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# Dynamic Load Balancing for Adaptive Mesh Ocean Circulation Model

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sponding atmospheric models, due to differences in the typical scales of motion. For example, the order of magnitude of the size of dynamic structures like fronts or eddies is a few tenths of kilometers in the ocean, while 5 or 10 times larger in the atmosphere. The horizontal resolution of OGCMs should allow the explicit representation of such structures, which leads to very important memory and CPU requirements (OGCMs typically use today an horizontal resolution of  $1/6^\circ$  to  $1/10^\circ$  – i.e. approximately 10 to 15 kilometers – and 20 to 50 vertical discretization levels, which represent an overall number of several million grid points). Moreover the time scales in the ocean are one order of magnitude larger than in the atmosphere, which implies integration of numerical models on longer time periods (from a few weeks to tenths of years, depending on the application), with a time step of a few minutes (typically 1 to 30, depending on the horizontal resolution and time integration scheme). These models are run of course on vector and/or parallel supercomputers (Cray C90 or T3E, Fujitsu VPP series ...), any simulation requiring several hundred or thousand hours of CPU-time. Note also that numerous ocean models use (or intend to) data assimilation methods, i.e. mathematical methods to mix optimally model forecasts and informations from observing systems, which increase once again the computational cost by one or two orders of magnitude.

## 1.2 Adaptive Meshing

In this context, it is clear that adaptive meshing could be of real interest for ocean modelers. It could reduce the computational cost of models by taking advantage of the spatial heterogeneity of oceanic flows and thus using a fine mesh only where and when necessary. Moreover, this would allow local zooms on limited areas of particular interest, e.g. for operational purposes. The finite element technique permits of course the use of such a non-uniform adaptive grid over the computational domain. However, this approach is mostly reserved, in the field of ocean modeling, to coastal or tidal models, and all major OGCMs use finite difference methods on structured grids. Starting from this consideration, Blayo and Debreu [3] are currently developing a Fortran 90 software package, which will furnish any finite difference ocean model the capability of adaptive meshing and local zooming. The method is based on the adaptive mesh refinement (AMR) algorithm proposed by Berger and Olinger [2], which features a hierarchy of grids at different resolutions, with dynamic interactions. Since the mesh refinement is adaptive over the time step, the number of grids as well as their sizes and resolutions vary during the simulation. Thus, the computational load is also varying in time, and a dynamic load balancing is necessary to implement efficiently this AMR method on a parallel computer.

## 1.3 Organization of the Paper

This paper will discuss the design, implementation and performance of a load balancing package and its integration into a code for large scale simulations of ocean circulation. It is organized as follows: in section 2, we describe some features of ocean models useful for the rest of the paper and present the AMR method. The model of Malleable Tasks with inefficiency factor is presented in section 3. Then, an adaptation of the level-by-level load balancing algorithm is detailed and analyzed in section 4. Some numerical experiments are reported

in section 5, in order to illustrate the behavior of the algorithm on a parallel machine. Finally, some conclusions and perspectives are discussed.

## 2 Ocean Models and Adaptive Mesh Refinement

### 2.1 About Ocean Models

The basic relations describing the ocean circulation are equations for conservation of momentum (Navier-Stokes equations), of mass (continuity equation), of heat and salt, and an equation of state expressing the water density as a function of temperature, salinity and pressure. Two main assumptions are usually considered: the Boussinesq's approximation (variations of density are small) leading to a non-divergent velocity, and the hydrostatic approximation. This leads to a set of equations known in oceanography as "primitive equations" (PE), which are solved by most OGCMs using finite difference techniques. Simpler models of ocean circulation (namely, quasigeostrophic models, shallow-water models, etc.) can also be derived from these equations by making additional assumptions.

Thus, an ocean model can be written in a symbolic way as:  $\frac{\partial X}{\partial t} = F(X)$  where  $t$  is the time,  $X$  is the state variable (including for example velocity, temperature, salinity and pressure for PE models) and  $F$  is a non-linear operator. The time discretization scheme is generally explicit, which leads in most cases to discretized equations of the form  $X(t + \delta t) = G(X(t), X(t - \delta t))$  where  $\delta t$  is the discretization time step. The algebraic operations involved by operator  $G$  are discretized versions of simple operators (gradient, jacobian or laplacian) and require only local calculations (i.e. computations at any grid point  $P$  use only the values at grid points in the immediate vicinity of  $P$ ). However, some models use also an implicit scheme for one of the equations (equation for barotropic – or depth-averaged – motion) to prevent using very small timesteps, which are necessary with explicit schemes due to the constraint of numerical stability in the representation of external gravity waves. This implies to solve at each time step a linear system  $AX(t + \delta t) = H(X(t), X(t - \delta t))$ .

### 2.2 Considerations about Parallelization

The parallelization of ocean models is performed by domain decomposition techniques (e.g. [4] [18]). The geographical domain is divided into subdomains, each of them being affected to a processor. Most of existing works usually consider as many subdomains as processors. The computation of the explicit terms is mainly local; it requires only, at the beginning of each time step, some communications between processors corresponding to adjacent subdomains, to exchange model variables along the common interfaces. On the other hand, linear systems for the implicit terms are not local, since they correspond to the discretized form of elliptic equations. Solving these global systems is performed either by a direct parallelization of classical methods, like SOR or preconditioned conjuguate gradient [17] or by domain decomposition techniques [13] [14]. An important point for the purpose of this work is to emphasize that ocean models are *regular* applications, in the sense that the volume of computations can be estimated quite precisely as a function of the grid size and the number of processors (at least, it can be measured by adequate benchmarks).

### 2.3 Adaptive Mesh Refinement (AMR)

The basic principle of AMR methods consists in locally refining or coarsening the computation mesh, according to some mathematical or physical criteria (like error estimates or eddy activity). Such techniques are widely used with finite element codes, but rather rarely with finite differences because such refinements lead to non-homogeneous grids and thus complicate the handling of the code. However, Berger and Olinger [2] proposed an AMR algorithm which avoids this drawback, by considering a locally multigrid approach. In their method, the refinement is not performed on a unique non-homogeneous grid, but on a hierarchy of grids, i.e. a set of homogeneous embedded grids of increasing resolutions, and interacting among themselves (see the principle on Figure 1). Without going deeply into more details (the reader can refer for example to [2] or [3]), the principle of the algorithm is as follows. Consider a hierarchy of grids, like in Figure 1: it consists in a root (or level-0) grid covering the entire domain of computation with coarse resolution  $\Delta h_0$  and coarse time step  $\Delta t_0$ , and a number of subgrids (or level-1 grids) with a finer resolution  $\Delta h_1 = \Delta h_0/r$  and a finer time step  $\Delta t_1 = \Delta t_0/r$  focused only on some subdomains ( $r$  is an integer called the refinement ratio). This structure is recursive, in the sense that any level- $l$  grid can contain finer level- $(l+1)$  subgrids, with  $\Delta h_{l+1} = \Delta h_l/r$  and  $\Delta t_{l+1} = \Delta t_l/r$  (of course until a maximum level  $l_{\max}$ ).

Time integration is performed recursively starting from the root grid. Any level- $l$  grid is advanced forward one time step  $\Delta t_l$ . The solutions at time  $t$  and  $t + \Delta t_l$  are used to provide initial and boundary conditions for the integration of the level- $(l+1)$  subgrids. These subgrids can then be advanced forward  $r$  time steps  $\Delta t_{l+1}$ , to provide a more precise solution at time  $t + r\Delta t_{l+1} = t + \Delta t_l$  on the regions covered by the subgrids. These solutions at level  $(l+1)$  are then used to improve the solution at level  $l$ , via an update procedure. This process is summarized in the following recursive algorithm, called by `INTEGRATE(0)`:

```
Procedure INTEGRATE( l )
  If l == 0 Then nbstep = 1
  Else nbstep = r
  Endif
  Repeat nbstep times
    Step on all grids at level l
    If level l+1 exists Then
      Compute boundary conditions at level l+1
      INTEGRATE( l+1)
      update level l
    Endif
  End Repeat
End Procedure INTEGRATE
```

The relevance of the grid hierarchy is checked regularly every  $N$  coarse time steps  $\Delta t_0$ . A criterion is evaluated at every grid point to determine whether the local accuracy of the solution seems sufficient or not. Subgrids can then be created, resized or removed.

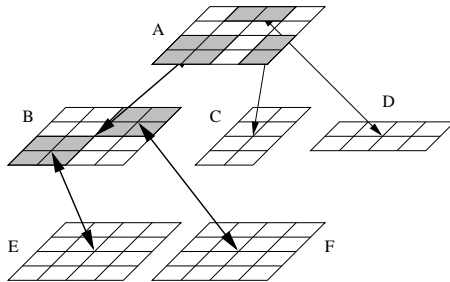


Figure 1: Example of grid hierarchy

A major interest of this AMR method is that it can be used without modifying the model. The model can be seen like a black-box (corresponding to the **Step** routine in the previous algorithm) needing only configuration parameters (domain of computation, mesh size  $\Delta h$ , time step  $\Delta t$ , initial conditions, boundary conditions) to be run. This is one of the key ideas of the package that we are presently developing.

In the parallel version of our package, the model will still be used as a black-box like in the sequential case, but with an additional parameter: the number of processors used to run the model. Since the different grids at a given level  $l$  can be run simultaneously, they will be affected to different groups of processors. The problem is then to determine, for a given hierarchy, which is the best grids-to-processors correspondence for computational efficiency. These assignments must be determined at every regridding step, because of the evolution of the grid hierarchy during the simulation. The model of Malleable Tasks presented in the next section is an efficient tool for solving such problems.

### 3 Parallel Implementation

#### 3.1 Malleable Tasks

Since the eighties, many works have been developed for parallelizing actual large scale applications like the oceanographic simulation considered in this paper. The solution of the scheduling problem (in its larger acception) is a central question for designing high-performance parallelization. It corresponds to finding a date and a processor location for the execution of each task of the parallel program. Among the various possible approaches, the most commonly used is to consider the tasks of the program at the finest level of granularity and apply some adequate clustering heuristics for reducing the relative communication overhead [9]. The main drawback of such an approach is that communications are taken into account explicitly (they are expressed assuming a model of the underlying architecture of the system which is very difficult to establish and often far from the actual asynchronous and non-deterministic behavior of the parallel execution). It is well-known that the introduction of explicit communications into the scheduling algorithms renders the problem harder than without communication [5]. Recently, a new computational model has been proposed [6]. *Malleable tasks* (denoted MT in short) are computational units which may

be themselves executed in parallel. It allows to take into account implicitly the influence of communications.

### 3.2 Motivation for Using MT

There are several reasons for introducing MT as a way to design efficient parallel applications. Let us summarize them below:

- MT allow to hide the complex behavior of the execution of a subset of tasks by using a parameter (the inefficiency factor  $\mu$  discussed later) to abstract the overhead due to the management of the parallelism (communications, idle times due to internal precedence constraints, etc.).
- MT reflect the structure of some actual parallel applications because they unify the usual single processor tasks and tasks which may require more than one processor for their execution. The structural hierarchy is then included into the model and the user does not specify when to use one model or another.
- MT simplify the expression of the problems in the sense that the user does not have to consider explicitly the communications. Thus, the same code may be used on several parallel platforms (only the  $\mu$  factors will change), leading to a better portability.

The idea behind this model is to introduce a parameter which will implicitly give the communication overhead. An average behavior has to be determined using the *inefficiency* factor that will be now discussed in more details.

### 3.3 Inefficiency Factor: Definition and Properties

The efficiency of the execution of a parallel application depends of many factors like the algorithm, the data size, the data partitioning between available processing units, communication volume, network latency, topology, throughput, etc.. Ideally, a parallel system with  $m$  processors could complete the execution of an application  $m$  times faster than a single processor. However, an actual  $m$  processors system does not achieve such speedup due to some overhead introduced by inter-processor communication (which is slow compared to basic processor computation) and synchronization between the tasks. The ratio between achieved speedup and theoretical speedup depends on the number  $m$  of processors and the size  $N$  of the application. It will be denoted by  $\mu(m, N)$  and called *inefficiency factor*. We give below the definition using a geometric interpretation.

**Definition 1** *The inefficiency factor is the expansion of the computational area while using  $m$  processors:  $t_m = \mu(m, N) \frac{t_1}{m}$  where  $t_1$  (resp.  $t_m$ ) is the time required to complete a malleable task of size  $N$  on one processor (resp. on  $m$  processors).*

Generally, the inefficiency factor increases with the number of processors and decreases with the size of the parallel task (at least until a certain threshold). The general shape of  $\mu$  function may be divided into three zones of consecutive

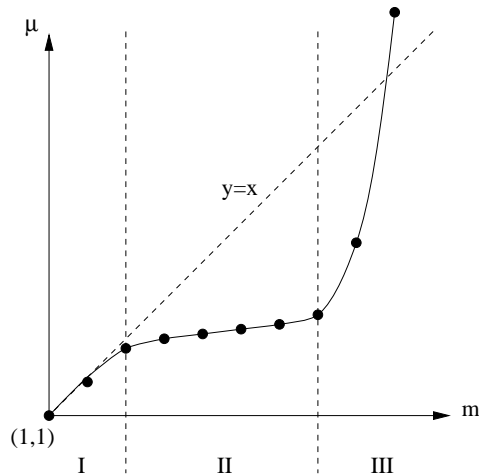


Figure 2: General qualitative shape of  $\mu$  function may be divided into three zones: (I) corresponds to the start-up overhead due to the management of the parallelism. (II) is the region where the addition of extra processors cost almost nothing, speed-ups here are generally linear. (III) corresponds to the degradation due to the lack of parallelism (for instance, for too small malleable tasks).

intervals of number of processors: (I) corresponds to the start-up overhead due to the management of the parallelism, where the efficiency may be quite bad. (II) is the region where the addition of extra processors cost almost nothing, speed-ups here are generally linear. (III) corresponds to a degradation due to the lack of parallelism (for instance, for too small MT). Some properties of  $\mu$  are presented below. They will be useful for bounding the time in the load-balancing heuristics.

**Property 1.**  $\mu(q, \cdot)$  is an increasing function of  $q$

**Proof 1** Adding a processor generally involves an extra cost for managing communications and synchronizations between this processor and the others. This comes directly from the Brent's Lemma [12].  $\square$

**Property 2.** Given a specific  $m$  processor system and a particular application,  $\mu(1, N) = 1$  and  $\mu(q+1, N) \frac{t_1}{q+1} < \mu(q, N) \frac{t_1}{q} \forall q, 1 \leq q \leq m-1$ .

**Proof 2** These relations are usual hypotheses in parallel processing. The first part comes from the sequential execution of a task. For the second relation, just remark that it is useless to solve the application with one more processor if  $t_{q+1}$  is greater than  $t_q$ .  $\square$

A direct corollary of this last property ensures that  $\frac{\mu(q, \cdot)}{q}$  is a decreasing function of  $q$ .



## 4 Load-Balancing Issues

### 4.1 Definition and Context

A parallel application is usually described as a set of *tasks* (which are computational units) plus their interactions which are represented as a graph, called the *precedence task graph* [12]. For the specific oceanographic application we are considering in this paper, the tasks correspond to solve the equations on a mesh at a given time step. The parameters of a task are the size of the mesh ( $n_1$  and  $n_2$  plus the height along the vertical axis, at each time step  $k$ ). This graph is not known in advance, it evolves in time and a mesh refinement leads to introduce a new series of tasks in the graph. In this sense, the problem is dynamic. Figure 3 represents the precedence graph corresponding to the calculation on the mesh and the corresponding data-flow dependency graph which includes also the data communications between tasks.

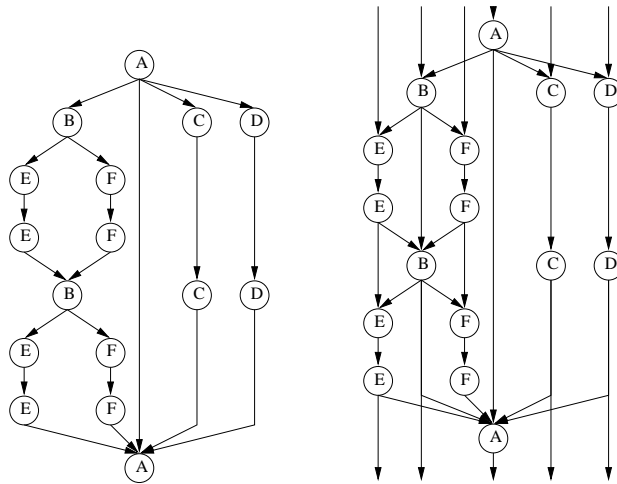


Figure 3: Precedence task graph and Data Flow graph.

According to most studies in parallel processing, we are interested in minimizing the total execution time. It is influenced by two contradictory criteria, namely the idle time (or load-balance) which decreases as the size of the grain decreases and the communication overhead which grows with the number of processors. The load-balancing problem corresponds formally to determine an application which associates to each (malleable) task a sub-set of the processors. The resulting scheduling problem (defined as determining the date of which each task will start its execution, synchronously on the sub-set of processors) should add some constraints; namely, a task is allocated only once and should not be preempted. One contribution of this work is to introduce the influence of the inefficiency factor into load-balancing policies.

The well-known *Gang scheduling* policy for the graph of figure 3 corresponds to allocate the total resources of the parallel system for executing each task [10]. Every task, one after the other, is executed by all processors. Gang will be used as a basis for comparison.

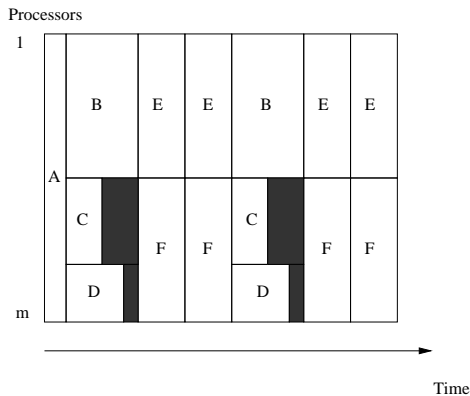


Figure 4: Level-by-level mapping

## 4.2 Level-by-level mapping

The well-known level-by-level load-balancing policy can also be adapted to MT with inefficiency. A complete presentation of the basic heuristic can be found in [11]. It was proposed for a classical model of tasks with no communication delays. The basic idea of the level-by-level mapping is to distribute the tasks level by level in the precedence task graph. The level  $i$  is defined as the sub-set of independent tasks (not related by the precedence constraints) at distance  $i$  from the root of the corresponding not-weighted task graph. In other words, this policy may be interpreted as a series of successive synchronized Macro-Gang schedulings with sets of independent malleable tasks. Figure 4 details the execution of the previous example with the level-by-level mapping.

For the mapping of each level many strategies may be applied:

- simply round a fraction of the number of processor in fonction of the grid size (one shelve)
- apply one of the theoretical result on malleable task, like the polynomial heuristic of guarantee 2 [6], a  $\sqrt{3}$  heuristic, pseudo-polynomial in the number of processor [7] or even a polynomial fully approximable scheme (with a very large constant factor) [8].

## 5 Experiments

### 5.1 The Ocean Model

The ocean model used for this study is a simple quasi-geostrophic box model, since our intention is to validate the MT approach and to design a good scheduling heuristic before running the simulation on the operational models. This type of model has been widely used in the ocean modeling community, and is known as a simple prototype of eddy-active large scale circulation in the mid-latitudes. Blayo and Debreu [3] already implemented the AMR method in such a multi-layered model. They demonstrated that the use of this method results in a very significant gain in CPU time (by a factor of 3) while conserving, within a

10 to 20% range, the main statistical features of the solution obtained with an uniformly high resolution. Moreover, it appears that one only simulation with the AMR method leads to better local predictions than classical nested grid techniques on regions of particular interest, wherever the region of interest is located, and for a comparable amount of computation.

In the present study, we use a barotropic (i.e. one layer) version of this model (see for instance [16] for a detailed description of such a model). Its governing equation can be written as:

$$\frac{\partial \Delta \psi}{\partial t} + J(\psi, \Delta \psi) + \beta \frac{\partial \psi}{\partial x} = \text{curl} \tau - r \Delta \psi + A \Delta^2 \psi \quad (1)$$

where  $\psi$  is the streamfunction,  $J$  is the Jacobian operator,  $\beta$  is the meridional gradient of the Coriolis parameter at the mid-latitude of the domain,  $\tau$  is the horizontal wind stress at the surface of the ocean,  $r$  is a bottom friction coefficient, and  $A$  is a lateral viscosity coefficient.

The discretization is performed on an uniform horizontal grid. The discrete laplacian is a standard five-point approximation, and the discrete jacobian is written following the classical Arakawa's stencil [1]. The time discretization of the vorticity equation (1) is explicit. A resolution of Poisson's equation  $\xi = \Delta \psi$  is performed at each time step to compute the streamfunction  $\psi$ , using a FACR (Fourier Analysis - Cyclic Reduction) method [15]. The figure 5 presents the evolution of the values of the  $\psi$  function on a 200x200 square domain with a static wind stream on the middle. The coefficients are the same order of magnitude as those of the North Atlantic area.

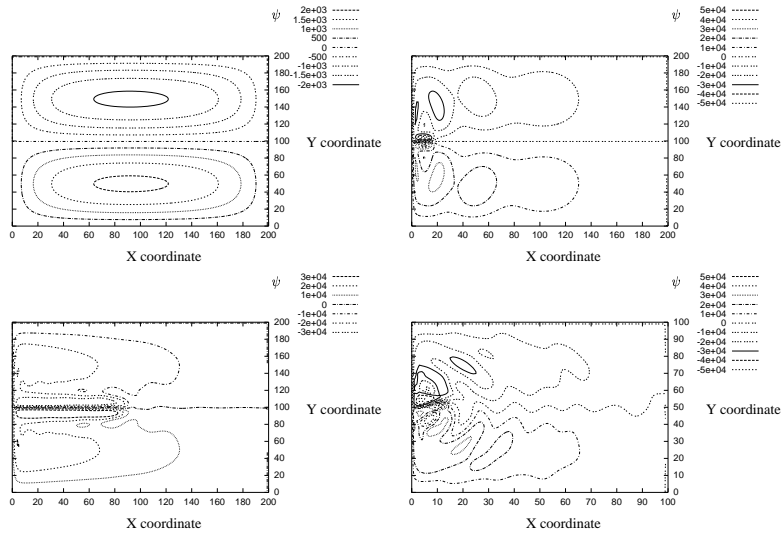


Figure 5: Evolution of the isovalues of the streamfunction  $\psi$  during initialization steps (10, 1000, 10000 and 50000 steps)

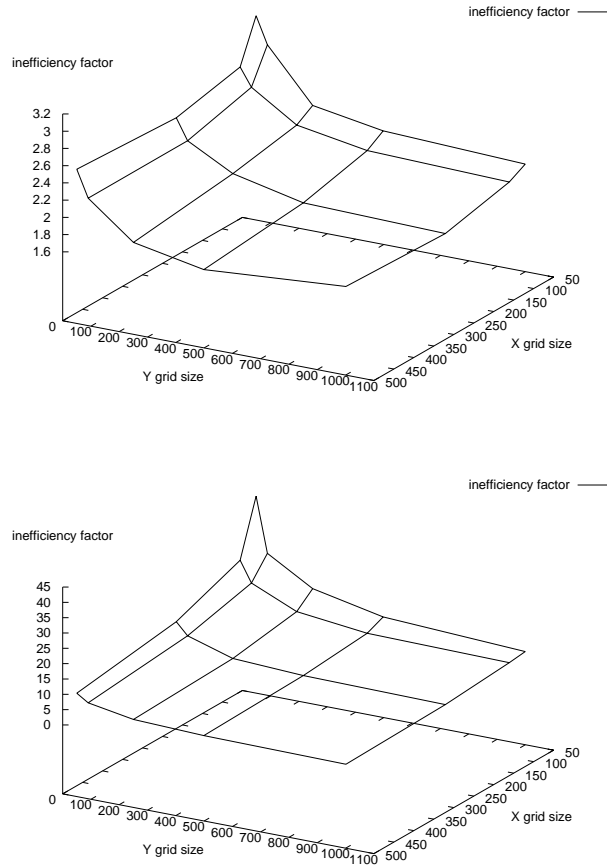


Figure 6: Inefficiency factor as a function of the number of gridpoints in  $x$ - and  $y$ -directions. Upper panel: 4 processors; lower panel: 15 processors

## 5.2 Evaluation of Inefficiency

The inefficiency factors  $\mu$  correspond to the penalty coefficients of the parallel time due to the overhead coming from the management of the parallelism. The parallel code was benchmarked in order to determine empirically an expression of the inefficiency factors for including them into the load-balancing policies. The  $\mu$  is measured using single grid parallel computation. The variation of these curves in regard to the number of processors confirms the same global behaviour. Two aspects were studied:

- The variation of  $\mu$  versus the grid size (cf Fig. 6) for a given number of proc. Note that for the different values of the number of processors, the general shape of the curves remains the same. The greater the number of processors, the greater the number of anomalies.
- Figure 7 depicts the typical behaviour of the  $\mu$  function versus the number

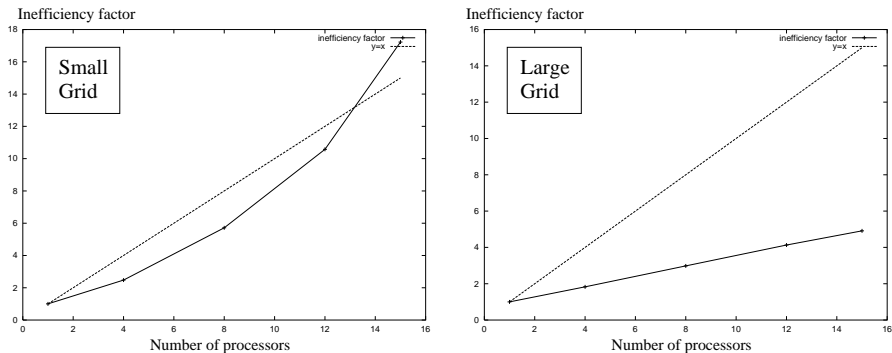


Figure 7: Inefficiency factor as a function of the number of processors. First panel: with anomaly (50x50 size); Left panel: without anomaly (500x500 grid size). The reference curve  $y = x$  is also plotted.

of processor for small and large grid. This behaviour shows clearly some limitations of speedup due to inefficiency for small grids (speed down) and a linear speedup of larger problems.

### 5.3 Experimental Results

We developed numerical experiments in order to validate our studies on an IBM-SP2 parallel machine with 16 nodes connected on a fast switch.

Since the oceanographic model considered here is rather simple, the present study should be interpreted as a preliminary study for comparing the load-balancing policies and demonstrating the interest of using MT model for actual applications.

The tests reported in Fig. 8 and 9 compare the execution time and the work of three load-balancing policies: gang, level-by-level with and without inefficiency factor for multiple runs of a test case.

Formally, the *work* is the makespan, times the number of involved processors. Note that the number of involved processors may be different to the number of available processors due to scheduling decision.

The scheduling decision of the level-by-level algorithm with inefficiency factor are different in Fig. 8 and Fig. 9. In Fig. 8, it schedules tasks on the available processors with the smallest rank at each level, minimizing the number of processors used. On the contrary, in Fig. 9 it alternatively chooses the available processors with largest or smallest rank, spreading both computation and memory usage.

On a small number of processors the differences among the policies algorithms are small. In fact, as the load imbalance produces large overhead, the gang scheduling performs better than level-by-level algorithms in some cases.

On a greater number of processor, the gang scheduling induces a large communication overhead and then is worse than the other strategies.

In Fig. 8, both level-by-level algorithms (with and without inefficiency factor) seem quite close in appearance on first panel, namely in achieved makespan.

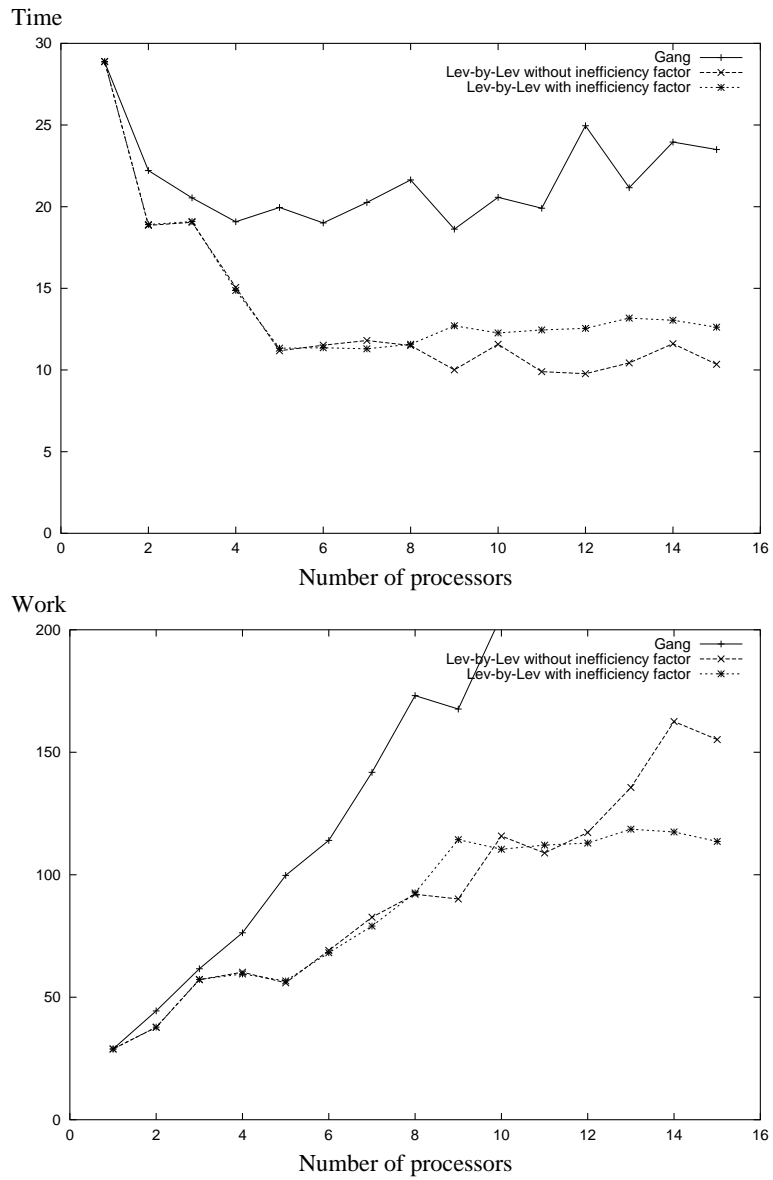


Figure 8: Time and Work achieved by Gang, Level-by-Level with and without inefficiency factor scheduling algorithms versus the number of processors on a small size adaptative simulation.

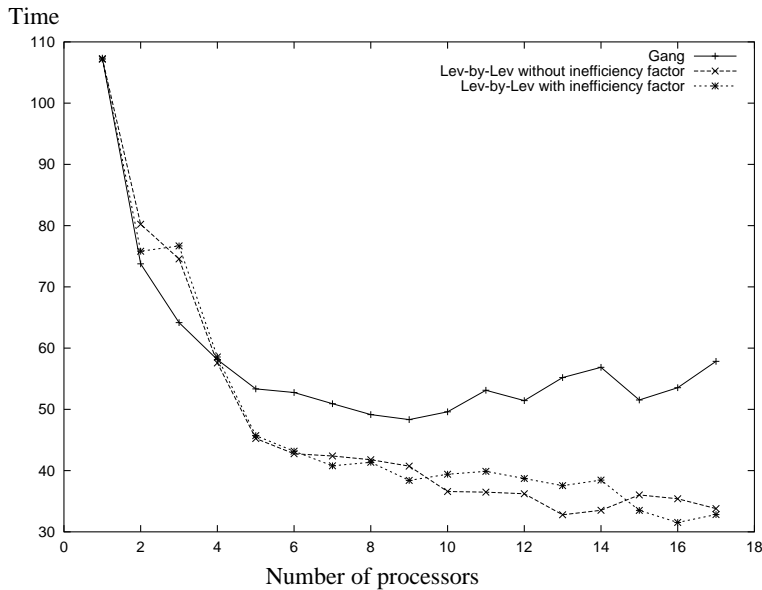


Figure 9: Time achieved by Gang, Level-by-Level with and without inefficiency factor scheduling algorithms versus the number of processors on a medium size adaptative simulation.

The level-by-level algorithm with inefficiency factor even achieves slightly worse makespan than the algorithm without inefficiency factor.

However the level-by-level algorithm with inefficiency factor never used more than 9 processors while it achieves almost the same time.

Fig. 8 shows clearly that the amount of computation can be bounded by the inefficiency factor policies without large performance penalty.

In Figure 9, the level-by-level algorithm with inefficiency factor achieved comparable speedup to level-by-level without inefficiency factor but never using more than 13 processors per level.

The good behavior of the level-by-level scheduling algorithm with inefficiency factor allows us to expect good results for future works using a larger number of processors. It shows clearly that this algorithm can manage efficiently simulation and avoid useless resource requirement. It also shows the usefulness of mapping algorithms scheduling more than one level at a time, like proportional mapping.

Note, since the adaptive simulation uses the oceanographic model as a black box, it is intrinsically synchronous; communications between MT only occur before and after computation. Thus the makespan is quite sensitive to the synchronization overhead among MT. This is especially true for the Gang scheduling and partially explains its poor efficiency. Gang scheduling is also highly sensitive to perturbations occurring in multi-user environment.

## 6 Conclusion and Future Works

We have presented in this work a new way to parallelize actual applications. It is based on the model of Malleable Tasks with inefficiency factor. We have

discussed some fundamental properties of MT and showed how they are suitable for designing load-balancing heuristics. Some adaptations of well-known heuristics have been done, and preliminary experiments on a test problem coming from oceanography was carried out to show the feasibility and potential of MT model.

Other short term perspectives are to develop other load-balancing heuristics, and implement them within operational models of ocean circulation.

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## References

- [1] Arakawa A., 1966: Computational design for long term integration of the equations of fluid motions. *J. Comp. Phys.*, **1**, 119-143.
- [2] Berger M. and J. Olinger, 1984: Adaptive mesh refinement for hyperbolic partial differential equations. *J. Comp. Phys.*, **53**, 484-512.
- [3] Blayo E. and L. Debreu, 1998: Adaptive mesh refinement for finite difference ocean models: first experiments. *J. Phys. Oceanogr.*, **29**, 1239-1250.
- [4] Bleck R., S. Dean, M. O'Keefe and A. Sawdey, 1995: A comparison of data-parallel and message passing versions of the Miami Isopycnic Coordinate Ocean Model (MICOM). *Paral. Comp.*, **21**, 1695-1720.
- [5] Hoogeveen J., Lenstra J.-K., and Veltman B., 1994: Three, four, five, six, or the complexity of scheduling with communication delays. *Operations Research Letters*, **16**, 129-137
- [6] Turek J., Wolf J. and Yu, P., 1992: Approximate algorithms for scheduling parallelizable tasks. *Fourth Annual ACM Symposium on Parallel Algorithms and Architectures*, 323-332
- [7] Mounié, G., Rapine, C., and Trystram, D., 1999: Approximate algorithms for scheduling parallelizable tasks. *Eleventh ACM Symposium on Parallel Algorithms and Architectures (SPAA'99)*, 23-32
- [8] Jansen, K. and Porkolab, L., 1999: Linear time approximation schemes for scheduling problems. *Tenth annual ACM-SIAM Symposium on Discrete Algorithms (SODA'99)*, 490-498
- [9] Gerasoulis A. and Yang T., 1994: DSC : Scheduling Parallel Tasks on an Unbounded Number of Processors. *IEEE Transaction on Parallel and Distributed Systems*, **5**, 951-967
- [10] Scherson I.D., Subramanian S.D., Reis V. L. M. and Campos L. M., 1996: Scheduling computationally intensive data parallel programs. *Placement dynamique et répartition de charge : application aux systèmes parallèles et répartis (École Française de Parallélisme, Réseaux et Système)*, Inria, 107-129



- [11] Kumar V., Grama A., Gupta A. and Karypis G., 1994: Introduction to Parallel Computing: Design and Analysis of Algorithms, *Benjamin/Cummings*
- [12] Cosnard M. and Trystram D., 1993: Algorithmes et architectures parallèles. InterEditions, collection IIA.
- [13] Debreu L. and E. Blayo, 1998: On the Schwarz alternating method for oceanic models on parallel computers. *J. Comp. Phys.*, **141**, 93-111.
- [14] Guyon M., M. Chartier, F.-X. Roux and P. Fraunie, 1995: First considerations about modelling the ocean general circulation on MIMD machines by domain decomposition method. Proceedings of the NATO advanced research workshop on *high performance computing in the geosciences*, les Houches, France, 21-25 June 1993, F.-X. Le Dimet Ed., NATO ASI Series C462, Kluwer Academic Publishers.
- [15] Hockney R.W., 1965: A fast direct solution of Poisson's equation using Fourier analysis. *J. ACM*, **12**, 95-113.
- [16] Pedlosky J., 1987: Geophysical fluid dynamics. Springer-Verlag, 710.
- [17] Smith R.D., J.K. Dukowicz and R.C. Malone, 1992: Parallel ocean general circulation modeling. *Physica D*, **60**, 38-61.
- [18] Wallcraft A.J. and D.R. Moore, 1997: The NRL layered ocean model. *Paral. Comp.*, **23**, 2227-2242.