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# Permeability anisotropy induced by the shear displacement of rough fracture walls.

H. Auradou,<sup>1</sup> G. Drazer,<sup>2</sup> J.P. Hulin,<sup>1</sup> and J. Koplik<sup>2</sup>

**Abstract.** The permeability anisotropy that results from a shear displacement  $\vec{u}$  between the complementary self-affine walls of a rough fracture is investigated. Experiments in which a dyed fluid radially injected into a transparent fracture displaces a transparent one are presented. A clear anisotropy is observed in the presence of shear displacements and allows us to estimate the ratio of the permeabilities for flows parallel and perpendicular to  $\vec{u}$ . A simple model which accounts for the development of channels perpendicular to  $\vec{u}$  qualitatively explains these results, and predicts a permeability decreasing (increasing) linearly with the variance of the aperture field for flow parallel (perpendicular) to the shear displacement. These predictions are then compared to the results of numerical simulations performed using a lattice-Boltzmann technique and to the anisotropies measured in displacement experiments.

## 1. Introduction

Subsurface fluid flow in low-permeability geological formations occurs primarily through networks of fractures [NAS, 1996; Adler and Thovert, 1999; Sahimi, 1995]. In order to model such systems, one needs to understand flow in single fractures which are the building blocks of the network. Among the many parameters which influence fluid motion at this scale, we focus here on the geometry of the fractures, specifically on the correlations between the roughness of the surfaces of the two opposite walls. A shear displacement of the latter alters these correlations and previous laboratory measurements demonstrated that it also affects the permeability, through variations of the local aperture of the fracture due to the wall roughness [Plouraboué *et al.*, 2000; Olsson and Brown, 1993].

In related experiments, a higher permeability was measured for flow perpendicular to the shear displacement than parallel to it [Gentier *et al.*, 1997; Yeo *et al.*, 1998]. Numerical studies found a similar behavior on explicitly anisotropic rough surfaces generated numerically [Thompson and Brown, 1991]; an indication of such anisotropic permeability was also observed in a previous work on self-affine fractures [Drazer and Koplik, 2002]. The purpose of the present paper is to combine experiments, numerical simulations, and analytic arguments to understand the permeability anisotropies that are induced by shear displacements of rough fracture walls.

A key geometrical feature of the fracture surfaces studied in this work is their statistical scale invariance known as *self-affinity* [Mandelbrot, 1983; Feder, 1988]. The analysis of the surfaces of both fault and fresh fractures has established that their roughness cannot be described by a finite set of typical wavelength values, but instead is self-affine. A self-affine surface is one that is *statistically* unchanged under the

scaling relation:  $x \rightarrow \lambda x$ ,  $y \rightarrow \lambda y$  and  $h(x, y) \rightarrow \lambda^\zeta h(x, y)$ . ( $(x, y)$  are coordinates in the mean surface plane,  $h(x, y)$  is the local height of the surface, and  $\zeta$  is known as the Hurst exponent.) The self-affine character of the fracture surfaces has been observed, over a significant range of length scales, in a broad variety of materials and for both natural and man-made fractures [Bouchaud, 2003]. In the case of fractured rocks, such as granite or basalt rocks, the Hurst exponent is found to be  $\zeta \approx 0.8 \pm 0.05$  over a broad range of length scales [Bouchaud, 2003], although values near 0.5 are found in other rocks [Boffa, 1998].

The key consequence of self-affinity in our context is the following scaling law for the variance of the surface heights:

$$\langle [h(\vec{x} + \vec{\lambda}) - h(\vec{x})]^2 \rangle \sim |\vec{\lambda}|^{2\zeta}, \quad (1)$$

which implies long-range correlations in the height-to-height fluctuations. It will be seen that these correlations strongly affect the spatial aperture distribution of the rough fractures studied in the present work, and therefore, the flow inside them.

In both experiments and numerical studies, we shall consider model fractures, made of two complementary rough self-affine surfaces: two perfectly matching surfaces separated by a distance  $a_0$  in the direction normal to their common mean plane and, in general, subject to a shear displacement  $\vec{u}$  parallel to this mean plane. In all cases, the gap between surfaces is large enough so that the two walls do not touch, thereby avoiding the effects of shear-induced dilation of the fracture gap on its permeability.

The experiments were performed using transparent models fabricated with moldings of actual rocks. In the experiments, a dye is injected radially from a point of the fracture and the anisotropy in the permeability is computed from that of the injection patterns. Only the ratio of the highest and lowest permeabilities can be determined in this way, but not their absolute values. These experiments are therefore appropriately complemented by numerical simulations which, in addition to the permeability, provide the complete flow field. The lattice Boltzmann technique has been used since it is convenient for determining the velocity field in these complex geometries [Drazer and Koplik, 2002; Kim *et al.*, 2003].

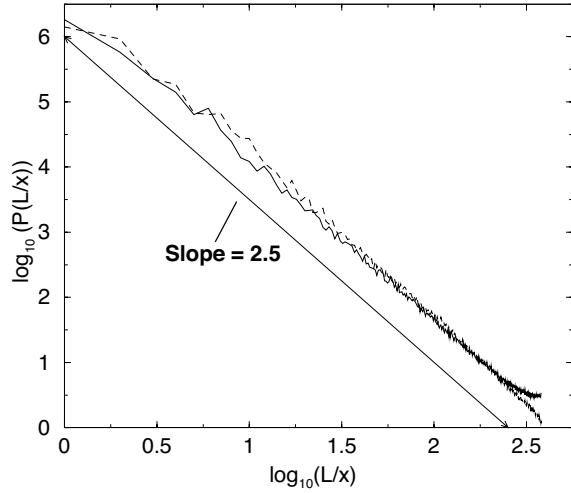
The paper is organized as follows: in Sec. 2.1, the laboratory procedure used to obtain the transparent cast of the fractured rock is described as well as the characterization of the surface roughness. The experimental set-up and the

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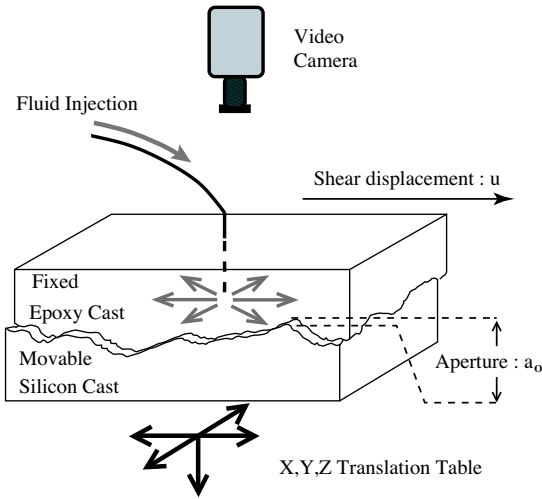
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image analysis methods are described in Sec. 2.2. In order to explain the experimental results described in Sec. 3, a theoretical analysis of the relation between the aperture field and the observed anisotropy in the permeability is developed in Sec. 4. The scaling laws obtained in this way are then compared to numerical simulations in Secs. 5 and 6. The same approach is finally applied to the experimental data in Sec. 7.



**Figure 1.** Power spectral density of the experimental fracture surfaces. The spatial frequencies are  $f = L/(n\Delta x)$ , for  $n = 1, 2, \dots, L/\Delta x$ , where  $L = 20\text{cm}$  is the fracture length and  $\Delta x = 250\mu\text{m}$  is the resolution length along the profile. The solid and dashed lines correspond to profiles obtained normal and parallel to the direction of crack propagation. The straight line has a slope -2.5.



**Figure 2.** Schematic view of the experimental set-up. The top wall is a transparent epoxy cast and has a fixed position. The bottom wall is a silicone replica of the top wall and can be moved down and laterally. The dyed fluid is injected in a radial, divergent configuration and it displaces a non-dyed fluid which initially saturates the fracture gap. The flow process is visualized with a camera set above the fracture. The two controlled lengths are indicated:  $a_0$ , the mean vertical separation between walls, and  $\vec{u}$ , the shear-displacement vector.

## 2. Experimental set-up and procedure

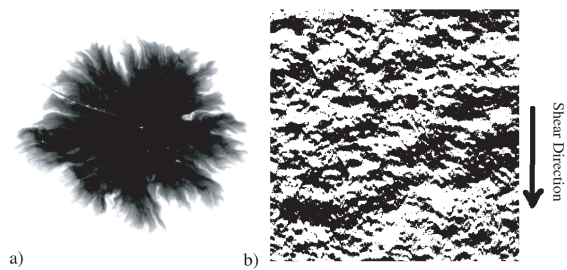
### 2.1. Model fracture and flow set-up

The experiments were performed on transparent moldings of fractured granite blocks extracted from a quarry in Lanhelin (Britanny). A tensile crack was produced by a compressive force applied on two opposite sharp edges carved in the middle of two facing sides (the initial block size is  $25 \times 25 \times 40 \text{ cm}^3$ ). Then, a silicon rubber molding of one of the fracture surfaces is made, and after that used as a template to fabricate transparent epoxy casts of the original surface. The epoxy cast is in turn used as a template to produce a second silicon rubber molding with a perfectly matching surface (dimensional deformations during polymerization are much smaller for the silicon than for the epoxy). This method is widely used and reproduction problems are negligible for the present application [Auradou *et al.*, 2001; Yeo *et al.*, 1998; Isakov *et al.*, 2001; Kostakis *et al.*, 2003]. The complementary epoxy and rubber surfaces are the walls of the model fracture used in the present work.

Prior to the experiments, 2D maps of the epoxy surfaces are obtained by means of a mechanical profilometer ( $770 \times 760$ ) points with a  $250\mu\text{m}$  grid spacing and a  $10\mu\text{m}$  vertical resolution [Boffa, 1998]. The difference in height between the highest and the lowest points of the surface is  $\simeq 17 \text{ mm}$  and the root mean square deviation of the surface heights is  $\simeq 3.5 \text{ mm}$ . As discussed in the introduction, *self-affine* surfaces have a roughness which does not contain any intrinsic wavelength, and it can be shown that the power spectral density  $P(f)$  of the heights, for a linear profile drawn on the surface, should verify the following scaling law with the spatial frequency  $f$  :

$$P(f) \sim A \times f^{-1-2\zeta}. \quad (2)$$

Such spectral densities  $P(f)$  obtained for the surface samples are plotted in Fig. 1, and follow indeed a power law as a function of  $f$ . The computed spectral densities showed no dependence, within experimental error, on the orientation of the profile used to compute them, which implies that the fractures are isotropic in their mean plane. By averaging spectra obtained for several orientations of the profiles, one obtains  $\zeta = 0.75 \pm 0.05$ , in agreement with previous studies of granite fractures [Boffa, 1998; Méheust and Schmittbuhl, 2000; Auradou *et al.*, 2001]. As indicated by the solid line with arrows at both ends in Figure 1,  $P(f)$  follows this power law over approximately two decades : the corresponding length scales range from  $50 \text{ mm}$  down



**Figure 3.** Example of the relation between flow anisotropy and aperture field in a sheared fracture, with  $a_0 = 1.25 \text{ mm}$  and  $u_y = 2.0 \text{ mm}$  (the direction of the shear displacement is indicated by the arrow). The normalized aperture variance is  $S^2 = 0.133$ . (a) Concentration map. (b) Aperture field computed using Eq. 4, where white corresponds to a local aperture  $a(x, y) < a_0$  and black to  $a(x, y) > a_0$ .

to 0.75 mm. The lower limit  $f_m$  of this range is set by the resolution used in the present map; previous measurements have shown that the self-affine description remains valid at lower length scales, of the order of 0.1mm or less [Boffa, 1998]. The upper limit  $f_M$  arises from the reduced number of wavelengths which describe the surface topography at large scales:  $f_M$  is thus related to the finite size of the sample, which has a large-scale topography dominated by a few large undulations. This statistical effect usually becomes apparent at length scales greater than one fourth of the system size [Méheust and Schmittbuhl, 2000].

## 2.2. Experimental procedure and image analysis

The objective of the experiments is to analyse the flow inside an open fracture by monitoring the radial spreading of an injected dye. Using a radial injection geometry allows to estimate variations of the permeability of the fracture with the flow direction from asymmetries in the shape of the invaded zone. The two complementary surfaces are initially brought into full contact, with their mean plane  $(x, y)$  horizontal. The top (transparent) surface is kept fixed thereafter (see Fig. 2 for details) and the lower one can be translated while remaining precisely parallel to its initial orientation. Specifically, it is first displaced downwards ( $-z$  direction) by a distance  $a_0$  to open the fracture and then laterally in the  $(x, y)$  plane to the required location. This lateral displacement  $\vec{u}$  is small enough so that the two surfaces do not come in contact.

The fracture model is then placed inside a leak tight basin which is filled with transparent fluid in order to saturate the model prior to the experiments. Transparent fluid is then injected through a hole in the top surface and flows radially outwards. Once a steady flow is reached, a dyed fluid is injected and pictures of the invaded region are taken at constant time intervals using a high resolution cooled 12 bits digital camera. Each pixel corresponds to a  $250\mu m \times 250\mu m$  area. The fluids used are water-glycerol solutions with 10% glycerol. 0.2-0.3 g/l of nigrosine are added to the dyed solution and the density of the two fluids is matched by adding an equivalent amount of NaCl to the transparent solution. In order to avoid density differences due to temperature variations the solutions are stored at the same temperature. In all experiments density differences between the two fluids were always smaller than  $25 \cdot 10^{-4} \text{ g/cm}^3$ .

The dye concentration map is determined for each image obtained during the experiment by assuming the validity of Beer-Lambert's law. In this case, the light intensity  $I(x, y)$  at a given point  $(x, y)$  of the image should vary exponentially both with the local aperture  $a(x, y)$  and the local concentration  $c(x, y)$  with:

$$I(x, y) = I_0(x, y) \exp(-2 \mu c(x, y) a(x, y)), \quad (3)$$

in which  $I_0(x, y)$  is the light intensity for transparent fluid and  $\mu$  is the extinction coefficient. The validity of this relation has been verified by opening the fracture without introducing a lateral shift so that the local aperture  $a(x, y)$  is constant and equal to the vertical displacement  $\Delta z$ . The corresponding light intensity  $I(x, y)$  was then determined for different dye concentrations  $c_0$  and different mean apertures  $a_0 = \Delta z$ . All the corresponding results collapse into a single curve by plotting the ratio  $I(x, y)/I_0(x, y)$  as a function of  $\Delta z \times c_0$  and, in a log-linear scale, the variation is linear, as expected, for  $a(x, y)c_0 < 0.15 \text{ g/m}^2$  ( $\mu = 2.9 \pm 0.4 \text{ m}^2/\text{g}$ ). Using this result, and Eq. 3, the local, instantaneous concentration  $c(x, y, t)$  during the displacement experiments can be determined from the following relation:

$$\frac{c(x, y, t)}{c_f} = \ln \left[ \frac{I(x, y, t)}{I_0(x, y)} \right] / \ln \left[ \frac{I_0(x, y)}{I_f(x, y)} \right]. \quad (4)$$

Figure 3a displays a typical concentration map obtained in this way for an experiment in which the fracture walls are translated relative to each other parallel to the arrow. In addition to the irregularity of the front boundary, already discussed by Auradou *et al.* [2001] and by Drazer *et al.* [2004], its global shape is clearly anisotropic with a strong elongation in the direction perpendicular to the shear displacement  $\vec{u}$ . A similar anisotropy is observed on the aperture field displayed in Figure 3b, with the appearance of elongated channels, also perpendicular to  $\vec{u}$ .

## 2.3. Quantitative characterization of anisotropy and relation to permeability field

The anisotropy of the invaded zone can be characterized quantitatively from the inertia tensor  $\mathbf{M}(t)$  of the concentration distributions.  $\mathbf{M}(t)$  is computed using the definition :

$$\mathbf{M}(t) = \frac{1}{c_f} \sum c(x, y, t) \begin{pmatrix} y_*^2 & -x_* \times y_* \\ -x_* \times y_* & x_*^2 \end{pmatrix}, \quad (5)$$

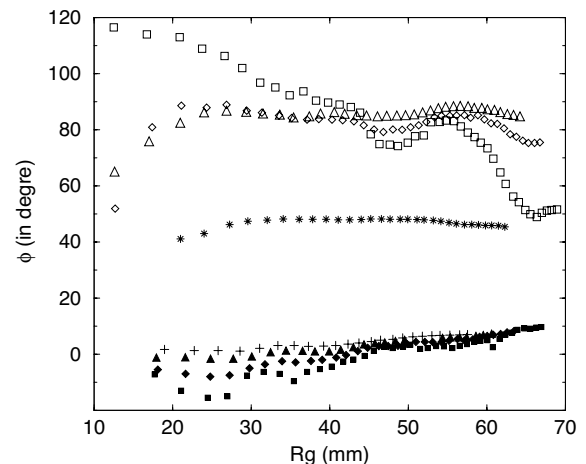
The sum is taken over all pixels in the image and  $x_* = x - x_c$  and  $y_* = y - y_c$  where  $(x_c, y_c)$  is the center of mass of the invasion pattern. The eigenvalues  $\lambda_+^2$  and  $\lambda_-^2$  of the tensor  $\mathbf{M}(t)$  and the corresponding principal axes are then determined: they characterize the magnitude of the anisotropy and the directions of fastest and slowest flow, respectively.

More quantitatively, the tensor  $M(t)$  may be used to estimate the ratio between the permeabilities for flow along these two directions. If we ignore the surface roughness and consider a two dimensional Hele-Shaw flow with weak anisotropy in the permeability,  $k_x \neq k_y$  but  $|k_x - k_y| \ll (k_x + k_y)$ , we can indeed compute the average elliptical shape of a convected tracer front as follows. Assume liquid is injected through a small hole of radius  $r_0$  at pressure  $p_0$ , and allowed to escape at the circular outer boundary  $r = r_1$  at atmospheric pressure  $p_1 \equiv 0$ . The velocities are

$$v_x = k_x \frac{\partial p}{\partial x} \quad \text{and} \quad v_y = k_y \frac{\partial p}{\partial y}, \quad (6)$$

which together with the incompressibility condition  $\nabla \cdot \vec{v} = 0$  gives

$$k_x \frac{\partial^2 p}{\partial x^2} + k_y \frac{\partial^2 p}{\partial y^2} = 0. \quad (7)$$



**Figure 4.** Angle  $\phi$  between the  $x$  axis and the direction of maximum flow velocity for different shear displacements.  $\vec{u}$  parallel to  $x$  : 0.25 mm ( $\square$ ), 0.5 mm ( $\diamond$ ), 0.75 mm ( $\triangle$ );  $\vec{u}$  parallel to  $y$  : 0.25 mm ( $\blacksquare$ ), 0.5 mm ( $\blacklozenge$ ), 0.75 mm ( $\blacktriangle$ ), 1 mm ( $+$ );  $\vec{u}$  at  $-45^\circ$  from  $x$ ,  $u = 1$  mm ( $*$ ).

To leading order in the permeability contrast, the pressure is readily found to be

$$p(r, \theta) = \frac{p_0}{\ln(r_0/r_1)} \left[ \ln(r/r_1) + \frac{k_x - k_y}{2(k_x + k_y)} \times \left( \frac{r^2 - r_0^2 - r_1^2 + r_0^2 \cos(2\theta)}{r_0^2 + r_1^2} \right) \right] \quad (8)$$

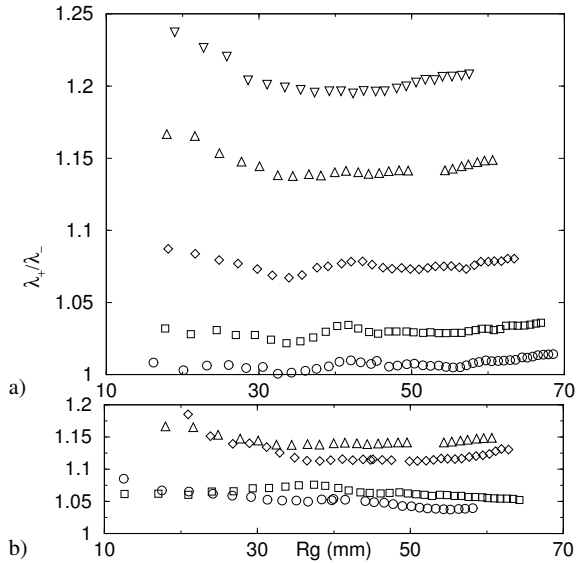
Thus, still to leading order in the permeability contrast, one has  $v_x \sim k_x x/r^2$ , and a similar relation for  $v_y$ . The location of the right-most end of the tracer ellipse,  $(x_R, 0)$ , has then a velocity  $dx_R/dt \sim k_x/x_R$  so that  $x_R^2(t) \sim k_x t$ ; similarly, the coordinate of the top end of the tracer ellipse verifies  $y_T^2(t) \sim k_y t$ . The ratio of the squares of the major and minor axes of the tracer ellipse, and therefore of the eigenvalues  $\lambda_+^2$  and  $\lambda_-^2$ , is equal to the ratio of the permeabilities for flow parallel to these axis:

$$\left( \frac{\lambda_+}{\lambda_-} \right)^2 = \frac{k_\perp}{k_\parallel} \quad (9)$$

### 3. Analysis of the experimental results

In this section we apply the procedures described above to determine the permeability anisotropy from the shape of the invasion pattern during the radial injection of a dyed fluid that displaces a transparent one. Several tracer injection experiments were performed for a fixed mean aperture  $a_0 = 1 \text{ mm}$ , and different shear-displacements in both  $x$  and  $y$  directions. Reference test experiments with no shear displacement between the fracture walls were also performed to check that aperture variations are statistically isotropic (in this case, a roughly circular shape of the invaded zone is expected for a radial injection).

The direction of the maximum flow velocity is associated to the principal axis of the inertia tensor  $\mathbf{M}(t)$  with the largest eigenvalue. Figure 4 displays the variation of the



**Figure 5.** Experimental variation of the ratio  $\lambda_+/\lambda_-$  as function of the radius of gyration  $R_g$  of the invaded zone for a shear displacement  $\vec{u}$  oriented along  $+y$  and with the following amplitudes :  $u = 0$  ( $\circ$ ),  $0.25 \text{ mm}$  ( $\square$ ),  $0.5 \text{ mm}$  ( $\diamond$ ),  $0.75 \text{ mm}$  ( $\triangle$ ),  $1 \text{ mm}$  ( $\nabla$ ). (b) Variation of  $\lambda_+/\lambda_-$  with  $R_g$  for a constant magnitude of the shear displacement  $\vec{u}$  ( $u = 0.75 \text{ mm}$ ) and oriented in the following directions:  $+x$  ( $\circ$ ),  $-x$  ( $\square$ ),  $+y$  ( $\triangle$ ),  $-y$  ( $\diamond$ ).

angle  $\phi$  of this axis with  $x$  as a function of the radius of gyration  $R_g$  of the invasion pattern.  $R_g$  verifies  $R_g^2 = \text{tr}(\mathbf{M})$  and increases approximately as the square root of time as required by mass conservation.

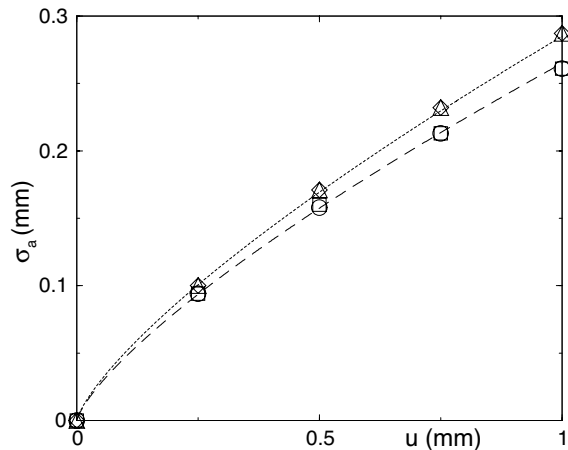
The data in Fig 4 correspond to different magnitudes and orientations of the shear displacement  $\vec{u}$ . In the case of non-zero shear displacements oriented parallel to the  $y$  axis (dark symbols), the angle  $\phi$  remains close to zero, with an average over all data equal to  $2.5^\circ$  and variations of the order of  $\pm 5^\circ$  around that value. In addition, the curves corresponding to larger amplitudes of the shear displacement ( $|\vec{u}| = u_y$  varying from  $0.5$  to  $1 \text{ mm}$ ) are all very similar. The elongation of the invaded zone is therefore, as expected, globally perpendicular to the lateral shift.

Analogous results are obtained for displacements parallel to the  $x$  axis (open symbols) and at an angle of  $-45^\circ$  from it ( $*$  symbols). In these cases,  $\phi$  is of the order of  $90^\circ$  and  $45^\circ$ , respectively, indicating again that the invaded zone is elongated perpendicularly to the displacement. This demonstrates that the flow anisotropy is directly connected to the lateral shift, with the preferential direction of flow oriented at an angle of  $90^\circ$  with respect to the direction of the shear displacement: this anisotropy does not arise therefore from any intrinsic feature in the fracture surface itself.

These results are in agreement with qualitative observations of the aperture fields, such as that presented in Fig. 3: the shear displacement creates ridges in the aperture field oriented in the perpendicular direction and partly inhibiting the flow. These conclusions are of course only valid if the shear displacement is different from zero. In the absence of such shear-displacement we observe, in some cases, a large dispersion in the measured values of  $\phi$ , as expected for a nearly circular invaded zone for which  $\phi$  is undetermined. In other experiments,  $\phi$  takes a better defined value, possibly due to a residual lateral shift.

While  $\phi$  indicates the orientation of the anisotropy, its magnitude is characterized by the ratio of the eigenvalues of the inertia tensor,  $\lambda_+/\lambda_-$ . The variation of  $\lambda_+/\lambda_-$  with the radius of gyration  $R_g$  of the invaded zone is displayed in Figure 5 :  $\lambda_+/\lambda_-$  reaches a roughly constant value for  $R_g > 30 \text{ mm}$ , suggesting that the growth of the invaded zone is self-similar. This asymptotic value of  $\lambda_+/\lambda_-$  represents then a robust parameter to characterize the anisotropy of the permeability for a given orientation and amplitude of the shear displacement.

Figure 5b also demonstrates that, for a fixed amplitude of the shear displacement, its orientation may influence  $\lambda_+/\lambda_-$ .



**Figure 6.** Variance of the aperture field,  $\sigma_a$ , as a function of the magnitude of the shear displacement,  $u$ , in directions :  $+x$  ( $\square$ ),  $-x$  ( $\circ$ ),  $+y$  ( $\triangle$ ),  $-y$  ( $\diamond$ ). Dotted line  $\sigma_a = 0.285 u^{0.75}$  and dashed line :  $\sigma_a = 0.265 u^{0.75}$ .

On the one hand, when the displacement is applied along a given orientation the ratio  $\lambda_+/\lambda_-$  takes similar values in the positive and negative direction. On the other hand,  $\lambda_+/\lambda_-$  depends on the orientation: while it is of the order of 1.05 for a shear displacement  $\vec{u}$  oriented along the  $x$  axis, it reaches 1.15 when  $\vec{u}$  is in the  $y$  direction. These fluctuations come from the statistical character of the distribution of the ridges that alter the flow: they vary not only from sample to sample, but also with the orientation of  $\vec{u}$ . On the other hand, for a given orientation, the variation of  $\lambda_+/\lambda_-$  with the magnitude of the shear displacement  $|\vec{u}| = u$  is smooth, suggesting that the location of the main ridges remains the same as  $u$  increases. These results on the ratio  $\lambda_+/\lambda_-$  may be transposed to the permeability ratio by means of equation 9, which allows then for comparisons with the numerical simulations discussed below.

#### 4. Theoretical discussion of the relation between permeability anisotropy and aperture field

The self-affine nature of the fracture walls affects the void distribution between them, and also plays an important role in fluid flow due to the hydrodynamic boundary conditions imposed there. Here we consider two complementary fracture surfaces with their mean planes parallel and horizontal, which can be separated vertically in order to open the fracture and shifted horizontally to introduce a shear displacement. For a vertical separation  $a_0$  and shear displacements  $u_x$  and  $u_y$  along the  $x$  and  $y$  directions, respectively, the local aperture is defined as the vertical distance separating the two walls and is given by :

$$a(x, y) = h(x, y) - h(x - u_x, y - u_y) + a_0. \quad (10)$$

The covariance of the aperture was obtained previously (using Eq. 1), and two distinct scaling regimes were found [Plouraboué *et al.*, 1995]. For distances smaller than the shear displacement, the aperture field was shown to have self-affine correlations with exponent  $\zeta$ , while for larger distances, the covariance decreases with the distance. The magnitude of the shear displacement thus represents a crossover length scale, and can be regarded as the typical size of features in the aperture distribution. Considering the self-affine scale invariance, the aperture variance,  $\sigma_a^2 = \langle (a(x, y) - a_0)^2 \rangle$ , for a shear displacement of size  $u$  is simply given by  $\sigma_a(u) = Cu^\zeta$ , where  $C$  is related to the amplitude of the roughness of the fracture wall. Figure 6 demonstrates this power law behavior for different orientations of the shear displacement directions. For all the chosen orientations,  $\sigma_a$  varies with  $u$  with an exponent  $\zeta = 0.75$ , in agreement with the Hurst exponent measured in Sec. 2.1 ( $\zeta = 0.75 \pm 0.05$ ).

In the following, we use the ratio  $S \equiv \sigma_a(u)/a$  as a parameter to quantify the normalized fluctuations in the local aperture introduced by the shear displacement: when  $S \ll 1$ , the aperture irregularities are a small perturbation on a nearly-constant aperture field (note that, in the absence of shear displacement between the surfaces, the aperture is constant and equal to  $a_0$  throughout the fracture) and their effect on the flow is negligible, but, as  $S$  approaches 1, the channel becomes quite convoluted and local aperture variations must be considered in order to correctly describe the flow properties.

A second important earlier result is that the aperture correlation length is anisotropic, being smaller in the shift direction than in the orthogonal direction by a numerical factor of  $2\zeta - 1 < 1$  [Roux *et al.*, 1998]. The effect of such an anisotropy on the flow in rough fractures was studied numerically by Thompson and Brown [1991], who demonstrated that the permeability along the direction of the largest correlation length is enhanced while flow parallel to the direction

of the smallest correlation length is inhibited. The effect was related to the occurrence of large ridges oriented along the largest correlation length direction, i.e. in the direction perpendicular to the shear displacement : in fact, such large structures are observed in our system, as can be seen in Figure 3.

This description can be used to motivate a model for the permeability anisotropy which considers the parallel ridges to provide parallel flow channels which are in series with respect to the direction of the shift [Zimmerman and Bodvarsson, 1996]. Flow parallel to the shift is then analogous to electrical current flow through resistors in series and the equivalent hydraulic aperture is  $\langle a^{-3} \rangle^{-1}$ , whereas flow perpendicular to the shift is analogous to resistors in parallel where the equivalent aperture is  $\langle a^3 \rangle$ . Specifically, considering that the distribution of apertures is Gaussian [Plouraboué *et al.*, 1995], one can evaluate the two averages involved, yielding the upper and lower bounds

$$\frac{\langle a^3 \rangle}{a_0^3} = 1 + 3S^2, \quad (11)$$

and

$$\frac{\langle a^{-3} \rangle^{-1}}{a_0^3} = 1 - 6S^2 - 9S^4 + O[S^6], \quad (12)$$

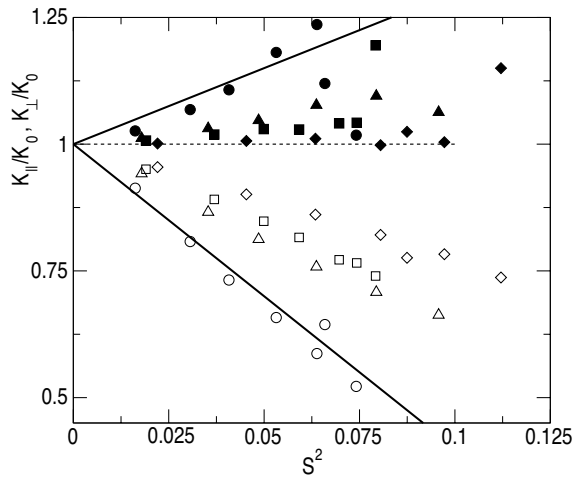
respectively, in the limit  $S \ll 1$ .

Before applying this prediction to the experimental data, we shall first evaluate them on the results of simulations performed on numerical models of self-affine fractures. These models allow for an easier testing of the variability of the results from a fracture sample to another. They also allow to determine separately the permeabilities for flow parallel and perpendicular to the displacement vector  $\vec{u}$ , while the experiments only allow to determine the ratio of these two permeabilities.

#### 5. Numerical procedure

A first issue is the choice of the flow computation technique. When the local aperture of the fracture varies slowly along the fracture plane, the lubrication, or Reynolds, approximation can be used to describe the flow in the fracture [Méheust and Schmittbuhl, 2000]. On the other hand, many studies have pointed out that, for natural fractures, this approximation fails to give an accurate description of the hydraulic properties [Yeo *et al.*, 1998; Nicholl *et al.*, 1999; Dijk and Berkowitz, 1999; Konzuk and Kueper, 2004]. In fact, when the roughness of the walls becomes non-negligible compared to the mean aperture of the fracture, e.g. when the surfaces are close to contact, the lubrication approximation fails, and a complete description of the flow field, which accounts for the flow velocity normal to the surface, is needed [Mourzenko *et al.*, 1995; Drazer and Koplik, 2000]. The lattice Boltzmann technique has therefore been selected to compute the flow because of its good adaptation to 3D complex geometries.

First, a self-affine surface is generated numerically, using a Fourier synthesis method [Drazer and Koplik, 2002], and the fracture pore space is modeled as the region between one such surface and a suitably translated replica. In order to mimic the experimental procedure, the two complementary surfaces are first assumed to be separated by a fixed distance  $a_0$  in the direction  $z$  normal to the mean plane of the fracture. Then, a shear displacement of the upper surface is introduced in the  $(x, y)$  plane. The maximum shear displacement investigated in each direction corresponds to the occurrence of the first contact point between the two surfaces and, therefore, the fracture remains open over its full



**Figure 7.** Normalized permeabilities as a function of the normalized aperture variance  $S^2$  from numerical simulations with variable shear displacements ( $0 < u < 1.6 \text{ mm}$ ) and a constant mean aperture ( $a_0 = 1.0 \text{ mm}$ ) for two fracture surface realizations. Open (closed) symbols : permeability  $k_{\parallel}/k_0$  ( $k_{\perp}/k_0$ ) parallel (perpendicular) to the shear displacement  $\vec{u}$  - Realization 1 : ( $\blacksquare, \square$ ) :  $u_x \neq 0$ ,  $u_y = 0$ , ( $\bullet, \circ$ ) :  $u_x = 0$ ,  $u_y \neq 0$ . Realization 2 : ( $\blacklozenge, \diamond$ ) :  $u_x \neq 0$ ,  $u_y = 0$ , ( $\blacktriangle, \triangle$ ) :  $u_x = 0$ ,  $u_y \neq 0$ . Upper (lower) solid lines = upper (lower) permeability bounds from Eq. 11 (Eq. 12).

area. In order to compare the numerical results to the experiments, both the Hurst exponent and the average magnitude of the surface roughness need to be matched to their experimental values: the average fluctuations in surface height over a distance of 1 mm was thus set to 0.25 mm, and the Hurst exponent  $\zeta$  was set to 0.8. Finally, the mean aperture was set to  $a \simeq 1 \text{ mm}$ , which is comparable to that used in the experiments. Using these values, the local height varies considerably over distances of the order of the aperture size so that computing the complete flow field inside the fractures is required.

In order to capture the three-dimensional character of the flow field inside the fractures, large cubic lattices, with as many as  $1024 \times 1024 \times 20$  sites, are used in the simulations. Hence, the lattice spacing  $\delta$  corresponds to 0.05 mm (the mean aperture  $a_0 = 1 \text{ mm}$  corresponds to 20 lattice sites). The Lattice Boltzmann method is then used to compute the flow field in the presence of a pressure gradient and using periodic boundary conditions in the plane of the surface (see *Drazer and Koplík* [2000] for details of the simulation method). The pressure gradient is applied both along and perpendicular to the shear displacement between surfaces. In addition, and to avoid the possible effects of any intrinsic anisotropy in the generation of the self-affine surfaces, the shear displacements are introduced in both the  $x$  and  $y$  directions within the mean plane. The Reynolds number in the simulations is  $\text{Re} = Ua/\nu \sim a^3 \rho \nabla P / 12\nu^2 < 1$  for  $a = 1 \text{ mm}$ , where we used the parallel plane approximation to estimate  $U$ . Inertia effects are therefore negligible [*Skjetne et al.*, 1999], and the flow is governed by the linear Stokes equations, which are invariant under velocity rescaling. Then, by computing the flow rate, we can obtain the permeability of the fracture and its dependence on the magnitude and orientation of the shear displacement.

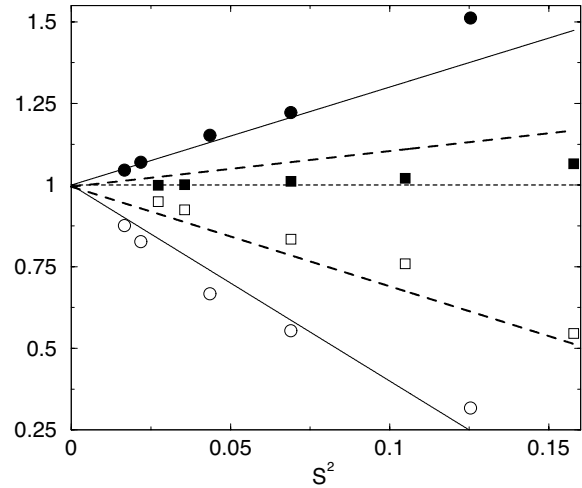
## 6. Analysis of the numerical simulations

Numerical simulations are performed in configurations reproducing as far as possible those encountered in the experiments. Several series of computations were performed on self-affine fracture models for different mean apertures ( $0.5 < a_0 < 2.0 \text{ mm}$ ) and shear-displacements  $\vec{u}$  ( $0 < u < 1.6 \text{ mm}$ ). The displacement  $\vec{u}$  is oriented along directions  $x$  or  $y$  in the mean plane of the surfaces and the size of the simulated fractures is typically  $25.6 \times 25.6 \text{ mm}$ . For given values of  $a_0$  and  $\vec{u}$ , computations are performed on 10 independent realizations in order to estimate the dispersion of the results (only a part of the corresponding data will be displayed for clarity but all data are consistent with the trends visible on the figures).

Figure 7 displays the variation of the normalized permeabilities  $k_{\parallel}/k_0$  (resp.  $k_{\perp}/k_0$ ) as a function of the normalized aperture variance  $S^2$  for a constant mean aperture  $a_0 = 1.0 \text{ mm}$  and for variable shear displacements either along  $x$  or along  $y$ .  $k_0$  is the isotropic permeability for  $u = 0$  and  $k_{\parallel}$  ( $k_{\perp}$ ) is the permeability for flow parallel (normal) to the shear displacement. The normalized variance  $S^2$  is used as the horizontal scale since, as already discussed in Sec. 4, it is the relevant parameter to characterize the dependence of the permeability on the shear-displacement (the magnitude  $u$  of the shear displacement ranges between 0 and 1.6 mm).

The key feature is the fact that  $k_{\perp}/k_0$  and  $k_{\parallel}/k_0$  both vary linearly with  $S^2$  but in opposite directions ( $k_{\perp}/k_0$  increases with  $S^2$  while  $k_{\parallel}/k_0$  decreases). Figure 7 also shows that this linear variation is observed for shear displacements  $\vec{u}$  oriented both parallel to  $x$  and  $y$  axes, which demonstrates that the permeability anisotropy is induced by the shear displacement and it is not associated to geometrical anisotropies present before the relative displacement of the walls.

These linear variations of the permeability qualitatively agree with the simple model presented in Sec. 4. In fact, in Sec. 4 we estimated the upper bounds for the variations of  $k_{\perp}/k_0$  and  $k_{\parallel}/k_0$  with  $S^2$  (see Eqs. 11 and 12), which are represented by solid lines in Fig. 7. As can be seen, the sign



**Figure 8.** Normalized permeabilities as a function of the normalized aperture variance  $S^2$  from numerical simulations with a fixed shear displacement ( $u_x = 1 \text{ mm}$ ) and a variable aperture ( $0.75 < a_0 < 2.0 \text{ mm}$ ) for two fracture surface realizations. Open (closed) symbols : permeability  $k_{\parallel}/k_0$  ( $k_{\perp}/k_0$ ) parallel (perpendicular) to the shear displacement  $\vec{u}$  - Realization 1 : ( $\blacksquare, \square$ ) . Realization 2 : ( $\bullet, \circ$ ). Upper (lower) solid lines = upper (lower) permeability bounds from Eq. 11 (Eq. 12). Dashed lines : linear fit of variation of  $k_{\perp}/k_0$  and  $k_{\parallel}/k_0$  with  $S^2$  over all realizations.

and order of magnitude of these variations are the same as those of the numerical simulations.

Another interesting result is the strong correlation between the variations of  $k_{\parallel}/k_0$  and those of  $k_{\perp}/k_0$ : fractures displaying a large reduction of the permeability  $k_{\parallel}$  for a given orientation and direction of the displacement  $\vec{u}$  generally also display a significant increase of  $k_{\perp}$  for the same displacement  $\vec{u}$ . This is for instance the case for the two fracture surface realizations displayed in Figure 7.

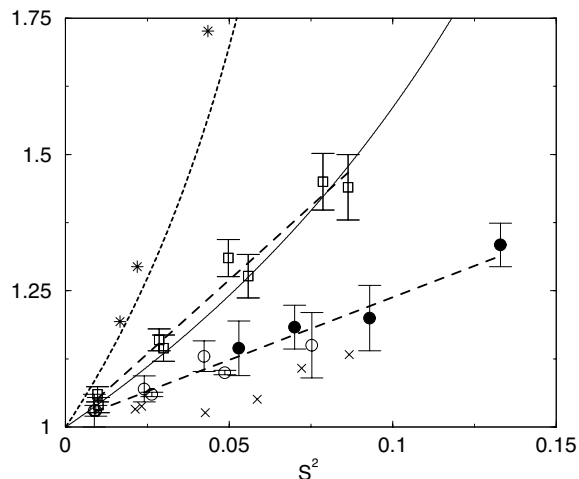
Finally, other simulations were performed in which  $S^2$  is modified by varying the mean aperture from  $a_0 = 2.0$  mm to  $a_0 = 0.75$  mm, while keeping the shear displacement constant,  $u = 1.5$  mm. In this case, the variance  $S^2$  increases when the aperture  $a_0$  decreases. The variations of  $k_{\parallel}/k_0$  and  $k_{\perp}/k_0$  as a function of  $S^2$  for sets of data obtained in this way for three different realizations of the fracture surface are plotted in Fig. 8. One observes the same general trend as when  $a_0$  is kept constant and  $u$  varies like in Fig. 7:  $k_{\parallel}/k_0$  decreases and  $k_{\perp}/k_0$  increases or remains constant as  $a_0$  is reduced and  $S^2$  increases. For a given realization of the surface, the variations of  $k_{\parallel}/k_0$  and  $k_{\perp}/k_0$  with  $S^2$  are linear and both remain within the bounds set by Eqs. 11 and 12. This confirms that the normalized aperture variance,  $S^2$ , is indeed the adequate parameter to characterize the disorder in the aperture field introduced by the shear displacement between the two surfaces.

Numerically, 10 self-affine surfaces were generated and were used to create aperture fields for different mean apertures and shear displacements. By fitting the average of the permeabilities over all realizations to a quadratic dependence on  $S$  we obtain:

$$\left\langle \frac{k_{\perp}}{k_0} \right\rangle = 0.995 + 1.09S^2, \quad (13)$$

and

$$\left\langle \frac{k_{\parallel}}{k_0} \right\rangle = 0.955 - 3.05S^2. \quad (14)$$



**Figure 9.** Permeability ratio  $k_{\perp}/k_{\parallel}$  as a function of normalized aperture variance  $S^2$ : ( $\square, \circ, \bullet$ ) experimental data; ( $\star, \times$ ) numerical data. -  $\square$ :  $u_x = 0, u_y \neq 0$ . -  $\circ$ :  $u_x \neq 0, u_y = 0, a_0 = 1.0$  mm, -  $\bullet$ :  $u_x = 2.0$  mm,  $1.25 < a_0 < 2$  mm. -  $\star$  and  $\times$ : numerical realizations displaying the strongest and the weakest anisotropy, respectively. - Dotted line: upper bound for the permeability ratio estimated from Eqs. 11 and 12. - Solid line: average  $\langle k_{\perp}/k_{\parallel} \rangle$  of the permeability ratio over all numerical realizations. - Dashed lines: linear fits of the experimental data.

Nevertheless, large fluctuations are observed from one realization to another: for some of them, the flow is only weakly perturbed by the wall roughness, while, for others if is strongly distorted and the permeability behaves as predicted by Eqs. 11 and 12.

## 7. Comparison between experiments and simulations

As discussed in Sec. 2.3, the analysis of the geometry of the invasion pattern in the injection experiments allows one to estimate the ratio  $k_{\perp}/k_{\parallel}$  of the permeabilities for flow perpendicular and parallel to the shear displacement  $\vec{u}$ . The variance  $S^2$  of the aperture can be computed for each set of values of  $a_0$  and  $\vec{u}$  from the profilometer maps of the fracture surfaces (see Sec. 2.1). It is therefore possible to determine experimentally both  $k_{\perp}/k_{\parallel}$  and  $S^2$  for each experiment. Figure 9 displays the variations of the permeability ratio as a function of  $S^2$ , as determined from the shape of the invasion front in a series of injection experiments (see Sec. 2.3). Three sets of experiments are presented, two of them correspond to shear displacements oriented along the x-direction or y-direction. No distinction between the positive and negative directions is made since, as pointed out in Sec. 3, the permeability ratio is roughly independent of the shear displacement sign but varies with its magnitude and orientation. The third set represents experiments performed with a fixed shear displacement,  $u_x = 2.0$  mm, and a variable aperture  $a_0$  ranging from 1.25 mm to 2.0 mm. As can be seen in Fig. 9, the permeability ratios measured with a variable aperture and a shear displacement oriented along the x-direction follows the same trend as those obtained with a fixed aperture and a variable shear displacement applied along the x-direction.

Independently of the shear-displacement direction the permeability ratio increases roughly linearly with  $S^2$  and its values remain below the upper bound provided by combining Eqs. 11 and 12. We also show that the permeability ratios obtained from the experiments always lie between the strongest and weakest anisotropies observed in the numerical computations. When  $S \sim 0.1$ , a departure of  $k_{\perp}/k_{\parallel}$  from a linear behavior is observed in the numerical simulations. This result is consistent with the model discussed in Sec. 4, which is clearly no longer valid for  $S \gtrsim 0.13$ , for which it predicts a closed fracture (zero permeability). A sharp decrease of the permeability along the shear displacement is thus expected and corresponds to a strong increase of  $k_{\perp}/k_{\parallel}$  when  $S$  can no longer be considered to be small compared to 1. This divergence is not observed experimentally since the method used for the anisotropy characterization is strictly valid for weak anisotropic permeabilities as discussed in Sec. 2.3.

Finally, we also present in Fig.9 a linear fit to the experimental data. The strongest effect is observed when the walls are shift in the y-direction and a linear regression indicates a linear increase of  $k_{\perp}/k_{\parallel}$  with  $S^2$  with a slope of  $5.4 \pm 0.3$ . In the x-direction, the permeability ratio also increases as  $S^2$  but with a slope of  $2.3 \pm 0.15$ .

## 8. Conclusions

The present study has confirmed both experimentally and numerically that the permeability of a fracture made of two complementary surfaces with a shear displacement  $\vec{u}$  is lower for flow parallel to  $\vec{u}$  than perpendicular to it. More precisely, the parallel permeability ( $k_{\parallel}$ ) decreases significantly while the perpendicular one ( $k_{\perp}$ ) increases or remains constant.

Qualitatively, this result appears to be related to the appearance of preferential channels of larger local aperture oriented perpendicular to  $\vec{u}$ : these channels are indeed visible

on aperture images reconstructed from experimental (or numerical) profilometry maps. Assuming such a geometrical structure allows to develop simple theoretical predictions for the relation between the permeability variations and the normalized variance  $S^2$  of the aperture field. The permeability for flow parallel (perpendicular) to  $\vec{u}$  was then shown to decrease (increase) approximately linearly with  $S^2$ . The absolute magnitude of the variation is also predicted to be two times larger for the parallel flow. These predictions are qualitatively correct and provide an estimation of the order of magnitude for the permeability variations and for the ratio of the permeabilities perpendicular and parallel to the shear displacement. In particular, both for the experiments and the numerical simulations, a linear variation of the permeability ratio with  $S^2$  is actually observed - showing that the variance  $S^2$  is a key parameter of the process. However there is a significant dispersion in the absolute value of the prefactors in these linear relations from one sample to another.

Further studies will be needed to understand the origin of the dispersion of the numerical values of the permeability variations and uncover the other characteristic geometrical parameters which may be involved in the process (finite size effects may be a limitation, particularly for the present simulations). Another important issue for practical applications is whether an estimation of the aperture variance can be obtained, for a given shift  $\vec{u}$  and mean aperture  $a_0$  of the fracture without requiring to map the full surface of the sample.

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