h\right] \\ &\simeq 4.5939(21)(7) \times 10^{-9}\text{J.s.kg}^{-1}. \end{aligned} \tag{2}$$

which is in agreement with the CODATA value. Here, the first figure in parentheses is the systematic errors discussed above when we take into account both the frequency calibration inaccuracy and the error on evaluating the atomic density in the collisional shift. The second figure is the 68% confidence interval of the fit determination.

The fact that ultra-cold bosons interact is a major drawback for precision measurements using atom interferometry. As we have seen, interactions result in a systematic shift as well as a decrease in measurement precision. In principle, the systematic shifts can be calculated. However, the interaction parameter U is hard to measure and is generally not known to better than $\sim 10^{-4}$. The atomic density is also subject to time fluctuations and is difficult to know to better than $\sim 10^{-3}$, reducing the absolute accuracy. We have
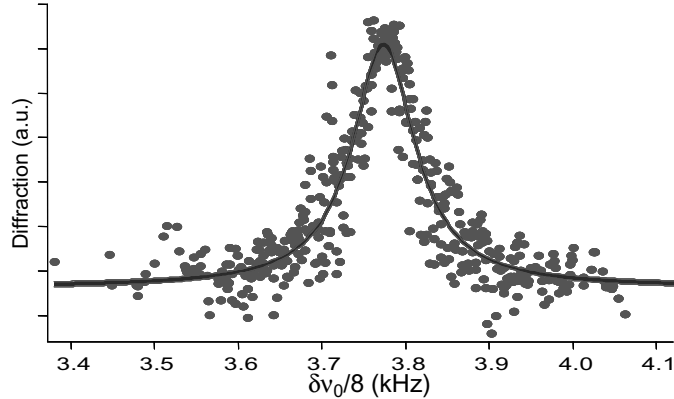
Fig. 5. Final spectrum (corrected for Doppler effect). The fit to this spectrum yields the centre frequency $\delta\nu_0$, from which we obtain the ratio h/m.

furthermore demonstrated, in an earlier experiment [1,65], that interactions produce a loss of coherence of the atomic samples at ultra-low, finite temperatures, limiting the maximum interrogation time of a coherent matter-wave atom interferometer. Finally, even at zero temperature, the mean-field energy due to interactions is converted into kinetic energy during free fall, giving rise to a faster ballistic expansion. This last effect will ultimately reduce interrogation times.

From these observations, we conclude that one should ideally use an interaction-free, ultra-cold atomic source for ultimate-precision atom interferometry in space. Using bosons, one could think of two ways of decreasing interaction effects. Close to a Feshbach resonance [66], one can control the interaction parameter U, which can be made equal to zero for a certain magnetic field [67,68]. However magnetic fields introduce further systematic shifts that are not controllable to within a reasonable accuracy. Alternatively, one could try to decrease the density of the sample of atoms, but the production of large atom number, ultra-low density Bose-Einstein condensate is a technical challenge not yet overcome [69].

A promising alternative solution is to use quantum-degenerate fermionic atomic sources [70]. The Pauli exclusion principle forbids symmetric 2-body collision wavefunctions, so at zero temperature a sample of neutral atomic fermions has no interactions. An ultra-cold fermionic source may still allow very long interrogation times, even if limited by the excess energy of the Fermi pressure, and would therefore be an ideal candidate for atom interferometry in space with ultimate precision and accuracy. On-ground experiments using ultra-cold fermions (Potassium 40) are now under development in our laboratory and around the world.

To conclude, we have shown that coherent atomic sources are very promising for high-precision atom interferometry measurements. Nevertheless, interac-

tions in quantum-degenerate bosonic gases cause phase shifts which are difficult to control and will ultimately limit the measurement accuracy. These shifts could be overcome either by precise control of the interaction properties, or by using fermionic, non-interacting samples of ultra-cold atoms. The use of atom interferometry with ultra-cold sources of atoms in micro-gravity are of great interest for tests of fundamental physics in space. There are potentially many experiments which would benefit from this technology, based on atomic clocks, interferometers and gravito-inertial sensors. In space, applications of these include ultra-precise definition of time, verification of the equivalence principle, measurement of the fine structure constant α and its drift in time, and tests of general relativity and post-newtonian gravitation theories. As an example, the Hyper project [27], which aims to measure the Lense-Thirring effect with orbiting atom-optical gyroscopes, would greatly benefit from using such coherent sources of cold atoms. In addition, sending an interplanetary probe equipped with ultra-cold atom interferometer into space would enable precise mapping of the Pioneer anomaly [71].

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