



**HAL**  
open science

# Potential of a moving test charge in a dusty plasma in the presence of grain size distribution and grain charging dynamics

Michael A. Raadu, Muhammad Shafiq

► **To cite this version:**

Michael A. Raadu, Muhammad Shafiq. Potential of a moving test charge in a dusty plasma in the presence of grain size distribution and grain charging dynamics. 2004. hal-00003135

**HAL Id: hal-00003135**

**<https://hal.science/hal-00003135v1>**

Preprint submitted on 22 Oct 2004

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Potential of a moving test charge in a dusty plasma in the presence of grain size distribution and grain charging dynamics

Michael A. Raadu\*and Muhammad Shafiq<sup>†</sup>

Royal Institute of Technology, Alfvén Laboratory,

Division of Plasma Physics, SE-100 44 Stockholm, Sweden

October 22, 2004

## Abstract

It is well known that the form of grain size distribution strongly influences the linear dielectric response of a dusty plasma. In previous results [IEEE Trans. Plasma Sci. 29, 182 (2001)], it was shown that for a class of size distributions, there is an equivalence to a Lorentzian distribution of mono-sized particles. The electrostatic response to a slowly moving test charge, using a second order approximation can then be found [Phys. Lett. A 305, 79 (2002)]. It is also well known [Phys. Plasmas 10, 3484 (2003)] that the dynamical charging of grains in a dusty plasma enhances the shielding of a test charge. It seems natural at this stage to seek the combined effects of grain size distribution and grain charging dynamics to a test charge moving through the dusty plasma. Here we consider the effects of both grain size distribution and dynamical grain charging to a test charge moving slowly in a dusty plasma by expressing the plasma dielectric response as a function of both grain size distribution and grain charging dynamics. Both analytical as well as the numerical results are presented. It is interesting to note that the previous results can be retrieved by choosing appropriate values for different parameters. This kind of study is relevant for both laboratory and space plasmas.

---

\*e-mail: michael.raadu@alfvenlab.kth.se

<sup>†</sup>e-mail: mohammad.shafiq@alfvenlab.kth.se

# 1 Introduction

There is currently considerable interest in understanding the physics of dusty plasmas which in addition to the electrons and ions, also contain a dust component. The dust grains become charged due to the collection of ions and electrons from the plasma and can typically acquire thousands of electron charges (due to high mobility of electrons, dust grains usually becomes negatively charged). Moreover, the dust charge does not necessarily remain constant and may continuously fluctuate due to varying plasma currents that flow onto the dust charge surface. The currents reaching the dust grain surface depend on the ambient plasma conditions and the floating potential of the dust particle. In this way the dust charge becomes a dynamic variable and fluctuates about an equilibrium charge state. In order to deal with the problem of charging dynamics, many papers have taken into account this dynamics and presented their results [1, 2, 3], the consequences of including a dust component have lead to a renewed interest in the problem of the test charge response. This is important for understanding the influence of a dust component on the interaction between charged particles. An important consequence of the potential excited by a moving charge is the energy loss and braking of the velocity due to the resultant electric field at the moving charge [4, 5].

We have investigated the response of a slowly moving test charge in a dusty plasma in the presence of charging dynamics [6, 7] and found that the dynamical charging of dust grains in a dusty plasma enhances the shielding of the test charge. The response potential up to the second order in test charge velocity was found and expressed analytically in terms of strength functions. A delayed shielding effect due to dynamical charging was also reported. The linearised dielectric theory was used and the equilibrium dust distribution was considered to be Maxwellian. Furthermore, the equilibrium dust particles were assumed to be similar and all the dust particles were treated as point particles. But this is not always true and in general, a size distribution of dust grains is to be expected both in artificial and natural plasmas [8, 9], for a Maxwellian distribution and a special class of physically reasonable size distributions, the dielectric response function was shown to be equivalent to that for monosized particles with a generalised Lorentzian

or kappa distribution [10] Recently, we have taken into account the test charge response of a dusty plasma with a grain size distribution [11, 12] and have shown that the form of grain size distribution strongly influences the linear dielectric response of a test charge in a dusty plasma. The analytical expressions for the response potential, using a second order approximation were found and the effects of collisions also investigated. More recently, A. M. Mirza et al. [13] extended this work further and presented analytical as well as numerical results for the slowing down of a pair of test charge projectiles moving through a multicomponent dust-contaminated plasma. In their analyses, they found that the energy loss for the Maxwellian distribution is larger compared to that for generalised Lorentzian distribution. They also found that for smaller values of the spectral index  $\kappa$ , the test charge projectile gains instead of losing energy.

In this paper, we have extended our previous work on grain size distribution [12] by taking into account the effect of charging dynamics and present analytical as well as numerical results for the response potential for a test charge moving through a multicomponent dusty plasma.

## 2 Plasma Dielectric for a Dusty Plasma

The linear response of the dusty plasma for an electrostatic disturbance can be determined through the choice of the plasma dielectric function. Here the dielectric will include a term for the dynamical charging of the dust grains and the effect of a specific choice for the size distribution will be taken into account.

### 2.1 Grain Size Distribution

Here we choose the size distribution  $h(a)$  used previously [10].

$$h(a)da = h_0 a^\beta \exp(-\alpha^3 a^3) da \quad (1)$$

where the constant  $h_0$  is defined by setting the integrated density to the dust density  $n_d$  ( $h_0 = 3n_d \alpha^{\beta+1} / \Gamma((\beta+1)/3)$ ). The distribution  $h(a)$  has a maximum at  $a = a_0 \equiv (\beta/3)^{1/3} \alpha^{-1}$ . If we let  $\alpha \rightarrow \infty$  with  $\beta \sim \alpha^3$  the distribution  $h(a)$  tends to a delta function

at  $a = a_0$  i.e. a monosized distribution with dust grain radius  $a_0$ . This limit is useful for comparing the general results that will be found here with earlier results for monosized distributions. The distribution  $h(a)$  can also be transformed to a distribution over grain mass  $m$  so that,

$$h(a)da = w(m)dm \equiv w_0 m^{(\beta-2)/3} \exp(-\mu m) dm \quad (2)$$

For small sizes  $h(a)$  has an approximate power law dependence on the size  $a$ . A power law dependence is a simple first approximation if the actual size distribution is not known. For large masses  $w(m)$  is dominated by an exponential decrease with mass (for  $\beta = 2$  the dependence is purely exponential, as assumed in early work on interstellar dust grains). These properties motivated this particular choice of size distribution [10]. Without charging dynamics this choice for  $h(a)$  leads to a dielectric response equivalent to a kappa distribution [10] with  $\kappa = (2\beta + 5)/6$ . (For the purely exponential mass dependence,  $\beta = 2$ , the index  $\kappa = 3/2$ .)

## 2.2 Charging Dynamics

Here we now include charging dynamics with frequencies  $\nu_0 \equiv \Omega_{u0}$  and  $\Omega_{v0}$  (defined by Melandsø *et al* [15]) that depend on grain size  $a$ . This leads to a response term with an integration over grain size with a differential “charging wavenumber”  $H_{dch}$  given by

$$H_{dch}^2(a) da \equiv 4\pi \frac{\Omega_{v0}}{\Omega_{u0}} ah(a) da$$

where  $\Omega_{u0}$  and  $\Omega_{v0}$  are the frequencies introduced by Melandsø *et al* [15] in the linearized equation for the grain charge perturbation  $q_{d1}$  with a plasma potential perturbation  $\phi_1$ ,

$$\frac{\partial q_{d1}}{\partial t} = -\Omega_{u0} q_{d1} - 4\pi \varepsilon_0 a \Omega_{v0} \phi_1 \quad (3)$$

In this equation  $\Omega_{u0} \equiv \nu_0(a)$  is the grain charge relaxation rate i.e.  $\tau_0 \equiv 1/\nu_0$  is the time scale for the grain charge to come into equilibrium with the undisturbed plasma. The total charging wave number  $K_{dch}$  may then be defined by

$$K_{dch}^2 \equiv \int_0^\infty H_{dch}^2 da$$

For a monosize distribution  $h(a)$  is a delta function and the above expression reduces to the standard definition [7]. Integrating for  $h(a)$  given by the equation (1) gives for  $H_{dch}$  the expression

$$K_{dch}^2 = 4\pi \frac{\Omega_{v0} n_d}{\Omega_{u0} \alpha} \frac{\Gamma\left(\frac{\beta+2}{3}\right)}{\Gamma\left(\frac{\beta+1}{3}\right)} \quad (4)$$

## 2.3 Plasma Dielectric

For a general size distribution with charge relaxation rate  $\nu_0(a)$  that is a function of the dust grain radius  $a$  the plasma dielectric function is,

$$D(K, K \cdot V_t) = 1 + \frac{K_{De}^2}{K^2} + \frac{K_{Di}^2}{K^2} + \frac{K_D^2}{K^2} \left[ 1 + \left( \frac{2\kappa}{2\kappa - 1} \right) \left( \frac{\hat{K} \cdot V_t}{V_{td}} \right) Z_\kappa \left( \frac{\hat{K} \cdot V_t}{V_{td}} \right) \right] + \frac{1}{K^2} \int_0^\infty \frac{H_{dch}^2(a) \nu_0(a)}{(\nu_0(a) - iK \cdot V_t)} da \quad (5)$$

where  $K_D$  and  $V_{td}$  are the effective Debye wave-number and effective thermal velocity for the dust as defined in [10]. For  $V_t < V_{td}$ , the plasma dispersion function  $Z_\kappa\left(\frac{\hat{K} \cdot V_t}{V_{td}}\right)$  is given as follows [16]:

$$Z_\kappa \left( \frac{\hat{K} \cdot V_t}{V_{td}} \right) = \frac{i\sqrt{\pi}}{\kappa^{3/2}\Gamma\left(\kappa - \frac{1}{2}\right)} \sum_{n=0}^{\infty} \left( -\frac{1}{i\sqrt{\kappa}} \right)^n \frac{\Gamma\left(\kappa + \frac{1}{2}(n+2)\right)}{\Gamma\left(\frac{1}{2}(n+2)\right)} \left( \frac{\hat{K} \cdot V_t}{V_{td}} \right)^n \quad (6)$$

Following the analysis of Melandsø *et al* [15], for a standard model of the dust charging process, explicit expressions can be found for the frequencies  $\Omega_{u0} \equiv \nu_0(a)$  and  $\Omega_{v0}$ . These may be written as [7],

$$\Omega_{v0} = \delta_{v0} \frac{a}{\lambda_{De}} \omega_{pi} \quad (7)$$

$$\Omega_{u0} = \delta_{u0} \frac{a}{\lambda_{De}} \omega_{pi} \quad (8)$$

where, assuming equal ion and electron temperatures, the numerical constants are  $\delta_{v0} = 2.793$  and  $\delta_{u0} = 1.795$ . Here we note that, for grain sizes comparable to the electron Debye length  $\lambda_{De}$ , these frequencies are of the order of the ion plasma frequency  $\omega_{pi}$ . The frequencies  $\Omega_{v0}$  and  $\Omega_{u0} \equiv \nu_0$  are simply proportional to the dust size  $a$  and the ratio  $\delta_0 \equiv \Omega_{v0}/\Omega_{u0} = \delta_{v0}/\delta_{u0} = 1.556$  is independent of the dust size. The last term in equation (5) can now be written using these expressions and the size distribution  $h(a)$  defined by equation (1). There is no obvious simple analytical expression for the resulting

integration, but for a slowly moving test charge the integral can be expanded as a power series in  $V_t$ . The individual terms can then be integrated in terms of the gamma function.

### 3 Response to a Moving Test Charge

For a test charge response in a plasma, the general expression for the electrostatic potential is given by [14]

$$\phi = \frac{q_t}{8\pi^3\epsilon_0} \int \frac{\exp[i\mathbf{K} \cdot \mathbf{r}]}{K^2 D(K, \mathbf{K} \cdot \mathbf{V}_t)} d\mathbf{K} \quad (9)$$

where  $V_t$  is the test charge velocity and  $D(K, \omega)$  is the plasma dispersion function. The explicit form of  $D(K, \omega)$  depends on the physics of the dusty plasma. Here  $D(K, \omega)$  is chosen to include the effects of a grain size distribution and charging dynamics.

For a slowly moving test charge ( $V_t < V_{td}$ ), we can expand the plasma dispersion function (equation (6)) up to first order and hence rewrite equation (5) for the dielectric up to second order in test charge velocity as

$$\begin{aligned} \frac{1}{K^2 D(K, K \cdot V_t)} &= \frac{1}{K^2 + K_{eff}^2} \left[ 1 + iA(\beta) \frac{K_D^2}{K^2 + K_{eff}^2} \left( \frac{\hat{K} \cdot V_t}{V_{td}} \right) \right. \\ &- B(\beta) \frac{K_D^2}{K^2 + K_{eff}^2} \left( \frac{\hat{K} \cdot V_t}{V_{td}} \right)^2 + i \frac{\alpha \lambda_{De}}{\delta_{u0} \omega_{pi}} \frac{K_1^2 K}{K^2 + K_{eff}^2} (\hat{K} \cdot V_t) \\ &\left. - C(\beta) \frac{\alpha^2 \lambda_{De}^2}{\delta_{u0}^2 \omega_{pi}^2} \frac{K_1^2 K^2}{K^2 + K_{eff}^2} (\hat{K} \cdot V_t)^2 + \mathcal{O}(V_t^3) \right]^{-1} \end{aligned} \quad (10)$$

with the definitions

$$K_1^2 = 4\pi\delta_0 \frac{n_d}{\alpha}, \quad K_{eff}^2 = K_{De}^2 + K_{Di}^2 + K_D^2 + K_1^2 f(\beta), \quad f(\beta) = \frac{\Gamma\left(\frac{\beta+2}{3}\right)}{\Gamma\left(\frac{\beta+1}{3}\right)}$$

and with

$$A(\beta) = \frac{\sqrt{\pi}}{\sqrt{\frac{1}{3}\beta + \frac{5}{6}}} \frac{\Gamma\left(\frac{1}{3}\beta + \frac{11}{6}\right)}{\Gamma\left(\frac{1}{3}\beta + \frac{4}{3}\right)}, \quad B(\beta) = \frac{4(\beta+4)}{2\beta+5}, \quad C(\beta) = \frac{\Gamma\left(\frac{\beta}{3}\right)}{\Gamma\left(\frac{\beta+1}{3}\right)}.$$

Here the term  $K_1^2 f(\beta)$  appearing in the definition of  $K_{eff}$  is identical to  $K_{dch}^2$  given by equation (4). The total charging wave number  $K_{dch}$  measures the contribution of charging dynamics to the total effective shielding wave number  $K_{eff}$ . In the above relations  $\beta$  is the power law index of the size distribution for small radii, related to the equivalent kappa

distribution by the relation  $\kappa = (2\beta + 5)/6$ . Expanding equation (10) for the inverse dielectric function up to second order in test charge velocity  $V_t$  and using in equation (1), we may express the electrostatic potential as  $\phi = \phi_1 + \phi_{ch}$  with

$$\begin{aligned} \phi_1 = & \frac{q_t}{8\pi^3\epsilon_0} \int \frac{\exp[iK \cdot r]}{K^2 + K_{eff}^2} \left[ 1 - \frac{iA(\beta)K_D^2}{K^2 + K_{eff}^2} \left( \frac{\hat{K} \cdot V_t}{V_{td}} \right) \right. \\ & \left. + \frac{B(\beta)K_D^2}{K^2 + K_{eff}^2} \left( \frac{\hat{K} \cdot V_t}{V_{td}} \right)^2 - \frac{A(\beta)^2 K_D^4}{(K^2 + K_{eff}^2)^2} \left( \frac{\hat{K} \cdot V_t}{V_{td}} \right)^2 \right] d\mathbf{K} \end{aligned} \quad (11)$$

and

$$\begin{aligned} \phi_{ch} = & \frac{q_t}{8\pi^3\epsilon_0} \int \frac{\exp[iK \cdot r]}{K^2 + K_{eff}^2} \left[ -i \frac{\alpha\lambda_{De}}{\delta_{u0} \omega_{pi}} \frac{K_1^2 K}{K^2 + K_{eff}^2} (\hat{K} \cdot V_t) \right. \\ & + C(\beta) \frac{\alpha^2 \lambda_{De}^2}{\delta_{u0}^2 \omega_{pi}^2} \frac{K_1^2 K^2}{K^2 + K_{eff}^2} (\hat{K} \cdot V_t)^2 - \frac{\alpha^2 \lambda_{De}^2}{\delta_{u0}^2 \omega_{pi}^2} \frac{K_1^4 K^2}{(K^2 + K_{eff}^2)^2} (\hat{K} \cdot V_t)^2 \\ & \left. - 2A(\beta) \frac{\alpha\lambda_{De}}{\delta_{u0} \omega_{pi}} \frac{K}{V_{td}} \frac{K_D^2 K_1^2}{(K^2 + K_{eff}^2)^2} (\hat{K} \cdot V_t)^2 \right] d\mathbf{K} \end{aligned} \quad (12)$$

It is to be noted that  $\phi_1$  is the same as we found earlier [12] except for the definition of  $K_{eff}$  which now includes the effect from charging dynamics in terms of  $K_1$ , while  $\phi_{ch}$  is the contribution which comes explicitly from the dust charging dynamics. The reader is referred to [12] for the results of equation (11) for  $\phi_1$ , while in the following we shall present the results for  $\phi_{ch}$ . The above equation (12) can be written in terms of strength functions as

$$\phi_{ch}(r, \lambda) = \frac{q_t}{8\pi^3\epsilon_0} \left[ V_t g_{11}(r) \cos \lambda + V_t^2 \left( g_{20}(r) + g_{22}(r) \cos^2 \lambda \right) + \mathcal{O}(V_t^3) \right] \quad (13)$$

where  $\lambda$  is the angle between the test particle velocity  $\mathbf{V}_t$  and the radial vector  $\mathbf{r}$ . The strength functions  $g_{ij}(r)$  are given by the following expressions

$$\begin{aligned} g_{11}(r) &= \pi^2 \frac{\alpha\lambda_{De}}{\delta_{u0} \omega_{pi}} K_1^2 \exp(-rK_{eff}) \\ g_{20}(r) &= \frac{\pi^2}{4K_{eff}} \frac{\alpha^2 \lambda_{De}^2}{\delta_{u0}^2 \omega_{pi}^2} K_1^4 \left[ \frac{4K_{eff}^2 C(\beta)}{K_1^2} \frac{1}{rK_{eff}} - 1 \right] \exp(-rK_{eff}) \\ &+ \frac{A(\beta)}{2V_{td}} \frac{\alpha\lambda_{De}}{\delta_{u0} \omega_{pi}} \frac{K_D^2 K_1^2}{r^3 K_{eff}^5} \left[ rK_{eff} (3 + r^2 K_{eff}^2) \Phi - (3 + 2r^2 K_{eff}^2) \Psi + 6rK_{eff} \right] \\ g_{22}(r) &= -\frac{\pi^2}{4K_{eff}} \frac{\alpha^2 \lambda_{De}^2}{\delta_{u0}^2 \omega_{pi}^2} K_1^4 \left[ \frac{4K_{eff}^2 C(\beta)}{K_1^2} \frac{(1 + rK_{eff})}{rK_{eff}} - rK_{eff} \right] \exp(-rK_{eff}) \end{aligned}$$



$$\begin{aligned}
& -\frac{A(\beta)}{2V_{td}} \frac{\alpha\lambda_{De}}{\delta_{u0}\omega_{pi}} \frac{K_D^2 K_1^2}{r^3 K_{eff}^5} \left[ rK_{eff} \left( 9 + 2r^2 K_{eff}^2 \right) \Phi \right. \\
& \left. - \left( r^2 K_{eff}^2 + 3 + rK_{eff} \right) \left( r^2 K_{eff}^2 + 3 - rK_{eff} \right) \Psi + 2rK_{eff} \left( 9 + r^2 K_{eff}^2 \right) \right]
\end{aligned}$$

where the following relations defining  $\Phi(rK_{eff})$  and  $\Psi(rK_{eff})$  in terms of exponential integrals [7] have been introduced,

$$\Phi(rK_{eff}) = \exp(rK_{eff}) E_1(rK_{eff}) - \exp(-rK_{eff}) E_i(rK_{eff}) \quad (14)$$

$$\Psi(rK_{eff}) = \exp(rK_{eff}) E_1(rK_{eff}) + \exp(-rK_{eff}) E_i(rK_{eff}) \quad (15)$$

$\Phi(y)$  and  $\Psi(y)$  (for  $y > 0$ ) are directly defined as principal parts of integrals (here, for real  $y$ , equivalent to taking the real part) as follows,

$$\Phi(y) = Re \left\{ - \int_0^\infty \frac{2t \exp(-yt) dt}{1-t^2} \right\} \quad (16)$$

$$\Psi(y) = Re \left\{ \int_0^\infty \frac{2 \exp(-yt) dt}{1-t^2} \right\} \quad (17)$$

From these definitions it follows that  $\Phi(y) = d\Psi(y)/dy$  and that  $\Psi(y) = d\Phi(y)/dy + 2/y$ . The functions  $\Phi(y)$  and  $\Psi(y)$  introduced here are closely related to the auxiliary functions  $f(y)$  and  $g(y)$  used in the analysis of the Sine and Cosine Integrals [17]. As for  $f(y)$  and  $g(y)$  asymptotic forms may be found for  $\Phi(y)$  and  $\Psi(y)$ ,

$$-\frac{1}{2} \Phi(y) \sim y^{-2} + 3! y^{-4} + 5! y^{-6} + 7! y^{-8} + O(y^{-10}) \quad (18)$$

$$\frac{1}{2} \Psi(y) \sim y^{-1} + 2! y^{-3} + 4! y^{-5} + 6! y^{-7} + O(y^{-8}) \quad (19)$$

## 4 Discussion

In equations (10) and (12) the combination of terms  $\tau \equiv \alpha\lambda_{De}/\delta_{u0}\omega_{pi}$  is equal to  $1/\nu_0(\alpha^{-1})$  (from equation (8) where  $\Omega_{u0} \equiv \nu_0$ ). Therefore  $\tau$  is the relaxation time for the charge on a dust grain with radius  $\alpha^{-1}$  to reach equilibrium with the ambient plasma, and  $V_t\tau$  is the distance travelled by the test charge in this time. As remarked above, if we let  $\alpha \rightarrow \infty$  with  $\beta \sim \alpha^3$  the distribution  $h(a)$  tends to a monosized distribution with dust grain radius  $a_0 \equiv (\beta/3)^{1/3}\alpha^{-1}$ . Putting  $\tau \equiv (\beta/3)^{1/3}\tau_0$ , the test charge response  $\phi_{ch}$  given by equation (13) may be shown to reduce to the known results for a monosized distribution with a charge relaxation time  $\tau_0$  [6, 7].

## 5 Acknowledgement

The authors would like to thank their colleagues at the Alfvén laboratory for useful discussions and suggestions. This work was partially supported by the Swedish Research Council.

## References

- [1] S. Ali, M. H. Nasim and G. Murtaza, Phys. Plasmas **10**, 941 (2003).
- [2] M. H. Nasim, P. K. Shukla and G. Murtaza, Phys. Plasmas **6**, 1409 (1999).
- [3] M. H. Nasim, A. M. Mirza, G. Murtaza and P. K. Shukla, Phys. Scr. **61**, 628 (2000).
- [4] M. H. Nasim, A. M. Mirza, M. S. Qaisar, G. Murtaza and P. K. Shukla, Phys. Plasmas **5**, 3581 (1998).
- [5] M. H. Nasim, M. S. Qaisar, A. M. Mirza, G. Murtaza and P. K. Shukla, Phys. Plasmas **7**, 762 (2000).
- [6] M. A. Raadu and M. Shafiq, AIP conference proceeding **649**, 430 (2002).
- [7] M. A. Raadu and M. Shafiq, Phys. Plasmas **10**, 3484 (2003).
- [8] T. K. Aslaksen and O. Havnes, J. Geophys. Res. **97**, 19175 (1992).
- [9] J. H. Oort and H. C. van de Hulst, Bull. Astron. Inst. Netherlands **10**, 187 (1946).
- [10] M. A. Raadu, IEEE Tran. on Plasma Sci. **29**, 182 (2001).
- [11] M. Shafiq and M. A. Raadu, AIP conference proceeding **649**, 422 (2002).
- [12] M. Shafiq and M. A. Raadu, Phys. Lett. A. **305**, 79 (2002).
- [13] A. M. Mirza, M. A. Sarwar and M. S. Qaisar, Phys. Plasmas **10**, 4253 (2003).
- [14] L. Stenflo and M. Y. Yu, Phys. Scr. **8**, 301 (1973).
- [15] F. Melandsø, T. Aslaksen and O. Havnes, Planet. Space Sci. **41**, 321 (1993).
- [16] R. L. Mace and M. A. Hellberg, Phys. Plasmas **2**, 2098 (1995).
- [17] M. Abramowitz, I. A. Stegun, *Handbook of Mathematical Functions*, National Bureau of Standard, (1964).