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# Analysis of the Drift Instability Growth Rates in Non-ideal Inhomogeneous Dusty Plasmas

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## Abstract

In this paper we introduce an algebraic form of the dispersion relation for a non ideal inhomogeneous dusty plasma in order to improve drastically the calculation of the drift instability growth rate. This method makes use of the multipole approximation of the Z dispersion function, previously published, and valid for the entire range. A careful analysis of the solutions spectra of this kind of polynomial equation permits us to calculate easily the growth rate of the drift instability for the ion-dust and dust acoustic mode. The value of the parallel to magnetic field wavelength for which the instability reaches the maximal value is carefully localized and discussed. The unstable dust-ion and dust acoustic mode are discriminated and analyzed in function of the density gradient, Te/Ti - ratio, and dust grain radius.

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## 1 Introduction

Plasma inhomogeneities across the magnetic field in the presence of finite - size charged grains causes a wide class of instabilities of an inhomogeneous dusty plasma called gradient instabilities. Such instabilities can be studied in the approximation on magnetic field where we have parallel straight field lines in order to simplify our treatment. We look for instabilities in the very low frequency regime where a new spectrum instabilities and waves appear, induced by the dust collective dynamics: Dust - Acoustic - Waves (DAWs), Dust - Ion - Acoustic - Waves (DIAWs), etc. The frequency of DAWs are around 10 Hz

as determined in the laboratory and lower in astrophysical plasmas [1,2]. In the case that grains are in the micron range we expect a non - ideal behavior due to the fact that the particulate are highly charged and intermolecular forces could play certainly an important role. In order to discuss this problem we compare the ideal properties with the simple hard - core model and in a next work we will use a better model by means of of the square - well model and the Padé rational approximant to the equation of state [3] for hard - sphere gas, that in our knowledge is more realistic as the simple application of the Van der Waals equation of state [4]. In this paper we show an analysis of the electrostatic waves and instabilities growth rates in a weakly non - ideal magnetized dusty plasma with density and temperature gradients, ignoring charge fluctuation. As introduced before, the non - ideal behavior is characterized by the hardcore model defined by

$$p = nT(1 + b_0n),$$

or in similar manner by the square - well model given by the Ree and Hoover expression [5 ].

## 2 Theoretical Model

In this paper we introduce a new numerical treatment in combination with a more realistic formulation of the equation of state to simulate weak non ideal effects in order to analyze inhomogeneous Vlasov - Dusty Plasma systems where a linearized dispersion relation is obtained. Due to the lower frequency range ( $\omega, k_z v_T \ll \omega_c$ ), enough energy can be transferred from the particle to the wave and instabilities can be generated. In order to get an adequate linear dispersion relation with a magnetic field given by  $\mathbf{B} = B_0 \hat{\mathbf{k}}$  for Maxwellian multi-species plasmas (electron, ion and dust), we introduce our well known and very accurate multipolar approximation [6] for the  $\mathbf{Z}$  dispersion function.

In the presence of a magnetic field we have the distribution function of the species  $\alpha$ , solution for the kinetic equation

$$\frac{df_\alpha}{dt} = \frac{q_\alpha}{m_\alpha} \nabla \phi \cdot \frac{\partial f_{o\alpha}}{\partial \mathbf{v}} \quad (1)$$

in the time dependent following form[7,8]

$$f(\mathbf{r}, \mathbf{v}, t) = \frac{q_\alpha}{m_\alpha} \int_{-\infty}^t \exp [i\omega(t - t')] \nabla \phi'(r(t')) \cdot \frac{\partial f_{o\alpha}}{\partial \mathbf{v}(t')} dt' \quad (2)$$

where  $\alpha = e, i, d$ . Now, the dispersion relation in terms of the dielectric susceptibilities, in the low frequency approximation ( $\omega, k_z v_T \ll \omega_c$ ) is

$$1 + \sum_{\alpha} \chi_{o\alpha} = 0 \quad (3)$$

where,

$$\chi_{o\alpha} = \frac{1}{(k\lambda_{D\alpha})^2} \left[ 1 + l_\alpha \frac{\omega}{\sqrt{2}k_z v_{T\alpha}} \mathbf{Z}(\xi_\alpha) I_0(z_\alpha) e^{-z_\alpha} \right] \quad (4)$$

with :

$$\begin{aligned} l_\alpha &= 1 - \frac{k_y T_\alpha}{m_\alpha \omega \omega_{c\alpha}} \left( \frac{d}{dx} \ln n_{o\alpha} + \frac{dT_\alpha}{dx} \frac{\partial}{\partial T_\alpha} \right) \\ z_\alpha &= \frac{k_y^2 T_\alpha}{m_\alpha \omega_{c\alpha}^2} \\ \xi_\alpha &= \frac{\omega}{\sqrt{2}k_z v_{T\alpha}^2} \end{aligned}$$

Further, in order to simplify our expressions, we use:

$$\frac{d}{dT_\alpha} \left( \frac{1}{v_{T\alpha}} \right) = -\frac{m_\alpha^{1/2}}{2 T_\alpha^{3/2}}; \quad \frac{dz_\alpha}{dT_\alpha} = \frac{k_y^2}{m_\alpha \omega_{c\alpha}^2}; \quad \frac{d\xi_\alpha}{dT_\alpha} = -\frac{\omega}{2k_z v_{T\alpha}^2 \sqrt{2m_\alpha T_\alpha}} \quad (5)$$

Now, using the following identity for the dispersion function  $\mathbf{Z}$

$$\boxed{\mathbf{Z}' = -2[1 + \xi_\alpha \mathbf{Z}(\xi_\alpha)],}$$

we obtain after several cumbersome algebraic manipulations the dielectric susceptibility in the form

$$\chi_{o\alpha} = \frac{1}{(k\lambda_{D\alpha})} \left[ 1 + \frac{\omega \mathbf{Z} I_{0\alpha} e^{-z_\alpha}}{\sqrt{2} k_z v_{T\alpha}} \left\{ 1 - \frac{k_y T_\alpha}{m_\alpha} \left( \frac{n'_{0\alpha}}{n_{0\alpha}} + T'_\alpha \left[ -\sqrt{\frac{m_\alpha}{T_\alpha^3}} \frac{v_{T\alpha}}{2} + \frac{\mathbf{Z}' \xi'}{\mathbf{Z}} + \frac{I'_0 z'_\alpha}{I_0} - z'_\alpha \right] \right) \right\} \right] \quad (6)$$

In order to put our dispersion relation in a dimensionless form, we introduce following suitable definitions:

$$\begin{aligned} \lambda_{D\alpha} &= \sqrt{\frac{T_\alpha}{n_{o\alpha} Z_\alpha^2 e^2}}; \quad K = k\lambda_{Di}; \quad \mu_\alpha = \frac{n_{o\alpha}}{n_{oi}} \\ \Theta_\alpha &= \frac{T_\alpha}{T_i}; \quad \omega_{c\alpha} = \frac{Z_\alpha e B}{m_\alpha}; \quad k\lambda_{D\alpha} = K \sqrt{\frac{\Theta_\alpha}{\mu_\alpha}} \\ \Omega &= \frac{\omega}{\omega_{pi}}; \quad \Omega_{c\alpha} = \frac{\omega_{c\alpha}}{\omega_{pi}}; \quad U_\alpha = \frac{v_{T\alpha}}{c_{si}} \end{aligned}$$

Now, using those results and assuming that  $\omega \ll \omega_{oi} \ll \omega_{od}$  we can write down Eq.(3) as

$$1 + \chi_{0e} + \chi_{0i} + \chi_{0d} = 0 \quad (7)$$

In the non ideal case (dust) we introduce a relation that in principle express the non ideal behavior of the system in terms of the pressure in the form

$$p = n_d^0 T_d (1 + b_d n_d^0) \quad (8)$$

given by the hard-core model. This model is taken for simplicity. A better model, as mentioned before, will be introduced in a future work. Now, following definitions are also useful

$$\frac{1}{L_p} = \frac{\nabla p_d}{p_d}; \quad \frac{1}{L_n} = \frac{\nabla n_d^0}{n_d^0}; \quad \frac{1}{L_d} = \frac{\nabla T_d}{T_d} \quad (9)$$

Those relations are very convenient by writing the full dispersion relation[4]. In fact we have

$$\frac{1}{L_p} = \frac{1 + 2b_d n_{0d}}{1 + b_d n_{0d}} \frac{1}{L_n} + \frac{1}{L_T}, \quad (10)$$

for the non-ideal case. For the ideal one, we use the well known relation  $p_{0j} = n_{0j} T_j$ , and in a similar way we get

$$\frac{1}{L_{p_j}} = \frac{1}{L_{n_j}} + \frac{1}{L_{T_j}} \quad (11)$$

where  $j = i, e$ . Two special cases can be worked out:

A) Density gradient equal to zero  $\nabla n_{0j} = 0$ , that means,  $L_{p_j} = L_{T_j}$ .

B) Temperature gradient equal to zero  $\nabla T_j = 0$ , that means,  $L_{p_j} = L_{n_j}$ .

Further we can introduce following relations in order to express dielectric susceptibilities in a suitable forms

$$\frac{n'_{0j}}{n_{0j}} = \frac{1}{L_{n_j}} \equiv \frac{1}{\Lambda_{n_j} \lambda_{Di}} \quad (12)$$

$$T'_j = \frac{T_j}{L_{T_j}} \equiv \frac{\Theta_j T_i}{\Lambda_{T_j} \lambda_{Di}} \quad (13)$$

Using those relations we arrive to the dispersion relation for the case B where we get:

$$\chi_{0e} = \frac{\mu_e}{K^2 \Theta_e} \left[ 1 + \frac{\Omega \mathbf{Z}_e I_{0e} e^{-z_e}}{\sqrt{2} K_z U_e} \left\{ 1 - \frac{K_y U_e^2}{\Omega \Omega_{0e}} \frac{1}{\Lambda_{n_e}} \right\} \right] \quad (14)$$

$$\chi_{0d} = \frac{\mu_d Z_d^2}{K^2 \Theta_d} \left[ 1 + \frac{\Omega \mathbf{Z}_d I_{0d} e^{-z_d}}{\sqrt{2} K_z U_d} \left\{ 1 - \frac{K_y U_d^2}{\Omega \Omega_{0d}} \frac{1}{\Lambda_{n_d}} \right\} \right] \quad (15)$$

$$\chi_{0i} = \frac{1}{K^2} \left[ 1 + \frac{\Omega \mathbf{Z}_i I_{0i} e^{-z_i}}{\sqrt{2} K_z U_i} \left\{ 1 - \frac{K_y U_i^2}{\Omega \Omega_{0i}} \frac{1}{\Lambda_{n_i}} \right\} \right] \quad (16)$$

where  $\Lambda_{n_d} = [(1 + 2b_d n_{0d}) / (1 + b_d n_{0d})] \Lambda_p$  and  $\Lambda_{n_j} = \Lambda_{p_j}$ .

In a similar way, it is possible to include the terms for case A, where we shall have

$$\Lambda_{n_j} = \Lambda_{p_j}. \quad (17)$$

Introducing now the multipolar approximation to  $\mathbf{Z}$  we can get a polynomial expression in the well known form[9]

$$\sum_i a_i \Omega^i / \sum_j b_j \Omega^j = 0 \quad (18)$$

where coefficients  $a_i$  and  $b_i$  are functions of the system parameters. Such an expression is easy to solve and with high accuracy to find roots of the numerator. An analysis of these solutions spectra permit us to give the imaginary parts  $\gamma = \text{Im}(\Omega)$  in function of  $1/K_y$ , which represent the growth rate instabilities.

### 3 Results and Conclusions

The quasi-neutrality equation for dusty plasmas can be approached by a simplified one due to the high state of charge of the dust grains

$$n_{oi} = Z_D n_{od} + n_{oe} \simeq Z_D n_{od} \quad (19)$$

and the electron susceptibility can be neglected in the dispersion relation. The range of the main parameters in the study of the low frequency oscillation of dust grains is established by the approximations that conduced to the simplified dispersion relation

$$\Omega, K_z U_d \ll \Omega_{cd} \quad (20)$$

Unstable dust oscillations ( $\text{Im}(\Omega) > 0$ ) are found for  $\Omega_{cd} \simeq 10^{-1}$ ,  $K_z U_d \simeq 10^{-2}$ . At the present time, we only give the results for the density gradient case (*i.e.*  $\partial/\partial T = 0$ ). For slightly inhomogeneous plasmas with normalized density gradient length  $\Lambda_n = n_{o\alpha}/(\lambda_{Di} \nabla n_{o\alpha}) \approx 10^2$ , the shape of the dust instability ( $\text{Im}(\Omega)_{\text{max}}$ ) curve as function of the perpendicular to magnetic field wavelength ( $1/K_y$ ) is similar to that for ions, previously studied [8].

The maximum value of the instability increases and narrows with the state of charge of the dust  $Z_D$  but decreases and get wider with the mass. For typical laboratory light dusty plasmas ( $m_d \sim 10^4 m_p$ ,  $Z_D \sim 10^3$ ) the instability of dust acoustic or electrostatic waves is narrower and smaller than that for ions In figure 1 the peak of the left corresponds to the typical shape of instability of slightly inhomogeneous plasmas, while the right region of instability appears for density gradient lengths of the order of a hundred of Debye lengths ( $\Lambda_n \equiv \Lambda \lesssim 10^2$ ). For higher density gradients ( $\Lambda = 5 \times 10^1$ ), this new instability region is wider and so high as the typical one. For even higher density gradients ( $\Lambda = 10^1$ ), figure 2 shows that the new right region gives a higher instability. This figure also shows the effect of the non ideality of the plasma. necessary condition for the exhibition of

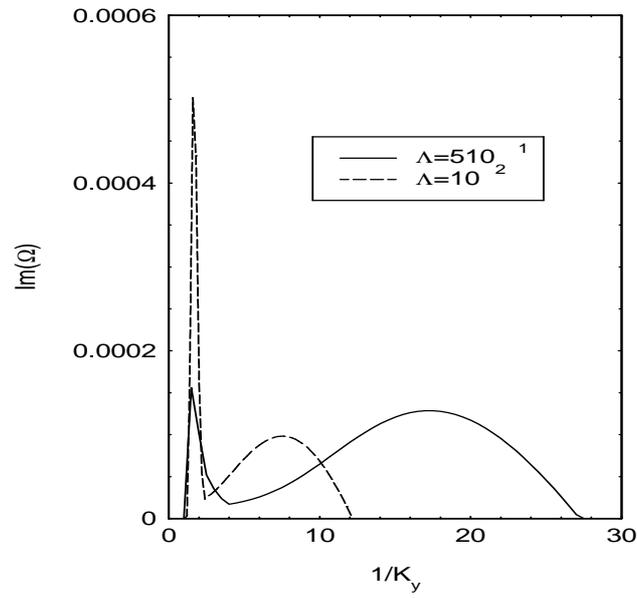


Figure 1: Normalized maximum growth rate as a function of normalized perpendicular wavelength for slightly inhomogeneous plasma ( $\Lambda = 10^2$ ) and for a relatively inhomogeneous one ( $\Lambda = 5 \times 10^1$ ).

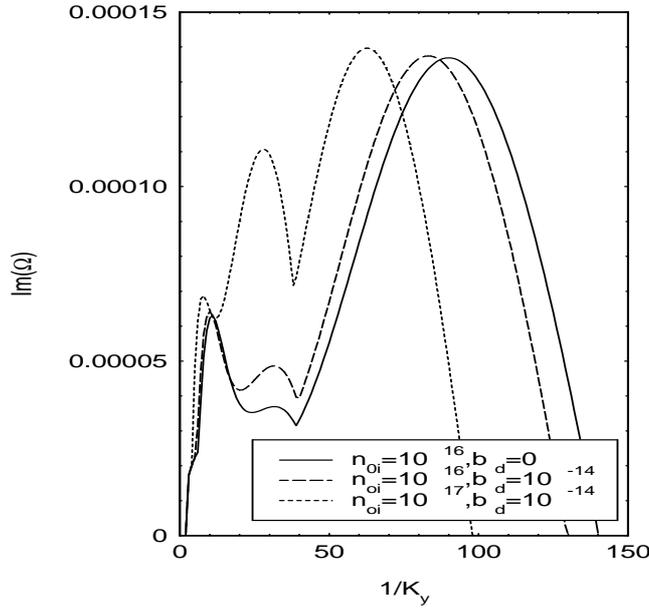


Figure 2: Normalized maximum growth rate as a function of normalized perpendicular wavelength for ideal and non ideal plasmas.

dust acoustic waves. For typical laboratory dust radius the  $b_{od}$  parameter of the hard core potential equation of state, is of the order of  $10^{-14}m^3$ . And for typical values of ion density of  $10^{16}m^{-3}$  (and corresponding  $n_{od}$ , by quasi-neutrality relation), it appears a new intermediate instability region which can reach a maximum for denser plasmas ( $n_{oi} = 10^{17}m^{-3}$ ) or larger dust particles ( $b_{od} \gtrsim 10^{-13}m^3$ ). This maximum is limited for the relation for dust collective behavior

$$r_d \ll \lambda_{Di} \quad (21)$$

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