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Didier Poilblanc, Douglas J. Scalapino. The Gap Function $\Phi(k,w)$ for a Two-leg t-J Ladder and the Pairing Interaction. *Physical Review B: Condensed Matter and Materials Physics (1998-2015)*, 2005, 71 (17), pp.174403. 10.1103/PhysRevB.71.174403 . hal-00003481

HAL Id: hal-00003481

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Submitted on 7 Dec 2004

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The Gap Function $\phi(k, \omega)$ for a Two-leg t-J Ladder and the Pairing Interaction

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(Dated: December 8, 2004)

Abstract

The gap function $\phi(k, \omega)$, determined from a Lanczos calculation for a doped 2-leg t-J ladder, is used to provide insight into the spatial and temporal structure of the pairing interaction. It implies that this interaction is a local near-neighbor coupling which is retarded. The onset frequency of the interaction is set by the energy of an $S = 1$ magnon-hole-pair and it is spread out over a frequency region of order the bandwidth.

PACS numbers: PACS numbers: 75.10.-b, 75.10.Jm, 75.40.Mg

ccsd-00003481, version 1 - 8 Dec 2004

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The momentum and frequency dependence of the superconducting gap function $\phi(k, \omega)$ provides information on the spatial and dynamic structure of the pairing interaction. Traditionally, electron tunneling has been used to explore the frequency dependence of the gap [1]. Presently, angular resolved photoemission spectroscopy [2] (ARPES) and scanning tunneling microscopy [3, 4] (STM) open the possibility of obtaining both k and ω information about the superconducting gap. However, approaches to extract this information on $\phi(k, \omega)$ from such experiments are still being explored. As has been previously discussed, a k and ω dependent gap function for finite t-J lattices can be obtained using Lanczos exact diagonalization [5, 6]. Here, using results for $\phi(k, \omega)$ obtained for a doped 2-leg t-J ladder, we explore what can be learned once data for $\phi(k, \omega)$ becomes available.

The Hamiltonian for a 2-leg t-J ladder can be written as

$$\begin{aligned}
H = & J_{\text{rung}} \sum_i \left(\mathbf{S}_{i1} \cdot \mathbf{S}_{i2} - \frac{1}{4} n_{i1} n_{i2} \right) + J_{\text{leg}} \sum_{i,\alpha} \left(\mathbf{S}_{i+1,\alpha} \cdot \mathbf{S}_{i,\alpha} - \frac{1}{4} n_{i+1,\alpha} n_{i\alpha} \right) \\
& - t_{\text{rung}} \sum_i \left(c_{i,1}^\dagger c_{i,2} + h.c. \right) - t_{\text{leg}} \sum_{i,\alpha} \left(c_{i+1,\alpha}^\dagger c_{i\alpha} + h.c. \right)
\end{aligned} \tag{1}$$

where $c_{i\alpha}$ are projected hole operators (spin indices are omitted) and $\alpha = (1, 2)$ labels the two legs of the ladder. We will consider the isotropic case in which $J_{\text{rung}} = J_{\text{leg}} = J$ and $t_{\text{rung}} = t_{\text{leg}} = t$ and our calculations will be carried out for a periodic 2×12 ladder at $1/8$ and $1/6$ hole doping.

As previously discussed, a gap function can be extracted by combining Lanczos results for the usual one-electron Green's function $G(k, \omega)$ with the Fourier transform of Gorkov's off-diagonal Green's function

$$F(k, t) = i \langle T c_{-k, -\sigma}(t/2) c_{k, \sigma}(-t/2) \rangle \tag{2}$$

Here, for a finite system, this expectation value is taken between the ground states for N and $N - 2$ particles and we choose the phase of F to be zero. We will take $N = 22$, $N - 2 = 20$ corresponding to an average filling $n \simeq \frac{21}{24} = 0.875$. The Dyson equations [7] relating G and F are

$$[Z(k, \omega) \omega - (\epsilon_k + X(k, \omega))] G(k, \omega) = 1 - \phi(k, \omega) F(k, \omega) \tag{3}$$

$$[Z(-k, -\omega) \omega - (\epsilon_k + X(-k, -\omega))] F(k, \omega) = -\phi(k, \omega) G(k, \omega) \tag{4}$$

with Z and X the usual Nambu self energies and $\phi(k, \omega)$ the gap function. Then, using the even ($k \rightarrow -k$) parity of $Z(k, \omega)$, $X(k, \omega)$, ϵ_k and $\phi(k, \omega)$ along with the even ω dependence of $Z(k, \omega)$, $X(k, \omega)$ and $\phi(k, \omega)$, we have

$$\phi(k, \omega) = \frac{F(k, \omega)}{F^2(k, \omega) + G(k, \omega)G(k, -\omega)}. \quad (5)$$

Alternatively, $Z(k, \omega)$ can be eliminated to obtain an expression for the superconducting gap

$$\Delta(k, \omega) = \frac{\phi(k, \omega)}{Z(k, \omega)} = \frac{2\omega F(k, \omega)}{G(k, \omega) - G(k, -\omega)}. \quad (6)$$

From a numerical calculation of $G(k, \omega)$ and $F(k, \omega)$, $\Delta(k, \omega)$ has been obtained for a 32-site, t-J cluster [5]. This extended previous work by Ohta *et. al* [8] who fit the spectral weight $Im F(k, \omega)$ to a $d_{x^2-y^2}$ BCS-Bogoliubov quasiparticle form in which the frequency dependence of the gap was neglected.

In contrast to the long range order of the superconducting ground state of a 2D lattice, a 2-leg ladder exhibits power law pair field correlations [10, 11] which decay as x^{-1/κ_ρ} . Here, κ_ρ is the Luttinger liquid parameter associated with the massless charge mode. This implies that for a ladder of length L , the off-diagonal Green's function $F(k, \omega)$ decays [12, 13, 14] as $(\xi/L)^{\frac{1}{2\kappa_\rho}}$. Here, the coherence length ξ is proportional to the inverse of the gap. For our doped ladder $L = 12$ is of order this coherence length so that we can probe the internal $k - \omega$ structure of a pair.

For $J/t = 0.4$ and $1/8$ doping, the k -dependence of the zero frequency gap $\phi(k_x, k_y, \omega = 0)$ is plotted in Fig. 1 for the bonding ($k_y = 0$) and antibonding ($k_y = \pi$) bands. For comparison, the solid curves correspond to $\phi_0 (a \cos k_x - \cos k_y)$ with $\phi_0 = 0.3$ and $a = 0.8$. This d-wave-like k -dependence of the gap function is similar to the behavior found in previous studies of both the t-J [8] and Hubbard ladders [9]. It implies that the spatial structure of the pairing interaction is dominantly a near-neighbor interaction.

The frequency dependence of the real and imaginary parts of $\phi(k, \omega)$ are shown in Fig. 2 for k values which are near the fermi surface of the bonding (red) and antibonding (green) bands respectively. In general, the gap function $\phi(k, \omega)$ is a complex frequency-dependent function $\phi_1 + i\phi_2$ with the imaginary part associated with dynamic decay processes. For our doped 2-leg ladder with $J/t = 0.4$, the magnitudes of the zero frequency gaps $\Delta_0 = \Delta(k_F, 0)$ obtained from eq. (6), for both the bonding and antibonding fermi points, are of order $0.15t$. The gap, Δ_0 , is reduced from $\phi(k_F, 0)$ due to the renormalization factor Z . The onset of

the imaginary part of $\phi_2(k, \omega)$ seen in Fig. 2(b) appears to occur somewhat below $3\Delta_0$. In addition, the peaking in $\phi_2(k, \omega)$ and the rapid rise in $\phi_1(k, \omega)$ as this onset frequency is approached suggest that a particular excitation mode occurs at a frequency Ω such that $\omega = \Delta_0 + \Omega$ determines the onset of $\phi_2(k, \omega)$.

The occurrence of an onset peak in $\phi_2(k, \omega)$ at threshold and the short-range nature of the interaction involving scattering of pairs from the bonding to antibonding band ($q_y \sim \pi$), imply that the pairing in this energy regime is mediated by an $S = 1$ channel. This follows from the form of the coherence factor which varies as $\frac{1}{2} \left(1 \pm \frac{\Delta(p+q, \omega) \Delta(p, \omega)}{E(p+q) E(p)} \right)$, with the plus sign associated with the charge channel and the minus sign with the $S = 1$ spin channel. For a “d-wave-like” gap with $\mathbf{q} = (k_F(\text{bonding}) - k_F(\text{antibonding}), \pi)$ and ω near threshold, the coherence factor goes to 1 for the spin channel and vanishes for the charge channel.

A plot of the low-energy $S = 1$ excitations for a doped ladder is shown in Fig. 3(a). The solid diamonds show a collective $S = 1$ bound magnon-hole-pair [15, 16] mode and the open symbols the $S = 1$ particle-hole continuum. A measure of the spectral weight of the $S = 1$ channel which couples to $\phi(k, \omega)$ is given by the d-wave projection of the spin fluctuation spectral weight

$$V_d(\omega) = \frac{1}{N^2} \sum_{k, k'} (\cos k_x - \cos k_y) (\cos k'_x - \cos k'_y) S(\mathbf{k} - \mathbf{k}', \omega) \quad (7)$$

with

$$S(q, \omega) = -\frac{1}{\pi} \text{Im} \left\langle S_{-q}^\alpha \frac{1}{\omega + E_0 + i\eta - H} S_q^\alpha \right\rangle. \quad (8)$$

Here S_q^α is the Fourier transform of the α spin component. A plot of $V_d(\omega)$ versus ω is shown in Fig. 3(b). The peak at $\Omega \simeq 0.15t$ arises from the bound magnon-hole-pair mode and the high frequency weight comes from the particle-hole continuum. We believe that the onset of $\phi_2(k, \omega)$ seen in Fig. 2 is due to a process in which a single particle excitation with energy $\omega = \Omega_m + \Delta_0$ emits a bound magnon-hole-pair and drops down to the gap edge while the remaining $\phi_2(k, \omega) \neq 0$ region arises from a coupling to the $S = 1$ continuum.

In order to further characterize the dynamic nature of the pairing interaction, we introduce

$$I(k, \Omega) = \frac{1}{\pi} \int_0^\Omega d\omega' \frac{\phi_2(k, \omega')}{\omega'}. \quad (9)$$

Then, since the gap function satisfies a dispersion relation, one has

$$\phi_1(k, 0) = I(k, \Omega \rightarrow \infty) + \phi_{\text{static}}(k). \quad (10)$$

Here, $\phi_{\text{static}}(k)$ represents a non-retarded contribution. For example, if one were to make a mean-field approximation in which

$$J \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j \right) \rightarrow -J \left(\langle \Delta_{i\delta} \rangle \Delta_{i\delta}^\dagger + h.c. \right)$$

one would have an effective attractive pairing interaction which is independent of frequency which would contribute to ϕ_{static} . In Fig. 4(a), we have plotted $I(k, \Omega)/\phi_1(k, 0)$ versus Ω and one sees that for $J/t = 0.4$, $I(k, \Omega)$ saturates at over 80% of $\phi_1(k, 0)$. This implies that the dominant part of the pairing interaction comes from dynamic processes. Also shown, Fig. 4(b), are similar results for $J/t=0.8$. In this, unphysically large, J/t regime only 60% of the $\omega = 0$ gap function is associated with a dynamic pairing interaction. This is similar to earlier results for the spatial structure of a pair [17, 18] in which near-neighbor sites for the two holes making up a pair were favored for large values of J/t . For physical values of J/t , next-near-neighbor (diagonal) hole-hole occupation was favored. The diagonal structure arises dynamically and reflects the retarded nature of the pairing interaction for physical values of J/t .

In summary, knowledge of the k - and ω -dependence of the gap function provides information on the spatial and temporal structure of the pairing interaction. Numerical solutions for the two-leg ladder show that the pairing interaction has a short-range, near-neighbor form, leading to momentum transfer processes which scatter pairs between the bonding and antibonding states. The d-wave-like momentum dependence of the gap function and the ω onset of $\phi_2(k, \omega)$ imply that the pairing interaction in this energy regime is mediated by the $S = 1$ channel. This channel contains both a bound magnon-pair state and a continuum of particle-hole excitations. We believe that the magnon-pair mode is responsible for the onset behavior seen in $\phi_2(k, \omega)$. The dispersion relation for $\phi_1(k, 0)$ shows that for physical values of J/t , the dominant part of the interaction is dynamic with contributions coming from both the magnon-hole-pair mode and the particle-hole continuum.

Acknowledgments

DJS would like to acknowledge support under NSF Grant #DMR02-11166. DP thanks IDRIS (Orsay, France) for use of supercomputer facilities.

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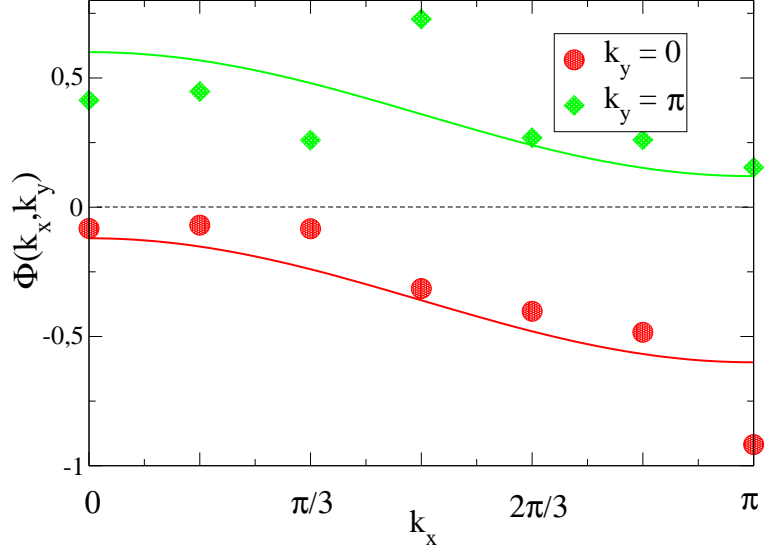


FIG. 1: The k_x dependence of $\phi_1(k_x, k_y, \omega = 0)$ for the bonding $k_y = 0$ (red) and antibonding $k_y = \pi$ (green) bands. For comparison, the solid curves show $\phi_0(a \cos k_x - \cos k_y)$ with $\phi_0 = 0.3$ and $a = 0.8$.

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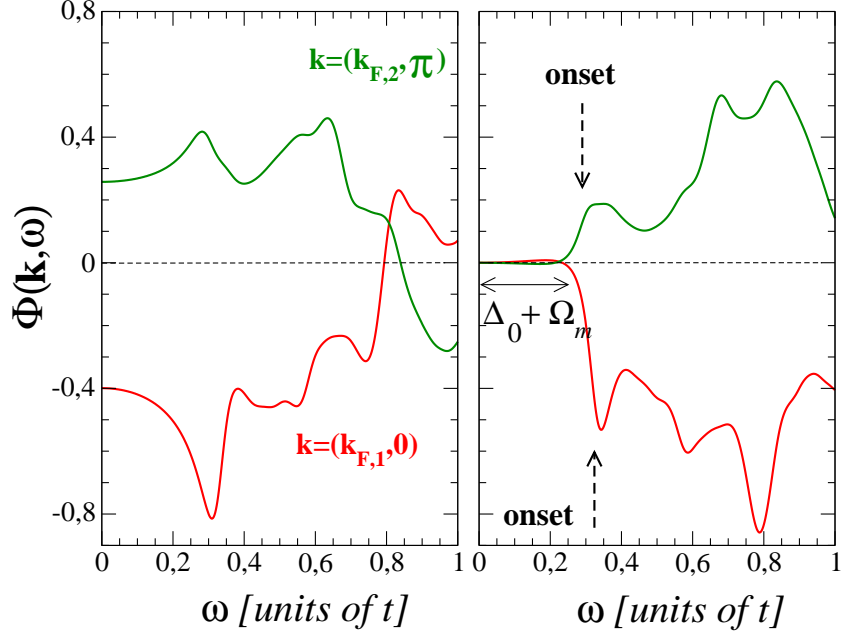


FIG. 2: Real (a) and imaginary (b) parts of the gap function $\phi(k, \omega)$ versus ω for a 2×12 ladder at values of k near the bonding (red) and antibonding (green) fermi momenta [$\sim (\frac{2\pi}{3}, 0)$ and $\sim (\frac{\pi}{3}, \pi)$ respectively] for $J/t = 0.4$. Here, $\Delta_0 \sim \Delta(k_F, \Delta_0)$ and Ω_m is the minimum energy for a bound magnon-hole-pair excitation.

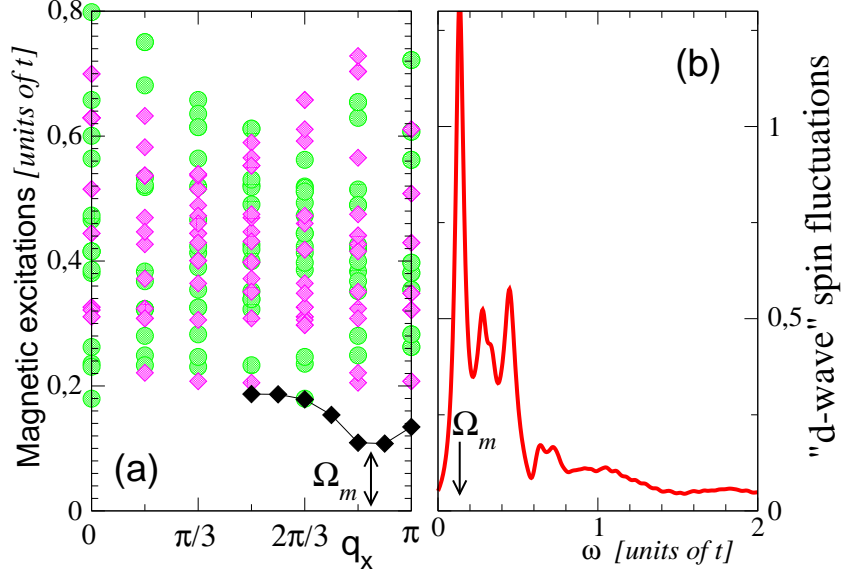


FIG. 3: (a) Low-energy electron-hole $S = 1$ excitations for a $1/8$ -doped $J/t = 0.4$ ladder for $q_y = 0$ (circles) and $q_y = \pi$ (diamonds) momenta. The solid diamonds denote a bound magnon-hole-pair collective mode [6]. (b) “d-wave” projection of the spin fluctuation spectral weight with Ω_m the minimum energy of the bound magnon-hole-pair collective mode (1/6 doping).

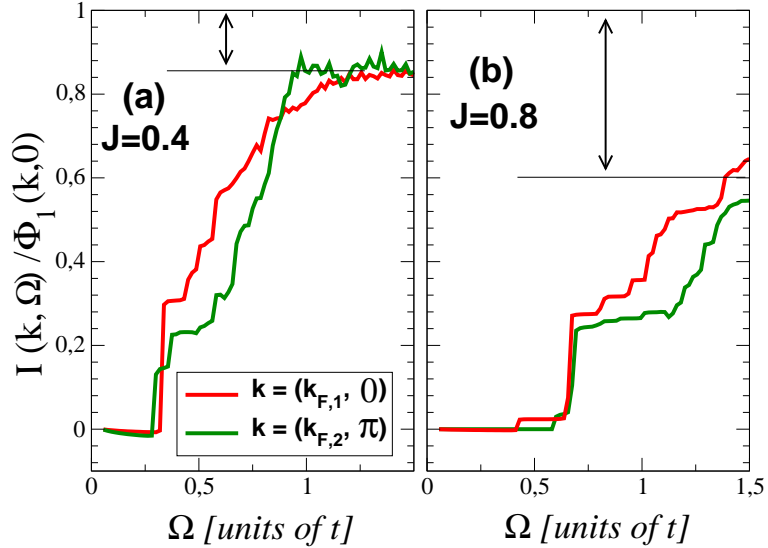


FIG. 4: The integrated contribution of $\phi_2(k, \omega)/\omega$ to $\phi_1(k, 0)$ for the bonding (red), antibonding (green) k -values shown in Fig. 2. (a) for $J/t = 0.4$ and (b) for $J/t = 0.8$. The non-retarded contributions are shown by arrows.