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Influence of the structural modulations and the Chain-ladder interaction in the $Sr_{14-x}Ca_xCu_{24}O_{41}$ compounds.

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We studied the effects of the incommensurate structural modulations on the ladder subsystem of the $Sr_{14-x}Ca_xCu_{24}O_{41}$ family of compounds using ab-initio explicitly-correlated calculations. From these calculations we derived t-J model as a function of the fourth crystallographic coordinate τ describing the incommensurate modulations. It was found that in the highly calcium-doped system, the on-site orbital energies are strongly modulated along the ladder legs. On the contrary the two sites of the ladder rungs are iso-energetic and the holes are thus expected to be delocalized on the rungs. Chain-ladder interactions were also evaluated and found to be very negligible. The ladder superconductivity model for these systems is discussed in the light of the present results.

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I. INTRODUCTION

The $Sr_{14-x}Ca_xCu_{24}O_{41}$ family of compounds have attracted a lot of attention in the last years due to the discovery of a superconducting phase in these quasiunidimensional systems. The $Sr_{14-x}Ca_xCu_{24}O_{41}$ are layered materials with alternated planes of two-legs spin ladders and planes of spin chains². The superconducting phase, observed in the highly calcium-doped systems, at low temperature and under pressure, is believed to be the realization of the predicted superconductivity in two-legs ladders systems³. Indeed, it is currently assumed that the $Sr_{14-x}Ca_xCu_{24}O_{41}$ ladder subsystem can be represented by a two-leg ladder isotropic t-J model with $J/t \sim 0.4$ and a hole doping ranging between 1⁴ and 2.8⁵ holes per formula unit (f.u.), that is between 0.07 and 0.2 hole per ladder site. Numerical as well as slave-bosons analytical calculations⁶, showed that in this parameter range, the superconducting pairing fluctuations are dominant. Hole-pairs are predicted to be bounded on the rungs and the superconductivity to be supported by pairs collective

The $Sr_{14-x}Ca_xCu_{24}O_{41}$ compounds present structural incommensurate modulations of both the chains and ladder subsystems. It was shown⁷ using ab-initio calculations, that, unlike what was usually assumed, the chain subsystem cannot be considered as a quasi-homogeneous system for which the structural modulations induce only small perturbative effects. In fact, the modulations caused by the influence of the ladders on the chain subsystem are responsible for the low energy physics of the latter, that is charge localization and spin arrangement. These ab initio calculations showed that the parameters of the second neighbor t-J+V model are strongly modulated. The major effects are (i) on the on-site magnetic orbital energies that fluctuate within a few electron-Volt range, (ii) on the first neighbor effective exchange integrals that range from ferromagnetic (as expected for such 90° oxygen-bridged copper sites) to antiferromagnetic (in the calcium highly-doped compounds). Moreover the calcium iso-electronic substitution, that acts as a chemical pressure, is responsible for large variations of the t-J model parameters. It results a quite different chain ground-state for the undoped and highly-doped systems. The undoped compound exhibits the formation of second-neighbor dimers caused by the electron-localization on the low orbital energy sites, while the large x compounds exhibit the formation of low spin clusters with antiferromagnetic local ordering.

One can thus wonder whether the structural modulations induce as important effects on the ladder electronic structure, as observed on the chain one. This point is of crucial importance since the existence of such an effect would strongly questioned the interpretation of the observed superconductivity. Another point, often addressed in the literature, is the importance of possible chain-ladder coupling. Indeed, in the highly calciumdoped compound, some of the chain oxygen atoms come close to the ladder coppers as apical atoms. The aim of the present work is to provide answers to these questions, by the means of ab-initio calculations. The method used is an embedded fragment spectroscopy method that explicitly computes both the correlation effects and the screening effects. A t-J model, explicitly dependent on the structural modulations, is derived for the ladders, from the ab-initio results. The calculations are done on both ladders and mixed, chain and ladder, fragments, in the low temperature phases of both the undoped⁸ and x = 13.6 calcium-doped compounds. It would have been of great interest to also run calculations in the superconducting phase, unfortunately structural data are not available under pressure. Nevertheless we will discuss possible extrapolation of the ambient pressure results.

The next section will be devoted to the computational details. Section 3 will discuss the t-J model, section 4 will discuss the chain-ladder interactions and finally the last section will be devoted to discussions and conclusions.

II. COMPUTATIONAL DETAILS

It is well known that, in strongly correlated systems, the magnetic and transfer interactions are essentially local and can be evaluated using embedded fragment spectroscopy calculations. The use of such technically finite systems allow to take advantage of abinitio multi-reference, correlated, spectroscopic methods¹⁰ that are able to treat explicitly the strongly correlated character of these systems. In the present work we used the Difference-Dedicated Configuration Interaction method¹¹. This technique is able to treat properly the open-shell character of the magnetic/hole orbitals, the strong electronic correlation of these orbitals, as well as the screening effects.

The effects of the rest of the crystal on the computed fragments are essentially the Madelung potential and the exclusion effects. They are respectively treated by an appropriate bath of point charges, and, on the fragment first-neighbor shells, by total-ion pseudo-potentials¹² (TIPs). Such approaches have been successfully used to study systems such as the high T_c copper oxides¹³ or the famous $\alpha' NaV_2O_5$ compound¹⁴.

The fragments are defined under the following criteria: they must include the magnetic centers, the bridging oxygens mediating the interactions, and the first coordination shell of the preceding atoms which is responsible for the essential part of the screening effects¹⁰. Two types of fragments are used in the present work:

- Cu_2O_7 fragments for the ladder legs and the ladder rungs interactions (see figures 1a and 1b),
- Cu_2O_8 for the chain-ladder interactions (see figure 1c).

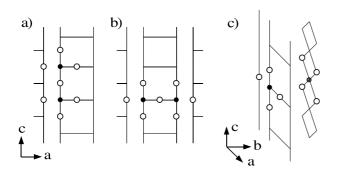


FIG. 1: Schematic representation of the computed fragments. a) for the ladder legs interactions, b) for the ladder rungs interactions and c) for the chain-ladder interactions. Circles represent the fragment atoms; the gray circles represent oxygen atoms, while the black circles represent the copper atoms.

On-site orbital energies (ε) and hopping (t) parameters of the t-J model are extracted from the first doublet-doublet excitation energies, and associated wavefunctions, of the above fragments with one magnetic electron. Exchange integrals (J) are deduced from the

singlet-triplet first excitation energies on the same fragments with two magnetic electrons. A least-square fit method is used in order to fit the effective parameters to the ab-initio results.

The calculations were done using the CASDI set of programs and the basis sets used can be found in reference¹⁵. The t-J model parameters were computed at 11 different positions along the ladders in order to get a good representation of their response to the incommensurate distortions. The computed value were then extrapolated to the whole system using the crystallographic description of incommensurate systems. It was done using a Fourier's series analysis, as a function of the fourth coordinate τ , associated with these systems incommensurate modulations¹⁶. All results of this work are given as a function of the τ coordinate of the copper atom (on-site parameters) and of the copper atom of lowest x and z coordinate for the interaction terms.

III. INFLUENCE OF THE STRUCTURAL MODULATIONS ON THE LADDER SUBSYSTEM

A. t-J+V model

Figure 2 displays the hopping and exchange parameters for the undoped and calcium-doped compounds. One first notices that, as was observed on the chains subsystem^{17,18}, the variations of the parameters are quite large. For instance, on the x = 13.6 ladder legs, the hoping modulations reach up to 160 meV. One should however point out that the nominal values of both the hoping and exchange integrals are much larger for the ladder subsystem than for the chain one due to the nearly 180° Cu-O-Cu angles. The consequence is that the relative variations of the parameters are much smaller on the ladder subsystems. Indeed, even for the hoping on the ladder legs of the $Ca_{13.6}$ compound, the relative variation reaches only 22% of the nominal value, while it was 100% for the chains nearest-neighbor (NN) hopings. It can thus be expected that despite their large absolute values, hoping and exchange fluctuations will not be as crucial for the ladders electronic structure, as for the chains subsystems one. One should even-though remember that these modulations are not small enough for their effect to be

Eccleston et al¹⁹ reported exchange values extracted from neutron scattering experiments, for the undoped compound. This values of 130 meV for the ladder legs and 72 meV for the ladder rungs are to be compared with our computed mean values of respectively 143 meV and 111 meV. While the order of magnitude are in good agreement the ratio between the average rungs and legs exchange is larger in our calculations $(J_{\perp}/J_{\parallel}=0.77)$ than in the experimental evaluation (0.55). One should however notice that the J_{\perp}/J_{\parallel} evaluation from neutron scattering does not take into account the parameters modulations along the ladders. Let us note that the

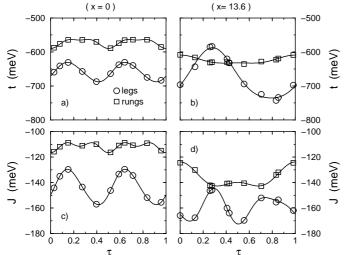


FIG. 2: Effective hopping (a and b) and exchange (c and d) parameters as a function of the modulation crystallographic coordinate τ . Figures a and c correspond to the undoped compound and figures b and d to the calcium-doped compound (x=13.6). The circles and squares are the computed points for the legs and rungs, the solid line are the fitted Fourier series.

 J_{\perp}/J_{\parallel} ratio for the parent ladder compound, $SrCu_2O_3$, was found to be larger with $J_{\perp}/J_{\parallel}=0.89^{20}$.

One can point out that both hoping and exchange absolute values increase with the calcium content. This is specially true for the ladder rung exchange that goes from a mean value of 112 meV for the x=0 compound to 136 meV for the x=13.6 compound.

Figure 3 reports the on-site orbital energies, $\varepsilon(\tau)$, for the x = 0 (c) and x = 13.6 (d) compounds. One sees immediately that the variations of these on-site energies are large, however much weaker in amplitude than for the chain subsystems. Indeed, for the undoped compound, ε varies in a 0.3 eV range while it is 1.2 eV¹⁷ for the chain subsystem. Similarly for the x = 13.6 compound the variations range is 1.6 eV for the ladders while it is 2.4 eV^{18} for the chains. This results are in agreement with the hoping and exchange results and are due to the facts that (i) the structural modulations are of weaker amplitude on the ladder subsystem than on the chain one, (ii) the parameters modulation of one subsystem, and specifically the on-site orbital energies, are essentially due to the structural modulations of the concerned subsystem itself¹⁷. It is remarkable that these onsite energies modulations are totally correlated with the Madelung potential modulation, both for the chain and ladder subsystems and both for the undoped and highly calcium doped compounds (see figure 4). Indeed, despite the fact that the local environments of the ladders and chain magnetic centers are very different the computed on-site orbital energies exhibit the same dependence to the Madelung potential: $\Delta \varepsilon = 0.42 \Delta V_{Mad.Pot.}$ (where

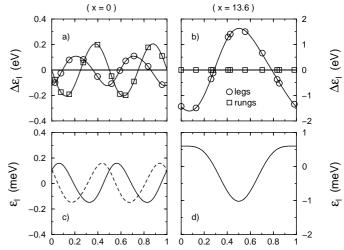


FIG. 3: On-site orbital energies differences between NN sites (a for x=0 and b for x=13.6). The circles correspond to the computed values on the legs fragments and the squares to the rungs ones. The lines are the fitted Fourier series. Deduced on-site energies are presented on figures c for x=0 and d for x=13.6. The solid and dashed curves correspond to the two legs of a ladder.

 $V_{Mad.Pot.}$ is the Madelung potential on the copper sites and delta refers to the NN differences).

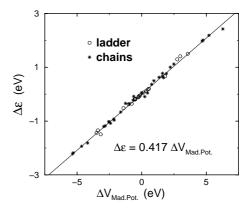


FIG. 4: Computed NN-difference of the on-site orbital energies as a function of the corresponding Madelung potential differences.

Comparing the ε values as a function of the calcium doping, one sees that as for the chain subsystem the calcium-doped compound presents much larger on site orbital energy fluctuations than the undoped system. This difference is in fact much larger than for the chain subsystem since the x=13.6 versus x=0 average orbital energy ratio is 5.3 for the ladders while it is only 2 for the chains. Another important point, that will be discussed in more details later on, is the fact that the ε variation range is weaker than the hoping integral in the x=0 system (0.3 eV versus 0.660 eV), while it is much larger

in the x=13.6 compound (1.6 eV versus 0.670 eV) and can be expected to induce in this case a hole localization on the ladders.

Another important remark concern the orbital energy difference between two sites belonging to the same rung. Indeed, for the undoped system this difference is strongly modulated, with a NN energy difference larger than between NN leg atoms (see figure 3a). For the highly doped compound however, the two copper sites of a same rung are iso-energetic. Indeed, the maximum absolute value of the computed orbital energy difference between two sites on the same rung is smaller than 1 meV. This energy degeneracy witness a hidden extra symmetry since the two copper sites of a rung respectively correspond to τ and $-\tau$. One retrieve this symmetry on figure 3d where the $\epsilon(\tau)$ function is symmetric around $\tau = 0.5$. The expected consequence of such a symmetry is the fact that the holes present in the ladder subsystem should be delocalized between the two sites of the ladder rungs, while localized in the \vec{c} direction.

IV. CHAIN-LADDER INTERACTIONS

In this section we will address the possibility of a chain-ladder coupling due to the structural modulations. Indeed, as mentioned in the introduction, in the highly calcium-doped compounds, the chains can be distorted in such a way that their oxygen atoms come into an apical position of the ladder copper atoms with a small $Cu_{ladder}-O_{chain}$ distance (2.7 Å). In the undoped compound however, the $Cu_{ladder}-O_{chain}$ distances always remain larger than 3.1 Å.

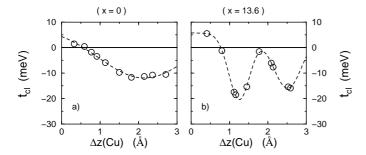


FIG. 5: Effective hoping between the chain and ladder closest copper sites as a function of the difference between the two copper coordinates along the \vec{c} axis $(\Delta z(Cu))$. Circles represent the computed points and the dashed lines are a guide for the eyes.

Figure 5 displays the effective hoping integrals between chain and ladder closest copper sites. One sees immediately that unlike what has been supposed in the literature, the chain-ladder hoping terms remain weak whatever the Cu_{ladder} – O_{chain} distances. Indeed, both the nominal values and the variation ranges (in meV units: [-11.7, 1.5] for the x = 0 system and [-18.5, 5.5] for the

x = 13.6 compound) are of the same order of magnitude for the undoped and calcium-doped compounds.

We also computed the chain-ladder magnetic couplings and the on-site orbital energy differences between the chain and ladder systems. The effective exchange were always found weaker than 1 meV, that is negligible. As far as the chain-ladder orbital energy differences are concerned, the results obtained from the present chainladder fragments are in full agreement with the results presented on the ladder subsystems and those from references 17,18 on the chain subsystems. One should point out that the present calculations allow us to set the chainladders relative energy levels, as will be discussed further. Let us also notice that the orbital energy differences between chain and ladder sites are not proportional to the Madelung potential differences on the related sites, unlike what was observed for the intra-subsystems orbital energy modulations (see figure 4).

As a conclusion on can say that both hoping and exchange values between the chains and ladders are found more than one order of magnitude weaker than the intrachain and intra-ladder interactions, for both compounds. The chain and ladder layers can thus be safely considered as non-interacting.

V. DISCUSSION AND CONCLUSION

One of the most controversial subject about the $Sr_{14-x}Ca_xCu_{24}O_{41}$ family of compounds is the chain to ladder hole transfer as a function of the calcium content. While it is clearly established that the hole transfer increases both with the calcium content and the applied pressure, the actual amount of transferred holes does not reach a consensus, even for the x=0 system. This subject is of importance for the understanding of the system properties since (i) it is supposed to be crucial for the superconducting phase³ and (ii) it is more generally directly related to the systems conduction properties since the conduction is supposed to occur in the ladder subsystem.

We evaluated the relative energy of the chain and ladder subsystems, responsible for the possible hole transfer, from the following parameters:

- the on-site orbital energies,
- the NN bi-electronic repulsion in both subsystems.

In references 17,18 , the NN bi-electronic repulsion V was computed for the chain subsystem using three centers fragment calculations. It was found to be almost independent of the structural modulations, temperature and calcium content. This can be understood by the fact that the NN repulsions are essentially dependent on the Cu-Cu distances and not on the Cu-O-Cu angles that dominate the modulations of the other effective interactions. We thus evaluated the NN repulsions on the ladders subsystem using the average computed chain value of 0.6 eV

and the standard Ohno formula 21 for the distance dependence

$$V(R) = \frac{V_0}{1 + R/a_0}$$

where R is the inter-site distance, a_0 is the Bohr radius and V_0 a R=0 effective constant. Extracting V_0 from the chain repulsions we get $V_0=3.7$ eV. We thus find

- for the intra-ladder repulsions : $V_{leg} \simeq V_{rung} \simeq 0.44 \text{ eV}$, and $V_{diag} \simeq 0.32 \text{ eV}$ (diagonal repulsion on a plaquette),
- for the inter-ladder repulsions : $V_{leg-leg} \simeq 0.6$ eV.

Using these approximations and the computed orbital energies we determined the relative energies of the chains and ladders as a function of τ (see figure 6). Since the ladder hole-doping remains small we considered that the repulsions where acting on all ladder sites. One sees im-

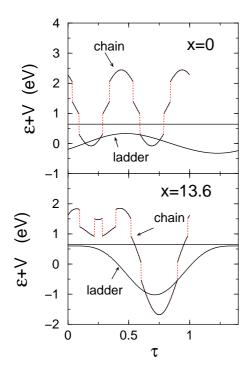


FIG. 6: Chain and ladder energies as a function of τ_{chains} . The site energies were evaluated as the sum of the orbital energies and the repulsion terms with neighboring atoms when the latters have lower orbital energies and are thus occupied prior to the considered one. The ladder fourth coordinate τ_{ladder} has been rescaled to $2c_{chain}/c_{ladder}$, so that the relative τ variation range of the two subsystems is proportional to the number of sites per f.u. in each sub-system. The horizontal line represents the Fermi level.

mediately that while for the x=0 compound the ladder energy curve is always about 0.65 eV lower than the first empty sites of the chains, for the x=13.6 compound, the most energetic ladder sites are at the Fermi level.

These results are coherent with a complete hole localization on the chain sub-system for the undoped compound and a small hole transfer to the ladders for highly doped systems. It seems however difficult to extract a precise value of the chain to ladder hole transfer (for the x=13.6 compound). Indeed the hole transfer can be expected to be sensitive to very small energetic variations, since the Fermi level is located in a region with large density of states, both for the chain and ladder subsystems. For the ladders subsystem, E_F is located on an energetic plateau. For the chain subsystem, E_F crosses the energy curve, unlike what happens in the undoped system where the Fermi level is located in an energetic gap. This may explain the large experimental range of values obtained for this hole transfer.

Another consequence of the Fermi level localization on a ladders energetic plateau, is that the holes are localized on (quasi) iso-energetic rungs — while they are localized on very energetically different sites in the chain subsystem. These rungs are not first neighbors, but in most cases second and in some cases third neighbors. The holes can thus be considered as evolving on a subset of the ladders with (quasi) iso-energetic sites. In such a model, the rungs belonging to the the real ladder located in between two hole-supporting rungs can be considered as bridging ligands. The resulting effective hoping and exchange integrals between the hole-supporting rungs are thus modulated according to the number and the characteristics of these in-between rungs. In such a model, these effective interactions between hole-supporting rungs can be expected to be rescaled to much lower values than the NN ones on the real ladder, and thus much lower values than the on-rung ones ($|t_{\rm leg}^{\rm eff}| \ll |t_{\rm rung}|$ and $|J_{\rm leg}^{\rm eff}| \ll |J_{\rm rung}|$).

We would like to conclude this work on a few opened questions on the super-conducting phase. First, the onsite energy variations are quite-large on the calciumdoped system, both for the chains and ladders subsystems. These modulations strongly increase as a function of the chemical pressure. Whether the applied pressure would act along the same line, it would induce an even larger distortion of the chains and ladders subsystems. Such a phenomenon would be quite incompatible with the usual super-conductivity theory in the ladder systems since it would induce an even larger modulation of the ladder on-site orbital energies. Another discrepancy between the present results and the super-conductivity predictions is that the super-conductive pairing is supposed to occur for (i) low ladder hole doping, (ii) $2|J_{rung}| >$ $|J_{leg}|$ (both are verified in the present case) and (iii) $|J_{rung}| > |t_{rung}|$ with is far from being verified since $|t_{rung}| > 4|J_{rung}|$. One thus wonder either what type of structural distortions under applied pressure could correct these features hindering the super-conductivity or whether the origin of the observed super-conductivity arise from another origin that the usually accepted homogeneous ladder t - J model.

Acknowledgments: the authors thank Dr. J. Etrillard for providing us with the undoped compound crystal-

			(a)			
x = 0	$arepsilon_l$	$\Delta \varepsilon_r$	t_l	t_r	J_l	J_r
a_0	0.0	0.0	-659.2	-572.2	-143.9	-111.6
a_1	-153.0	204.3	28.3	12.4	-13.7	2.5
a_2	8.9	3.4	2.0	-5.1	-1.0	-2.5
$arphi_1$	0.819	0.875	0.655	0.750	0.904	0.750
$arphi_2$	0.786	0.688	0.089	0.500	0.982	0.500
(b)						
x = 13.0	ε_l	$\Delta \varepsilon_r$		t_r	J_l	J_r
a_{i}	0.0	0.0	-672.5	-624.7	-159.3	-136.4
a	-811.4		73.9	11.5	-0.2	8.0
a_{i}	2 -209.5		-11.1	4.0	-10.5	4.5
a_{i}	3				4.4	
φ	0.500	0.000	0.284	0.001	0.284	0.995
φ	0.004	0.000	0.529	1.000	0.537	0.997
φ :	3				0.972	

TABLE I: Analytic fit of the t-J model, a) for the x=0 undoped compound, b) for the x=13.6 calcium-doped compound. All energies are given in meV.

lographic structures, Dr. Maynau for providing us with the CASDI code. The present calculations were done at the CNRS/IDRIS computational facilities under project n 1104.

Appendix

The computed results were fitted as a function of the fourth crystallographic coordinate τ , using a Fourier series, according to the following expression

$$a_0 + \sum_n a_n \cos\left[2\pi n(\tau - \varphi_n)\right] \tag{1}$$

Only terms with a non negligible contribution to the series were retained.

The results are summarized in tables Ia for the x=0 and Ib for x=13.6.

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