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# Nonperturbative functional renormalization group for random field models: the way out of dimensional reduction

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We have developed a non-perturbative functional renormalization group approach for the random field  $O(N)$  model (RFO(N)M) that allows us to investigate the ordering transition in any dimension and for any value of  $N$  including the Ising case. We show that the failure of dimensional reduction and standard perturbation theory is due to the non-analytic nature of the zero-temperature fixed point controlling the critical behavior, non-analyticity which is associated with the existence of many metastable states. We find that this non-analyticity leads to critical exponents differing from the dimensional reduction prediction only below a critical dimension  $d_c(N) < 6$ , with  $d_c(N = 1) > 3$ .

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After decades of intensive investigation, the nature and properties of the phase transition in systems with quenched disorder remains a much debated topic. Among the controversial issues are the critical behavior and the phase diagram of the random field Ising model (RFIM) [1]. For the RFIM, the diagnosis is well established: standard perturbation theory “exactly” (*i.e.* to all orders) predicts an equivalence between the critical behavior of the RFIM in dimension  $d$  and that of the corresponding pure Ising model in dimension  $d - 2$  (“dimensional reduction”) [2, 3], which contradicts rigorous results showing that the lower critical dimension of the RFIM is 2 and not 3 [4]. The failure of dimensional reduction (DR) and conventional perturbation theory is attributed to the existence of a “complex energy landscape” characterized by many metastable states [5]. However, limited progress has been made for developing a theory able to cope with metastability in random field systems, especially for the physically relevant case of the RFIM in  $d = 3$ . The most significant step has been the formulation of a perturbative renormalization group (RG) approach for the RFO( $N > 2$ )M near the lower critical dimension  $d = 4$  [6]. Already at leading order, the calculation includes an infinite set of marginal operators near the zero-temperature fixed point controlling the critical behavior, hence the name functional renormalization group (FRG). This perturbative FRG has been further developed to study equilibrium and out-of-equilibrium properties of elastic manifolds in disordered medias [8, 9, 10, 11], and it has been shown that the large scale behavior of these systems is controlled by non-analytic renormalized actions, with non-analyticities encoding the effect of the many metastable states at zero temperature.

In this letter, we develop a non-perturbative FRG for the RFO( $N$ )M by combining the ideas of the perturbative FRG for disordered systems with the formalism of the non-perturbative RG for the effective average action, based on an exact equation (ERGE) [12]. We implement a tractable approximation scheme that allows us to recover the perturbative results in the relevant limits

(most importantly, the perturbative FRG at first order in  $\epsilon = d - 4$  for  $N > 2$  [6, 7]) and to describe, approximately but non-perturbatively, the ordering transition in the whole  $d - N$  diagram, including  $N = 1$  and  $d = 3$ . This provides a comprehensive picture of the critical behavior of random field systems. We find in particular that this latter is controlled by a zero-temperature fixed point at which the renormalized effective average action is non-analytic (albeit in a more complex way than in the random elastic manifold case [8–11]); however, the DR prediction for the critical exponents breaks down only below a critical dimension  $d_c(N) < 6$ , with  $d_c(N = 1) > 3$ .

We start with the standard effective hamiltonian for the RFO( $N$ )M in  $d$  dimensions with an  $N$ -component field  $\chi(x)$  coupled to a random field  $h(x)$  with zero mean and variance  $\overline{h_\mu(x)h_\nu(y)} = \Delta\delta_{\mu\nu}\delta(x - y)$ . For convenience, we derive the ERGE for disorder averaged functions within the replica formalism, but it could similarly be obtained by using *e.g.* the dynamic formulation. We thus consider the “replicated” action

$$S_n[\{\chi_a\}, \{J_a\}] = \int d^d x \left\{ \frac{1}{2T} \sum_{a=1}^n \left[ \partial\chi_a \cdot \partial\chi_a + \tau\chi_a^2 + \frac{u}{12}\chi_a^4 \right] - \frac{\Delta}{2T^2} \sum_{a,b=1}^n \chi_a \cdot \chi_b - \sum_{a=1}^n \chi_a \cdot J_a \right\}, \quad (1)$$

where we have introduced sources acting separately on each replica, which therefore explicitly breaks the permutation symmetry between the replicas. One can associate to the above action the thermodynamic potential  $W_n[\{J_a\}] = \log Z_n[\{J_a\}]$  where  $Z_n[\{J_a\}] = \int \prod_{a=1}^n \mathcal{D}\chi_a \exp(-S_n[\{\chi_a\}, \{J_a\}])$ , and its Legendre transform, the effective action  $\Gamma_n[\{\phi_a\}] = -W_n[\{J_a\}] + \int d^d x \sum_{a=1}^n \phi_a \cdot J_a$  which is the generating functional of the vertex functions. In the following, we drop the subscript  $n$ .

To investigate the phase diagram and critical behavior of the model, we use an ERGE for the effective average action [12]. An effective average action  $\Gamma_k$  at the running scale  $k$  is obtained by integrating out fluctuations

with momenta  $q \gtrsim k$  via the introduction of an infra-red (IR) cutoff function  $R_k(q)$ ;  $\Gamma_k$  continuously interpolates between the bare action, eq. (1), at the microscopic scale  $k = \Lambda$  and the usual effective action when  $k \rightarrow 0$ . It follows an exact flow equation,

$$\partial_t \Gamma_k[\{\phi_a\}] = 1/2 \text{Tr} \partial_t R_k(q) (\Gamma_k^{(2)}[\{\phi_a\}, q] + \mathbb{1} R_k(q))^{-1} \quad (2)$$

where  $\partial_t$  is a derivative with respect to  $t = \ln(k/\Lambda)$ ,  $\mathbb{1}$  is the unit matrix with elements  $(2\pi)^d \delta^{(d)}(q - q') \delta_{ab} \delta_{\mu\nu}$ ,  $\Gamma_k^{(2)}$  is the tensor formed by the second functional derivative of  $\Gamma_k$  with respect to the fields  $\phi_a^\mu(q)$  and  $\phi_b^\nu(q')$ , and the trace involves an integration over momenta as well as a sum over replica indices and  $N$ -vector coordinates.

Eq. (2) is a complicated functional integro-differential equation that cannot be solved exactly but provides a convenient starting point for non-perturbative approximation schemes. One such scheme that efficiently deals with the momentum dependence of the vertex functions is the derivative expansion [12]. However, disordered systems require more because inversion of the matrix involving  $\Gamma_k^{(2)}$  in eq (2) is a difficult task as far as the replica indices are concerned for non-integer values of  $n$ . As in the perturbative FRG approach [10], we follow the route that consists in expanding all functions of the replica fields  $\{\phi_a\}$  in increasing number of free replica sums. Illustrated for the potential  $\mathcal{U}_k(\{\phi_a\})$  at the running scale  $k$  (*i.e.* the effective average action for uniform fields), this gives:

$$\mathcal{U}_k(\{\phi_a\}) = \sum_{a=1}^n U_k(\phi_a) - \frac{1}{2} \sum_{a,b=1}^n V_k(\phi_a, \phi_b) + \dots \quad (3)$$

where  $U_k, V_k, \dots$  are continuous functions of their arguments and satisfy the replica permutation symmetry. When all replica fields are equal, each free replica sum brings a factor of  $n$  and the procedure amounts to an expansion in powers of  $n$  (with  $n \rightarrow 0$ ).

Within this framework, the simplest truncation for  $\Gamma_k$  that already contains the main ingredients for a non-perturbative approach of the RFO(N)M is the following:

$$\Gamma_k[\{\phi_a\}] = \int d^d x \left\{ \sum_{a=1}^n \left( \frac{1}{2} Z_{m,k} \partial \phi_a \cdot \partial \phi_a + U_k(\phi_a) \right) - \frac{1}{2} \sum_{a,b=1}^n V_k(\phi_a, \phi_b) \right\} \quad (4)$$

with one single wave function renormalization for all fields,  $Z_{m,k}$ , which is defined as the derivative w.r.t.  $q^2$  of  $\Gamma_k^{(2)}$  evaluated at  $q = 0$  for a (non-zero) field configuration  $\phi_m$  that minimizes the 1-replica potential  $U_k$ : this is the so-called pseudo first-order of the derivative expansion [12]. With the above truncation that keeps only the first two terms of the expansion in the free replica

sums, the ERGE for  $\Gamma_k$ , eq. (2), can be reduced to coupled partial differential equations for the *functions*  $U_k(\phi)$  and  $V_k(\phi, \phi')$ , whereas a running anomalous dimension is defined as  $\eta_k = -\partial_t \log Z_{m,k}$ . The details will be given elsewhere.

To study the critical behavior associated with the ordering transition and search for fixed points (FP) of the flow equations, we introduce as usual renormalized dimensionless quantities. However, anticipating that the putative FP is expected at zero temperature [13, 14], it is convenient to make explicit the flow of a running temperature and the associated exponent. For simplicity, let discuss first the RFIM. We define a renormalized disorder correlation function  $\Delta_k(\phi, \phi') = \partial_\phi \partial_{\phi'} V_k(\phi, \phi')$  and a renormalized disorder strength  $\Delta_{m,k} = \Delta_k(\phi_m, \phi_m)$ . A running temperature can now be defined by  $T_k = Z_{m,k} k^2 \Delta / (\Lambda^2 \Delta_{m,k})$ : when  $k = \Lambda$ , it reduces to  $T_\Lambda = T$  (since from eq. (1),  $Z_\Lambda = 1/T$  and  $\Delta_{m,\Lambda} = \Delta/T^2$ ). An associated running exponent is obtained from  $\theta_k = \partial_t \log T_k$ . By using the definition of  $\eta_k$ , one may alternatively introduce an exponent  $\bar{\eta}_k = -\theta_k + 2 + \eta_k$  and compute it from the equation  $\bar{\eta}_k - 2\eta_k = \partial_t \Delta_{m,k}$ . Dimensionless quantities (noted by lower cases) appropriate for looking for a zero temperature FP are then:  $\varphi = (T_k Z_k k^{-(d-2)})^{1/2} \phi$ ,  $u_k(\varphi) = T_k k^{-d} U_k(\phi)$ ,  $v_k(\varphi, \varphi') = T_k^2 k^{-d} V_k(\phi, \phi')$ , and  $\delta_k(\varphi, \varphi') = \partial_\varphi \partial_{\varphi'} v_k(\varphi, \varphi')$ . The procedure can be extended to the RFO(N)M. It is however more convenient in this case to introduce the variables  $\rho = \varphi^2/2$  and  $z = \varphi \cdot \varphi' / (4\rho\rho')^{1/2}$ . In scaled form, the flow equations for  $u_k(\rho)$  and  $v_k(\rho, \rho', z)$  read (for simplicity, we drop the subscript  $k$  for all quantities but  $T_k$ ):

$$\begin{aligned} \partial_t u = & (2 - d + \eta - \bar{\eta})u + (d - 4 + \bar{\eta})\rho u_\rho \\ & + 2v_d[(N - 1)\delta_T l_1^d(w_T) + \delta_L l_1^d(w_L)] \\ & + 2v_d T_k[(N - 1)l_0^d(w_T) + l_0^d(w_L)] \end{aligned} \quad (5)$$

$$\begin{aligned} \partial_t v = & (4 - d + 2\eta - 2\bar{\eta})v + (d - 4 + \bar{\eta})(\rho v_\rho + \rho' v_{\rho'}) \\ & - v_d \left\{ (N - 1) \left[ (2\rho v_\rho - z v_z) \delta_T l_2^d(w_T)/\rho + (2\rho' v_{\rho'} - z v_z) \delta_T l_2^d(w_T')/\rho' + v_z^2 l_{1,1}^d(w_T, w_T')/(2\rho\rho') \right] \right. \\ & + (1 - z^2) \left[ v_{zz} (\delta_T l_2^d(w_T)/\rho + \delta_T l_2^d(w_T')/\rho') + 2\rho v_{\rho z}^2 l_{1,1}^d(w_T', w_L)/\rho' + 2\rho' v_{\rho' z}^2 l_{1,1}^d(w_T, w_L')/\rho - (v_z^2 + 2z v_z v_{zz} - (1 - z^2) v_{zz}^2) l_{1,1}^d(w_T, w_T')/(2\rho\rho') \right] \\ & \left. + 2 \left[ \delta_L (v_\rho + 2\rho v_{\rho\rho}) l_2^d(w_L) + \delta_L (v_{\rho'} + 2\rho' v_{\rho'\rho'}) l_2^d(w_L') + 4\rho\rho' v_{\rho\rho'}^2 l_{1,1}^d(w_L, w_L') \right] \right\} - T_k v_d \\ & \left\{ (N - 1) \left[ (2\rho v_\rho - z v_z) l_1^d(w_T)/\rho + (2\rho' v_{\rho'} - z v_z) l_1^d(w_T')/\rho' \right] + (1 - z^2) v_{zz} \left[ l_1^d(w_T)/\rho + l_1^d(w_T')/\rho' \right] + 2 \left[ (v_\rho + 2\rho v_{\rho\rho}) l_1^d(w_L) + (v_{\rho'} + 2\rho' v_{\rho'\rho'}) l_1^d(w_L') \right] \right\} \end{aligned} \quad (6)$$

where the indices  $\rho, \rho', z$  indicate derivatives w.r.t.  $\rho, \rho', z$ , and  $v_d^{-1} = 2^{d+1}\pi^{d/2}\Gamma(d/2)$ ;  $w_{T,L} = w_{T,L}(\rho)$ ,  $w'_{T,L} = w_{T,L}(\rho')$ , and similarly for  $\delta_{T,L}$  and  $\delta'_{T,L}$ , where  $w_T(\rho) = u_\rho(\rho)$  and  $w_L(\rho) = u_\rho(\rho) + 2\rho u_{\rho\rho}(\rho)$  are the transverse and longitudinal masses, whereas  $\delta_T(\rho) = 1/(2\rho)v_z(\rho, \rho, 1)$  and  $\delta_L(\rho) = 2\rho v_{\rho\rho'}(\rho, \rho, 1)$  are the transverse and longitudinal disorder correlation functions when  $\varphi = \varphi'$ . For brevity we have omitted the arguments  $\rho$  and  $\rho', z$  of all functions. Finally,  $l_p^d(w)$  and  $l_{p,p'}^d(w, w')$  are the so-called dimensionless threshold functions, that essentially encode the non-perturbative effects beyond the standard one-loop approximation: their definition and properties are discussed at length in ref. [12]. For the present work, we choose for IR cutoff function  $R_k(q) = Z_{m,k}(k^2 - q^2)\Theta(k^2 - q^2)$  where  $\Theta$  is the Heaviside function. When  $N = 1$  and  $z = \pm 1$ , one recovers the RFIM ( $\rho$  and  $\rho'$  being used in place of  $\varphi$  and  $\varphi'$ ).

The above flow equations are supplemented by equations for  $\eta_k$  and  $\bar{\eta}_k$ . For lack of space, we just give here the equation obtained for  $2\eta_k - \bar{\eta}_k = -\partial_t \Delta_{m,k}$  in the case of the RFIM:

$$\begin{aligned} 2\eta_k - \bar{\eta}_k = & 2v_d \left\{ l_4^d(u_m'') u_m'''^2 - 4l_3^d(u_m'') u_m''' \delta_m' \right. \\ & + l_2^d(u_m'') (\delta_m'' + \frac{3}{2} \delta_m'^2 - \frac{u_m'''}{u_m''} \delta_m - \frac{1}{4} \Sigma_m) + l_1^d(u_m'') \frac{\delta_m'}{u_m''} \\ & \left. - T_k [l_2^d(u_m'') u_m''' \delta_m' - l_1^d(u_m'') (\frac{1}{2} \delta_m'' - \frac{u_m'''}{u_m''} \delta_m' + \frac{1}{2} \tilde{\Sigma}_m)] \right\} \end{aligned} \quad (7)$$

with  $\Sigma(\varphi) = \lim_{\varphi \rightarrow \varphi'} (\partial_\varphi - \partial_{\varphi'})^2 (\delta(\varphi, \varphi') - \delta(\varphi, \varphi))^2$  and  $\tilde{\Sigma}(\varphi) = \lim_{\varphi \rightarrow \varphi'} (\partial_\varphi - \partial_{\varphi'})^2 \delta(\varphi, \varphi')$ ; the subscript  $m$  indicates that the functions are evaluated for fields equal to  $\varphi_m$  and primes indicate derivatives w.r.t.  $\varphi$ . Note the appearance of the “anomalous” terms  $\Sigma_m$  and  $T_k \tilde{\Sigma}_m$  that can only differ from zero when a non-analyticity (a “cusp”) in  $\varphi - \varphi'$  appears in the renormalized disorder function  $\delta(\varphi, \varphi')$  when  $\varphi' \rightarrow \varphi$ . Actually, if  $\delta(\varphi, \varphi')$  is analytic when  $\varphi' \rightarrow \varphi$ , the flow equations for  $u(\varphi)$  and  $\delta(\varphi, \varphi)$  can be closed by taking from the beginning the replica symmetric limit: this analytic behavior in the vicinity of replica symmetry is precisely what is implied by the standard perturbation theory. *In our formalism, breakdown of DR thus implies the emergence of a non-analyticity in the renormalized disorder correlation function.* Note also that if a FP is found to eqs. (5,6), and provided that  $\theta = 2 - \eta + \bar{\eta} > 0$ , it is at zero temperature and temperature is irrelevant (albeit dangerously): indeed, in the vicinity of the FP,  $T_k$  flows to zero as  $k^\theta$  when  $k \rightarrow 0$ . In most of the following, we will consider directly the  $T = 0$  limit, which allows to drop all terms proportional to  $T_k$  in the above equations.

Because of their structure, the above non-perturbative FRG equations reproduce all perturbative one-loop results in their range of validity, in particular the  $\epsilon = 6 - d$  expansion at first order and the  $N = \infty$  limit; a stronger property is that one also recovers the per-

turbative FRG equation at first order in  $\epsilon = d - 4$  for the RFO( $N > 2$ )M [6]; In this case,  $d = 4$  being the lower critical dimension, the FP occurs (as for the pure system) for a value of  $\rho_m$  that goes to infinity as  $1/\epsilon$ . One can thus organize a systematic expansion in powers of  $1/\rho_m$ . The longitudinal mass becomes very large around  $\rho_m$ , and by using the known asymptotic properties of the threshold functions for large arguments [12], one can derive the flow equations for  $\rho_m$  (obtained from eq. (5) and the condition  $u'(\rho_m) = 0$ ) and for the function  $R(z) = v(\rho_m, \rho_m, z)/(2\rho_m^2)$  (recall that since  $\delta_{T,m} = 1$  by construction,  $R'(1) = 1/\rho_m$ ). This latter reads

$$\begin{aligned} \partial_t R(z) = & (\epsilon + 2\eta)R(z) - C_4/2 \left\{ (N-1)[R'(z)^2 \right. \\ & + 2R'(1)(2R(z) - zR'(z))] + (1-z^2)[-R'(z)^2 \\ & \left. + (1-z^2)R''(z)^2 + 2(R'(1) - zR'(z))R''(z)] \right\} \end{aligned} \quad (8)$$

where  $C_4 = 2v_4 l_2^4(0) = (16\pi^2)^{-1}$ , irrespective of the choice of the IR cutoff function; in addition, the exponent  $\eta$  is given by  $\eta \simeq C_4 R'(1)$ . The above equations are identical to the FRG equations at order  $\epsilon$  first derived by D. Fisher [6].

The main advantage of the non-perturbative FRG that we have developed is that the mechanism by which DR and conventional perturbation theory break down can be studied in the whole  $d - N$  diagram. Although we have not yet obtained the full numerical solutions to eqs. (5,6), partial solutions and analyses lead us to propose the following picture. (1) Except when  $N = \infty$  and  $d \geq 6$ , the analytic FP found in perturbation theory is never stable (more precisely once unstable). (2) The stable FP is characterized by a renormalized disorder correlation function, involving 2 fields  $\varphi$  and  $\varphi'$ , which is non-analytic near  $\varphi = \varphi' \simeq \varphi_m$ : for instance, in the RFIM  $\delta(\varphi, \varphi')$  is non-analytic in  $(\varphi - \varphi')$  and for the RFO( $N$ )M near  $d = 4$ ,  $R(z)$  is non analytic in  $(1 - z)$ ; more generally, the non-analyticity appears in the variable  $(\varphi - \varphi')^2$ . (3) The power exponent characterizing the non-analyticity increases discontinuously with  $N$  and  $d$ : e.g., near  $d = 4$  for  $N > 2$  there is a cusp  $R'(z) \sim (1 - z)^{1/2}$  for  $2 < N \leq 18$ , a “sub-cusp”  $R'(z) \sim (1 - z)^{3/2}$  for  $18 < N \lesssim 18.045$ , and so on, and at large  $N$  the exponent varies as  $N/2$ ; a similar trend is observed with increasing  $d$ , and in  $d = 6 - \epsilon$ , the exponent goes as  $1/\epsilon^2$  (a result quite similar to that of Feldman [7]). (4) Only when a cusp is present do the critical exponents change from their DR value, so that there is a critical line  $d_c(N)$  (or  $N_c(d)$ ) separating the two regions of the  $d - N$  plane in which the exponents are equal ( $d > d_c(N)$ ) or not ( $d < d_c(d)$ ) to the DR predictions. To locate this line, it is sufficient to study the flow of the second derivative  $\partial^2 v / \partial((\varphi - \varphi')^2)^2$  when  $\varphi = \varphi'$  under the assumption of analytic behavior: the appearance of a cusp, or equivalently of a term in  $|\varphi - \varphi'|^3$  in  $v$ , is then signalled by a divergence in the second derivative. To simplify the analysis, we have used an expansion in pow-

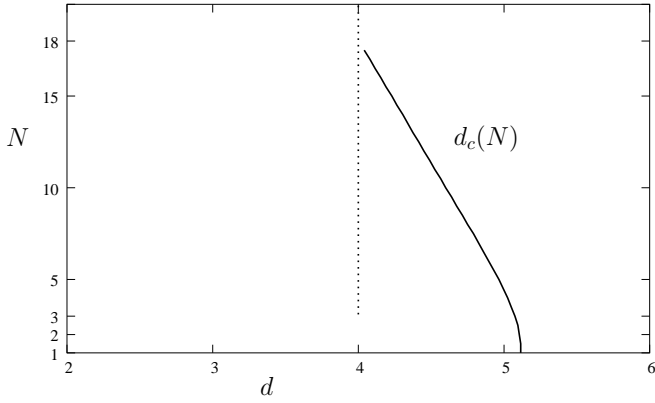


FIG. 1: Location of the critical line  $d_c(N)$  below which a cusp appears and exponents differ from their DR values.

ers of the fields (including all terms up to  $\phi^4$ ) around  $\varphi_m$ :  $u = u_2(\rho - \rho_m)^2$ ,  $v = 2v_1(\sqrt{\rho\rho'}z - \rho_m) + v_2(\rho + \rho' - 2\rho_m)^2 + v_3(\rho - \rho')^2 + v_4(\sqrt{\rho\rho'}z - \rho_m)^2 + v_5(\sqrt{\rho\rho'}z - \rho_m)(\rho + \rho' - 2\rho_m)$ , where  $v_1 = 1$  by construction. The result is shown in fig. 1: when  $d \rightarrow 4$ , one recovers that the critical  $N$  is 18 (see above and ref. [6]) and for the RFIM one finds  $d_c(N) \simeq 5.1$  (within the present approximation). (5) Below  $d_c(N)$ , the  $T = 0$  “cuspy” FP is associated with critical exponents differing from their DR value; the simplest illustration is for the RFO( $N > 2$ )M at first order in  $\epsilon = d - 4$ : see fig. 2 where we display  $\eta$  and  $\bar{\eta}$  normalized by their DR value  $\bar{\eta} = \eta = \epsilon/(N - 2)$  as a function of  $N$  (for  $N = 3, 4, 5$ , they agree with those of ref. [7]). (6) For any finite temperature  $T_k$ , the cusp is rounded: this can be inferred, *e.g.*, from the term  $T_k \tilde{\Sigma}_m$  in eq. (7) that must stay finite as one approaches the FP. As in the random elastic manifold problem [9, 11], temperature prevents the flow from being non-analytic at any finite scale and one may expect that the rounding of the cusp involves a boundary layer as  $T_k \rightarrow 0$  and one approaches the FP.

The present non-perturbative FRG approach of random field systems provides a consistent and global pic-

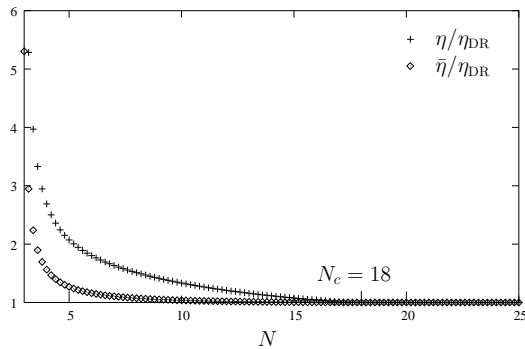


FIG. 2: Exponents  $\eta$  and  $\bar{\eta}$  (normalized to their DR expression) vs  $N$  for the RFO( $N > 2$ )M at first order in  $\epsilon = d - 4$ .

ture of the critical behavior associated with the ferromagnetic ordering transition. The failure of DR and standard perturbation theory comes from the existence of many metastable states, but the mechanism by which this occurs is rather subtle: metastability results from an interplay between ferromagnetic ordering and disorder, and it plays a role at large scale because the fixed point occurs at  $T = 0$ . This interplay leads to an effective renormalized *random potential* (beyond the bare random field term) that displays many minima. The renormalized disorder correlation function, which is the second derivative of the second cumulant of this random potential, acquires a non-analyticity (a cusp in low enough  $d$ ) as it flows to the  $T = 0$  fixed point. The physics of such cusps has been discussed in the context of random elastic manifolds [8, 9, 10, 15]; but in the present case the cusp only occurs when the system is in the vicinity of the minimum of the *non-random potential*. Work is in progress to obtain the full numerical solution of the flow equations and compute the critical exponents, as well as investigate the connection of the present theory with replica symmetry breaking approaches [16, 17] and possible formation of bound states [17, 18].

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