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Quantum electrodynamical modes in pair plasmas

Mattias Marklund, Padma K Shukla, Gert Brodin, and
Lennart Stenflo

Department of Physics, Umeå University, SE-901 87 Umeå, Sweden

Abstract. We predict the existence of new nonlinear electromagnetic wave modes in pair plasmas. The plasma may be either non-magnetized or immersed in an external magnetic field. The existence of these modes depends on the interaction of an intense circularly polarized electromagnetic wave with a plasma, where the nonlinear quantum vacuum effects are taken into account. This gives rise to new couplings between matter and radiation. We focus on pair plasmas, since the new modes are expected to exist in highly energetic environments, such as pulsar magnetospheres and the next generation of laser-plasma systems.

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1. Introduction.

Quantum electrodynamics (QED) offers new phenomena with no classical counterparts, such as the Casimir effect. Similarly, and related to the Casimir effect, is so called photon-photon scattering (see, e.g., [1, 2, 3, 4]). The effective interaction between photons in a quantum vacuum is mediated by virtual electron-positron pairs, and therefore cannot occur within standard Maxwell electrodynamics. Photon-photon collisions have attracted much interest over the years, both from an experimental and an astrophysical point of view (see [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20] and references therein). The effect of photon-photon scattering could be of fundamental importance in high-intensity laser pulses, in ultra-strong cavity fields, in the surroundings of neutron stars and magnetars, and in the early Universe. However, the presence of plasmas in many highly energetic systems makes their theoretical analysis less tractable than the pure quantum vacuum model. Anyhow, we here present a theory of electromagnetic wave interaction in plasmas, taking photon-photon scattering into account. It is shown that under certain circumstances the weak QED effects will act to generate distinct new wave modes. Specifically, we focus on pair plasmas, and argue that our new modes could be of importance in the next generation of laser-plasmas, as well as in pulsar magnetospheres.

2. Basic equations.

The weak nonlinear self-interaction of photons in the quantum vacuum can be expressed in terms of the Heisenberg-Euler Lagrangian [1]

$$\mathcal{L} = \epsilon_0 \mathcal{F} + \kappa \epsilon_0^2 (4\mathcal{F}^2 + 7\mathcal{G}^2), \quad (1)$$

where $\mathcal{F} = -F_{ab}F^{ab}/4 = (E^2 - c^2B^2)/2$, $\mathcal{G} = -F_{ab}\widehat{F}^{ab}/4 = c\mathbf{E} \cdot \mathbf{B}$, and $\widehat{F}^{ab} = \epsilon_{abcd}F^{cd}/2$ is the dual of Maxwell's field strength tensor F_{ab} . Here $\kappa \equiv 2\alpha^2\hbar^3/45m_e^4c^5 \approx 1.63 \times 10^{-30} \text{ ms}^2/\text{kg}$, α is the fine-structure constant, \hbar is the Planck constant divided by 2π , m_e is the electron mass, and c is the speed of light in vacuum. The Lagrangian (1) is valid when

$$\omega \ll \omega_e \equiv m_e c^2 / \hbar, \quad |\mathbf{E}| \ll E_S \equiv m_e c^2 / e \lambda_c \quad (2)$$

respectively. Here e is the elementary charge, λ_c the Compton wavelength, ω_e the Compton frequency, and $E_S \simeq 10^{18} \text{ V/m}$ the Schwinger field strength. The first inequality states that the individual photons should not create real electron–positron pairs out of vacuum fluctuations, while the second states that the collective energy of many photons should not create real electron–positron pairs.

Using Eq. (1), the dispersion relation, in the absence of matter fields, for photons in a background electromagnetic field \mathbf{E} , \mathbf{B} is [5, 15]

$$\omega(\mathbf{k}, \mathbf{E}, \mathbf{B}) = c|\mathbf{k}| \left(1 - \frac{1}{2}\lambda|\mathbf{Q}|^2\right), \quad (3)$$

where

$$|\mathbf{Q}|^2 = \varepsilon_0 \left[E^2 + c^2 B^2 - (\hat{\mathbf{k}} \cdot \mathbf{E})^2 - c^2 (\hat{\mathbf{k}} \cdot \mathbf{B})^2 - 2c\hat{\mathbf{k}} \cdot (\mathbf{E} \times \mathbf{B}) \right], \quad (4)$$

and $\lambda = \lambda_{\pm}$, where $\lambda_+ = 14\kappa$ and $\lambda_- = 8\kappa$ for the two different polarisation states of the photon. Furthermore, $\hat{\mathbf{k}} \equiv \mathbf{k}/k$.

We may add the matter fields to the Lagrangian (1). Introducing the vector potential A^b , such that $F_{ab} = \partial_a A_b - \partial_b A_a$, Euler–Lagrange's equations give us the sourced Maxwell equations

$$\partial_b F^{ab} = 2\epsilon_0 \kappa \partial_b \left[(F_{cd} F^{cd}) F^{ab} + \frac{7}{4} (F_{cd} \widehat{F}^{cd}) \widehat{F}^{ab} \right] + \mu_0 j^a, \quad (5)$$

where j^a is the four current. Using the Lorentz gauge $\partial_b A^b = 0$, Eq. (5) yields

$$\left[1 - 2\epsilon_0 \kappa (F_{cd} F^{cd})\right] \square A^a = 2\epsilon_0 \kappa \left[F^{ab} \partial_b (F_{cd} F^{cd}) + \frac{7}{4} \widehat{F}^{ab} \partial_b (F_{cd} \widehat{F}^{cd}) \right] + \mu_0 j^a, \quad (6)$$

where $\square = \partial_a \partial^a$ is the d'Alembertian.

For a circularly polarized electromagnetic wave $\mathbf{E}_0 = E_0(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}) \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$ propagating along a constant magnetic field $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$, the invariants satisfy

$$F_{cd} F^{cd} = -2E_0^2 \left(1 - \frac{k^2 c^2}{\omega^2}\right) + 2c^2 B_0^2, \quad F_{cd} \widehat{F}^{cd} = 0, \quad (7)$$

where \mathbf{k} is the wave vector and ω the frequency of the circularly polarized electromagnetic wave. Using these expressions, Eq. (6) reduces to

$$\square A^a = -4\epsilon_0 \kappa \left[E_0^2 \left(1 - \frac{k^2 c^2}{\omega^2}\right) - c^2 B_0^2 \right] \square A^a + \mu_0 j^a, \quad (8)$$

which, together with the dynamical equations for the particle current j^a , is our main equation.

3. Unmagnetized plasmas

With $B_0 = 0$, the effect due to the presence of a plasma can be written in terms of a modified wave operator

$$\square \rightarrow \square - \frac{\omega_p^2}{c^2}, \quad (9)$$

where the plasma frequency is given by [21,22]

$$\omega_p = \sum_j \gamma_j^{-1/2} \omega_{pj} = \sum_j \left(\frac{n_{0j} q_j^2}{\epsilon_0 m_j \gamma_j} \right)^{1/2}, \quad (10)$$

where the sum is over particle species, q_j is the charge, m_j is the rest mass, n_{0j} denotes the particle density in the laboratory frame, and the relativistic factor of each particle species is $\gamma_j = (1 + q_j^2 E_0^2 / m_j^2 c^2 \omega^2)^{1/2}$. Making a harmonic decomposition of the fields, we see that Eq. (9) gives

$$\square - \frac{\omega_p^2}{c^2} = \frac{\omega^2 - \omega_p^2}{c^2} - k^2. \quad (11)$$

In the low frequency limit, $\omega^2 \ll k^2 c^2$, we obtain from (6) and (11) the nonlinear dispersion relation

$$\omega^2 = \frac{2\alpha}{45\pi} \left(\frac{E_0}{E_S} \right)^2 \frac{k^4 c^4}{\omega_p^2 + k^2 c^2}. \quad (12)$$

Next we focus our attention on a pair plasma. For an equal density (n_0) electron–positron plasma, with ultra-relativistic particle motion ($\gamma_e \gg 1$), we use the approximation $\omega_p^2 \approx 2\omega_{pe}^2 (\omega/\omega_e)(E_S/E_0)$, where $\omega_e = m_e c^2/\hbar$ and $\omega_{pe} = (e^2 n_0 / \epsilon_0 m_e)^{1/2}$. We then obtain [23]

$$\omega^3 = \frac{\alpha}{45\pi} \left(\frac{\omega_e}{\omega_{pe}} \right) \left(\frac{E_0}{E_S} \right)^3 \frac{k^4 c^4}{\omega_{pe} + (E_0/E_S)(k c \omega_e / 2 \omega \omega_{pe}) k c}. \quad (13)$$

from Eq. (12). We note that the Compton frequency ω_e is much larger than ω_{pe} for virtually all plasmas, and corresponds to electron densities up to $\sim 10^{38} \text{ m}^{-3}$.

If the amplitude of the vector potential varies slowly, we can derive a nonlinear Schrödinger equation by taking the media response into account. We may take the scalar potential $\phi = 0$. The weakly varying vector potential amplitude $\mathbf{A} = \mathbf{A}(t, z)$ then satisfies [24]

$$i \left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) \mathbf{A} + \frac{v_g'}{2} \frac{\partial^2 \mathbf{A}}{\partial z^2} + a(|\mathbf{A}|^2 - A_0^2) \mathbf{A} = 0, \quad (14)$$

where $v_g = \partial\omega/\partial k$, $v_g' = \partial^2\omega/\partial k^2$, $a = -\partial\omega/\partial A_0^2$, and $A_0^2 = E_0^2/\omega^2$. We note that the response from the medium through ω_{pe} depends on several parameters.

4. Magnetized plasmas

In the presence of a magnetic field, the plasma contribution to the dispersion relation can be obtained by substituting

$$\square \rightarrow \square - \sum_j \frac{\omega \omega_{pj}^2}{\omega \gamma_j \pm \omega_{cj}}, \quad (15)$$

for the d'Alembertian. The sum is over the plasma particle species j ,

$$\omega_{cj} = \frac{q_j B_0}{m_j}, \quad (16)$$

is the gyrofrequency, and

$$\gamma_j = (1 + \nu_j^2)^{1/2}, \quad (17)$$

is the the gamma factor of species j , with ν_j satisfying [21, 22]

$$\nu_j^2 = \left(\frac{eE_0}{cm_j} \right)^2 \frac{1 + \nu_j^2}{[\omega(1 + \nu_j^2)^{1/2} \pm \omega_{cj}]^2}. \quad (18)$$

Making a harmonic decomposition of the fields, and looking for low-frequency modes in an ultra-relativistic pair plasma, we use the approximations $\omega \ll kc$ and $\gamma_e \gg 1$, at which Eq. (6) together with (15) gives [25]

$$\frac{k^2 c^2}{\omega^2} \approx \frac{4\alpha}{45\pi} \left[\left(\frac{E_0}{E_S} \right)^2 \frac{k^2 c^2}{\omega^2} + \left(\frac{cB_0}{E_S} \right)^2 \right] \frac{k^2 c^2}{\omega^2} \mp \frac{\omega_{pe}^2}{\omega \omega_e} \frac{E_S}{E_0}. \quad (19)$$

In the limit of no photon–photon scattering, i.e. $\alpha \rightarrow 0$, we recover the modes found in Ref. [22].

Magnetized pair plasmas can be found in the surroundings of pulsars and strongly magnetized stars, e.g. in the form of accretion disks. At a distance from the star's surface, the magnetic field will be weak, being essentially dipole in character, and the first term in the square bracket of Eq. (19) will be the dominant QED contribution.

Close to neutron stars or magnetars, the magnetic field strengths are in the range $10^6 - 10^{11}$ T [26, 27], and, depending on the frequency of the circularly polarized wave, the second term in the square bracket of Eq. (19) may dominate the behavior of the wave mode. If $cB_0 \gtrsim E_S$, we have

$$\omega \approx \mp \omega_e \frac{E_0}{E_S} \left[1 - \frac{4\alpha}{45\pi} \left(\frac{cB_0}{E_S} \right)^2 \right] \left(\frac{kc}{\omega_{pe}} \right)^2. \quad (20)$$

5. Discussion and conclusion

Most situations in which photon–photon scattering can be important are of an extreme nature. Examples of environments where the effects may either be dynamically significant, or measurable, are the next generation of high power lasers and possibly their combination with plasmas into laser–plasma systems [12, 20], high field superconducting cavities [14], and astrophysical environments, such as pulsar magnetospheres [26] and the vicinity of magnetars [27]. In astrophysical environments, effects such as photon splitting or magnetic lensing have been suggested to take place [5, 6, 7, 16]. Even in cosmology, the effects of photon–photon scattering could be detectable using precision observations of the cosmic microwave background [28, 19].

However, plasmas may in many circumstances be a prominent component of the physical systems considered above. For example, it is currently believed that the highest experimental field strengths could be obtained using laser–plasma systems [29]. Therefore, the addition of plasmas to the dynamics of photon–photon scattering adds an important piece to our understanding of the nonlinear quantum vacuum, and as shown here, could provide a unique signature of photon–photon scattering. It remains to be seen whether this can be realized in a laboratory or in astrophysics.

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