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# Transport and vortex pinning in micron size superconducting Nb films

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We have carried out Hall measurements on thin films of Nb in the flux flow regime. The Hall bars were several microns in scale. Oscillations with magnetic field in the transverse and longitudinal resistances between the depinning field  $B_d$  and the upper critical field  $B_{c2}$  are observed below  $T_c$ . The Hall effect may even change sign. The transverse and longitudinal resistances are interpreted in terms of current-driven motion of vortices in the presence of a few impurities. Simulations from time-dependent Ginzburg-Landau equations (TDGL) confirm this argument.

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The conductivity in the flux-flow regime of type-II superconductors is determined by the dynamics of vortices. Since vortex motion in an applied external magnetic field is intrinsically at least two dimensional, to understand the transport the full conductivity tensor is needed [1, 2, 3]. In past studies of vortex matter [4], the current-voltage curves have been observed to have interesting effects such as aging and memory phenomena. Together with effects of hysteresis in the magnetization curves, these have been interpreted as the consequences of the motion of interacting vortices in a complex energy landscape with random potential. This provides a general explanation of memory effects, slow relaxation rates and sensitivity to initial conditions. In this paper we present experimental evidence of similar memory and pinning effects in Hall and resistance measurements of type-II thin films, using samples of micron dimension well below  $T_c$ . By relating these results to simulations of the dynamics of vortices interacting with one another and a vortex scattering potential, we will deduce a length

scale for the vortex pinning potential.

Up to now, observed anomalies in the Hall effect close to  $T_c$  in bulk samples have been explained as being related to vortex motion damping [5] or plastic flow of the vortex lattice [6]. As we diminish the sample size, one would expect to reach a regime where the motion of a finite number of vortices must be considered. In order to simplify full vortex dynamics, in which one must consider motions of the vortex lines, we have fabricated planar samples with thickness  $L_z = 800 - 1000\text{\AA}$ , comparable to the coherence length  $\xi$  at 4K. In this case the vortex core can be considered as a disk. We took a conventional superconductor, Nb, with samples as pure as possible to minimize pinning. Nb has the advantage that the bulk vortex structure has been studied in detail by neutron scattering [7] and microscopy [8]. The epitaxial samples (100) were grown by DC magnetron sputtering on R-plane sapphire at 600°C and the eight-lead Hall bars processed by photolithography and reactive-ion etching (the bar area is  $L_x \times L_y = 50\mu\text{m} \times 5\mu\text{m}$ ; the resistance probes, which are micron wide, are separated by  $L_p = 10\mu\text{m}$ ). DC Hall resistance  $R_{xy}$  and magnetoresistance  $R_{xx}$  measurements were made in the flux-flow regime with current reversal in the  $B$  transverse configuration. To eliminate the effects of contact misalignment, the Hall resistance

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was obtained by subtracting the positive and negative magnetic field data. The films had  $T_c = 8.4\text{K}$  and a resistive ratio  $\rho_{xx}(300\text{K})/\rho_{xx}(10\text{K}) = 5.1$  with a carrier density  $n_e \simeq 8.5 \times 10^{22}\text{cm}^{-3}$ , obtained from the expression  $R_{xy} = B/en_e L_z$  for the Hall resistance in the normal phase. The low resistance contacts on the Hall bars were made by Indium bonding. The measurements were made with a Quantum Design Magnetic Property Measurement System adapted for transport measurements, giving high stability as a function of magnetic field and temperature. Figure 1 shows magnetoresistance as a function of magnetic field at  $T = 4\text{K}$ , well below  $T_c$  at current  $I = 200\mu\text{A}$ . The resistance  $R_{xx}$  is zero for fields below  $B_d$ , the depinning field for vortices, and then increases to reach its normal state value at  $B_{c2}$ . From the upper critical field experimental value  $B_{c2}(4\text{K}) \simeq 1.3\text{T}$ , we use the Ginzburg-Landau theoretical expression  $B_{c2}(T) = \phi_0/(2\pi\xi^2(T))$ , where  $\phi_0$  is the flux quantum, to deduce approximately the zero temperature coherence length  $\xi_0 \simeq 115\text{\AA}$ . From the carrier density  $n_e$ , the zero temperature London penetration length can be estimated using the expression  $\lambda_0 = \sqrt{m_e/\mu_0 n_e e^2}$ , which gives  $\lambda_0 \simeq 180\text{\AA}$ , where  $m_e$  is the electron mass, and we obtain  $\kappa \simeq 1.6$ . The electronic mean free path  $l_e$  is computed from the normal resistance  $R_{xx}$  at 4K using the Drude formula  $l_e \simeq (1.264 \times 10^4 \Omega) n_e^{-2/3} L_p / (R_{xx} L_z L_y)$ [9]. The value of the normal resistance at 4K is  $R_{xx} \simeq 1.7\Omega$ , based on Fig. 1 data, which gives  $l_e \simeq 76\text{\AA}$  at this temperature, of the same order as the coherence length. The relatively large value of  $l_e$  for these thin films shows the good quality of the samples. The transverse resistance displays a more striking behaviour; it oscillates strongly between the depinning field  $B_d$  and  $B_{c2}$ . Above  $B_{c2}$  the Hall resistance recovers the behavior of the normal state: it is linear with field, with slope inversely proportional to the Nb carrier density. In the field range where  $R_{xy}$  oscillates, this resistance may even be negative. As seen in Fig. 2, in general a minimum of  $R_{xy}$  corresponds to a maximum of  $dR_{xx}/dB$ . If  $R_{xy}$  were dominated by a parasitic resistive component, this would tend to give maxima in  $R_{xy}$  where the oscillating part of  $R_{xx}$  is maximal, ie where  $dR_{xx}/dB$  is between extrema. Subtraction of positive and negative magnetic field Hall data was important to avoid such parasitic effects. We note that to observe the oscillations the magnetic field had to be swept very slowly: typically at least 12 hours per scan for a given temperature.

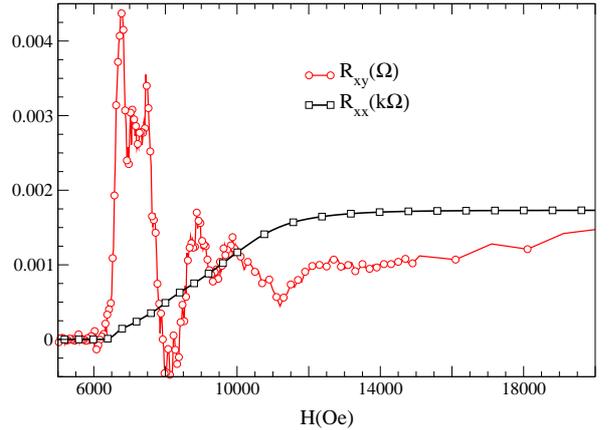


FIG. 1: Resistances for a sample size  $L_p \times L_y = 10\mu\text{m} \times 5\mu\text{m}$ ,  $T=4\text{K}$ ,  $I=200\mu\text{A}$ . For convenience  $R_{xx}$  is scaled by a factor  $10^{-3}$ .

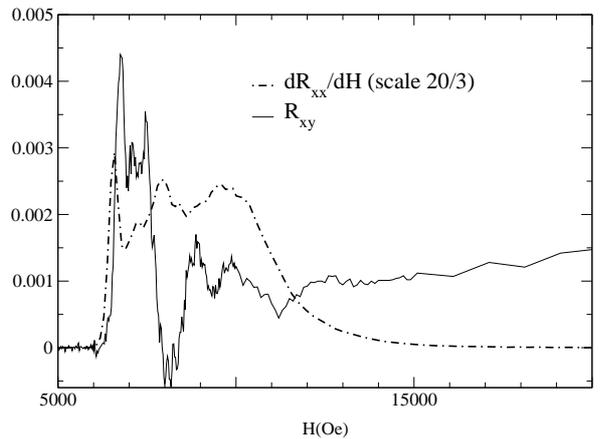


FIG. 2: Comparison of  $R_{xy}$  and  $dR_{xx}/dH$ . The parameters are the same as for Figure 1.

Figure 3 shows  $R_{xy}$  versus B for 2 different temperatures. As we heat the sample (from 4K) the oscillations in  $R_{xy}$  diminish as  $T$  approaches  $T_c$  over the narrowing field range between the depinning and the upper critical fields. Measurements at different longitudinal currents are shown in Fig. 4, from which we conclude that the oscillations are a low current phenomena: at  $800\mu\text{A}$  they have almost disappeared. At low currents, the sign may be inverted (compare the data for 100 and  $200\mu\text{A}$ ). In fact we observe inverted curves in different field scans at the same current (see insert of Fig. 4). Between these

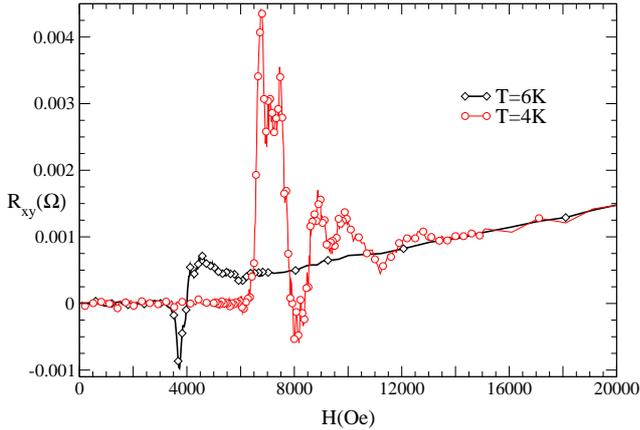


FIG. 3: Transverse resistance for different temperatures,  $I=200\mu A$ .

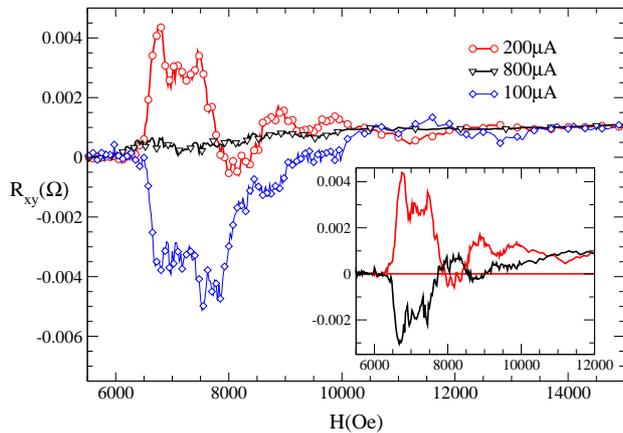


FIG. 4: Transverse resistance for different currents,  $T=4K$ . In insert, different scans for the same current  $200\mu A$ , showing history dependence.

scans the sample is kept at low temperature but was subject to different currents and fields. What is surprising is that while the oscillating part of the Hall conductivity is inverted, the pattern is very similar. The sign inversion invites comparison to the anomalous sign change observed in the Hall resistance near  $T_c$  in the bulk [5], described phenomenologically in terms of a vortex velocity which has a component opposite to the direction of the superfluid flow. Zhu et al. [6] related the sign reversal of the mixed-state Hall resistivity close to the critical temperature to thermal fluctuations and vortex-vortex

interactions and associated this inversion of sign to incoherent motion of vortices, i.e. plastic flow of vortex lattice. Here the sign reversal is of an oscillating transverse Hall effect and is most visible at low temperatures rather than as a smooth change of sign close to  $T_c$ , but we retain from the comparison that it must be related to the dynamics of interacting vortices. *Periodic* sharp oscillations in the resistance have been observed in superconducting Nb films with a square lattice of artificial pinning centers and are associated with commensurability of the vortex and antidots densities [10, 11]. Thus the oscillations could be due to pinning. The inversions we see indicate that for a fixed configuration of defects the overall sign depends on the initial vortex configuration. The two striking features of our results: sharp oscillations and sensitivity to initial conditions, lead us to propose a dynamical model including both vortex interaction and a quenched pinning potential. In order to describe the evolution in time of the local superconducting and normal flows, and local induced magnetic field, we use the TDGL equations [12, 13, 14, 15, 16, 17]. They are time- and space- dependent nonlinear differential equations coupling the superconducting order parameter and the vector potential and are useful to predict qualitative effects of vortex dynamics in the mixed phase of type-II material. The TDGL equations are derived from the extremum of the superconductor free energy in presence of an external magnetic field  $\mathbf{B}_e$  perpendicular to the sample:

$$\mathcal{F} = \frac{a}{2}|\psi|^2 + \frac{b}{4}|\psi|^4 + \frac{1}{2m_s} \left| \left( \frac{\hbar}{i} \nabla - q_s \mathbf{A} \right) \psi \right|^2 + \frac{1}{2\mu_0} \mathbf{B}^2 - \frac{1}{\mu_0} \mathbf{B}_e \cdot \mathbf{B} \quad (1)$$

$\psi(\mathbf{r}, t)$  is the order parameter of the superconducting phase and we choose a gauge where the scalar potential is zero. The coefficient  $a$  is proportional to  $(T - T_c)$  and is negative in the superconducting region.  $b$  does not depend on the temperature and is positive.  $m_s = 2m_e$  and  $q_s = 2q_e$  are the mass and charge of the Cooper pair. The steady state solution in the mixed phase in the absence of current is an Abrikosov vortex array. The interactions between vortices are included inside the non linear terms of the differential equations. Inside the vortex cores the magnetic field is maximum and superconducting density minimum. When a homogeneous current  $\mathbf{J}$  is applied to one direction the vortex array moves in the trans-

verse direction in the absence of pinning. The current is introduced as a boundary condition. It is the sum of the contribution from the normal and superfluid components:  $\mathbf{J} = \hat{\sigma}\mathbf{E} + \mathbf{J}_s$ ,  $\mathbf{J}_s = q_s/m_s\Re[\bar{\psi}(-i\hbar\nabla - q_s\mathbf{A})\psi]$ . The conductivity tensor  $\hat{\sigma}$  is the inverse of the classical resistivity tensor  $\hat{\rho}$ . The uniform equilibrium value of  $\psi$  [Eq. (1)] in the absence of a magnetic field can be simply written as  $\psi_0 = \sqrt{-a/b}$ , with  $a = -a'(1 - T/T_c)$ ,  $a'$  is a positive constant.  $a'$  and  $b$  are related to the zero temperature coherence and penetration lengths by  $\xi_0 = \hbar/\sqrt{2m_s a'}$  and  $\lambda_0^2 = m_s b/(\mu_0 q_s^2 a')$ . Within the domain of applicability of the TDGL theory, all material parameters can be reduced to the following units:

$$\begin{aligned} \text{lengths } L &\rightarrow L/\xi_0, \\ \text{temperature } T &\rightarrow T/T_c, \\ \text{magnetic field } B &\rightarrow B/B_{c2}(0), \\ \text{potential vector } \mathbf{A} &\rightarrow \mathbf{A}/B_{c2}(0)\xi_0 \\ \text{wavefunction } \psi &\rightarrow \psi/\psi_0 \end{aligned} \quad (2)$$

This allows one to study numerically general features of Nb films with only a few dimensionless parameters like  $\kappa$  or reduced temperature  $t = T/T_c$ . The time  $\tau$  is defined in units of  $\tau_0 = \mu_0 \kappa^2 \xi_0^2 \sigma_n n_n(T)/n_e$ , where  $n_n$  is the normal electron density, and  $\sigma_n$  the normal regime conductivity. The dimensionless equation of motion for  $\psi$  reads:

$$\frac{\partial\psi}{\partial\tau} = \frac{1}{\eta} \left[ - \left( \frac{1}{i}\nabla - \mathbf{A} \right)^2 \psi + (1-t) \left( 1 - |\psi|^2 \right) \psi \right], \quad (3)$$

where  $\eta$  is a relaxation rate proportional to the product of a dimensionless constant  $m_e/(\kappa^2 \hbar \mu_0 \sigma_n)$  and a numerical value. This numerical value can be estimated from the Bardeen-Cooper-Schrieffer theory[18, 19]. Taking the experimental values (see Table 1), the dimensionless constant is close to 20. In references[15, 19, 20],  $\eta$  varies from 0.8 to 12, and we will choose its numerical value around unity to optimize time convergence. From the Maxwell equations we obtain:

$$\begin{aligned} \frac{\partial\mathbf{A}}{\partial\tau} &= \hat{\rho} (-\kappa^2 \nabla \times (\nabla \times \mathbf{A}) \\ &+ (1-t)\Re \left[ \bar{\psi} \left( \frac{1}{i}\nabla - \mathbf{A} \right) \psi \right] \right), \end{aligned} \quad (4)$$

where  $\rho_{x,x} = \rho_{y,y} = 1$  and  $\rho_{y,x} = -\rho_{x,y} = \alpha B$  are the normal state components of the resistivity tensor in di-

mensionless units. The coefficient  $\alpha = \sigma_n B_{c2}/q_e n_e$  is small ( $\alpha \simeq 10^{-3}$  with the values of Table 1). In the following, we choose  $\kappa = 2$  (to be clearly type II and close to the experimental value) and  $T=0$ . Equations 3 and 4 are discretized on a grid of size  $N\xi_0 \times N\xi_0$  with time step  $\Delta\tau = 0.04$ . It ensures that the equations converge to a unique steady solution. Taking different time steps around this value does not change the numerical results. Instead of using a discretized version of the vector potential  $\mathbf{A}_{i,j}(\tau)$ , we consider the link variables  $U_{i,j}^\mu(\tau) = \exp(-iA_{\mu,i,j}(\tau)a_\mu)$ ,  $\mu = x, y$ , which preserve the gauge invariance properties of the continuous model [16, 21]. The resulting equations are accurate to second order in space and time steps. From the time dependence of the link variables, we obtain the instantaneous electric fields and the time averaged resistances.

We calculate with an external field  $B_e$  perpendicular to the sample and a bulk current  $\mathbf{J}$  along the  $x$  axis, in units of  $J_0 = q_e \hbar n_s(T)/(2m_e \xi_0)$ , with  $n_s(T) = n_e - n_n(T)$  the superconducting electron density. At zero temperature, given the experimental values (Table 1), this gives a current density  $J_0 \simeq 6.85 \times 10^{13} \text{A m}^{-2}$ . The wavefunction and magnetic field are periodic along the  $x$  axis and  $\psi$  vanishes on boundaries of the transverse direction, where the magnetic field takes the values  $B_e \pm \Delta B$ ,  $\Delta B = J L_y / 2\kappa^2$  being the contribution from the current introduced here as a boundary condition. At the beginning of the numerical simulation ( $t = 0$  and  $B_e = 0$ ), we take as initial conditions  $\psi = 1$  in the bulk of the sample and small random values around zero for the potential vector components  $A_x$  and  $A_y$ . After an equilibration time, a few thousand time steps  $N_0$ , we switch on the current and measure the resistances, over a period of time  $\tau = N_t \tau_0$  for each value of the external field  $B_e$ , where  $N_0$  and  $N_t$  are the number of iterations of the equations. Every time we increase the external magnetic field by a small amount, we let the system approach the steady state during  $N_0$  time steps before recording the resistance values.

We model the ‘‘defects’’ in the sample (grain boundaries, thickness variations, the specific geometry....) by considering a square lattice of points where *either* the wavefunction vanishes  $\psi \sim 0$ , *or* it is fixed to a non-zero value depending on the superfluid density  $\psi \sim |1 - B/B_{c2}|$ . In the first case the impurities decrease the local condensate density and attract vortices while in the second case they repel. Because the pinning potential is

periodic, it is characterised by a distance between centers denoted by  $L_{imp}$ .  $L_{imp}$  is the distance between vortex scattering centers, to be distinguished from  $l_e$  which is the distance between elastic scattering centers for electrons. For simplicity, we chose a periodic structure for the impurities instead of a random one. This is in order to study the influence of the impurity density, considered as a single parameter, on the voltage oscillations. In the simulations  $L_{imp}$  varies from 3.33 (36 impurities,  $N = 20$ ) to 12.5 (1 impurity,  $N = 25$ ). In Fig. 5 we show the simulations for two different impurity concentrations together with the zero impurity case, shown to indicate the noise in the calculations. The oscillations in the transverse resistance, which are similar for the two forms of pinning potential, strongly resemble those seen in the experiments. From the numerical data, and from the range of system size studied ( $N=20-25$  coherent lengths), we can extract a characteristic period  $\delta B$ , or equivalently a length  $L_c \equiv 1/\sqrt{\delta B}$ , and plot it as a function of  $L_{imp}$ , as shown in the insert of Fig. 5. It is seen to decrease with increasing  $L_{imp}$ , i.e.  $\delta B$  increases with increasing purity. From the dominant period of the experimental oscillations (Fig. 1) we can extract from the insert of Fig. 5 an effective distance for vortex pinning  $L_{imp} \simeq 15 - 20\xi_0 = 0.17 - 0.23\mu m$ . We use  $\xi_0 = 115\text{\AA}$  and assume we can apply the numerically derived relation between  $L_{imp}$  and  $L_c$  to our experimental sample. Indeed, the curve  $L_c(L_{imp})$  depends on the parameters of the equations 3 and 4, in particular  $\kappa$ , the temperature and the ratio between the system size and the coherent length. Strictly speaking, the TDGL theory may not be accurate in the field range where we extracted  $L_c$ , but we expect to obtain a reasonable estimate of  $L_c$  for  $\kappa$  around the experimental value 1.6.

In Fig. 6 we calculate  $R_{xy}$  for two different currents: just as in the experiment (Fig. 4) if the current increases, the oscillation amplitude decreases. Furthermore we observe sign inversion due to differing initial conditions as shown in Fig. 7. The simulations then reproduce the experimental observation of a strong memory effect: while there is correlation in the positions of extrema, the nature of each extremum (maximum or minimum) depends on the initial condition at  $B_d$ , but is conserved through the scan. These simulations, which do *not* have the geometric features of the contacts, show that the effects seen experimentally are properties of *generic small* devices, and are not just an effect of the specific Hall bar geometry. It is

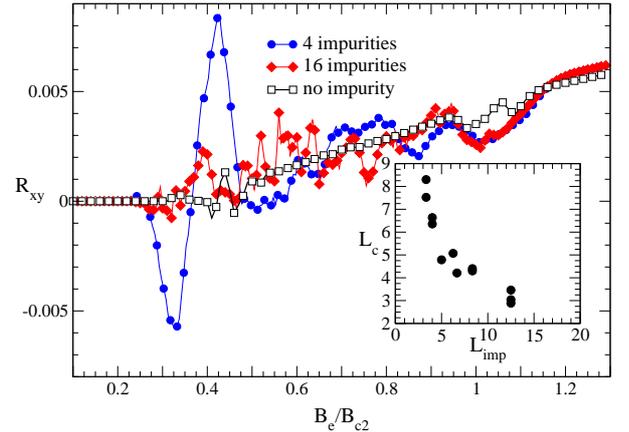


FIG. 5: Calculated transverse resistances for different impurity densities. In insert is  $L_c$  versus  $L_{imp}$  in units of the coherence length  $\xi_0$ . Parameters, whose definitions are in the text, are  $N = 25$ ,  $J = 0.03$ ,  $N_t = 50000$ , and  $\eta = 0.5$ .

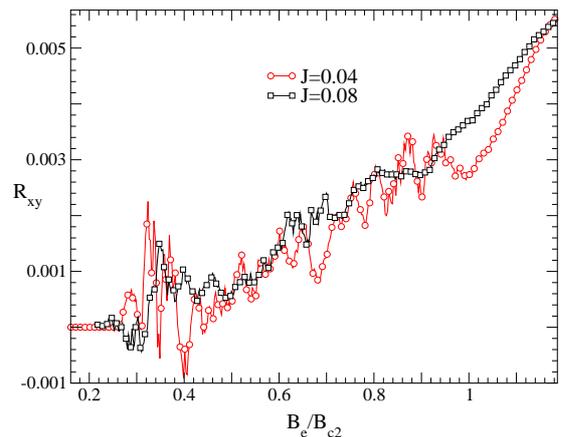


FIG. 6: Calculated transverse resistances for 2 different currents. Parameters are:  $N = 20$ , 9 impurities,  $N_t = 50000$ , and  $\eta = 0.8$ .

not excluded, however, that the boundaries of the device contribute to the pinning potential.

In conclusion, we have shown that micron scale Hall bars of thin films display strong oscillations in the transport in the flux-flow regime, in particular in the Hall voltage at low current. We have shown numerically that these oscillations may be explained by the effects of pinning potentials. The transverse voltage is proportional to the average vortex velocity in the longitudinal direction, which is much smaller than the velocity transverse to the

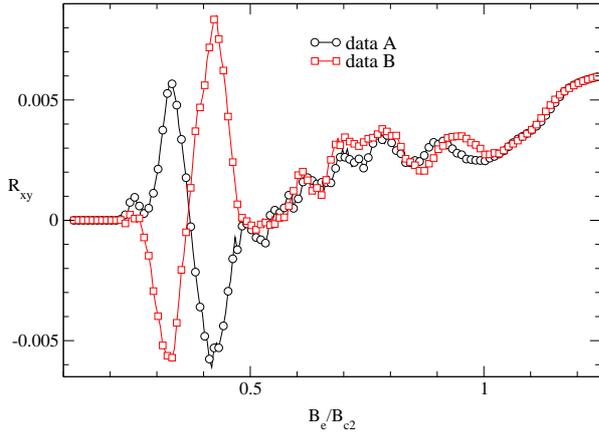


FIG. 7: Sign inversion seen for different random initial conditions A and B. Parameters are:  $N = 20$ , 4 impurities,  $J = 0.03$ ,  $N_t = 50000$ , and  $\eta = 0.8$ .

current ( $R_{xy}/R_{xx} \simeq 10^{-3}$ ).  $R_{xy}$  is therefore sensitive to the presence of bulk impurities which act as scattering centers for the vortices. As the transverse component of the vortex velocity is much larger than the longitudinal component, a small change in the velocity will have a much larger *relative* effect on its longitudinal component, hence  $R_{xy}$ . We explain the sign inversion seen experimentally (Fig. 4) and numerically (Fig. 7) by the fact that each vortex can be scattered by an impurity in two opposite directions, depending on its initial coordinates. The effect is amplified by the collective behavior of other vortices which tend to follow the same direction due to the stiffness of the vortex array. The surprising memory effect, that the inversion continues for the whole field scan, is reproduced by our model of quenched impurities, at least for the lower field range  $B/B_{c2} \lesssim 0.6$ . The abnormal Hall effect here is due to pinning of correlated vortices [22] and not to the dynamics of a single vortex. Thus even potentials of short range, as used in the simulations, are sufficient to influence the transport. Low current transport measurements are then a useful probe of vortex pinning potentials. These potentials are responsible for the complex behavior, as seen here and in other studies of bulk samples [4] that also showed aging and memory phenomena in the current-voltage characteristics for example. From the experimental oscillations (Fig. 1) and assuming the characteristic curve  $L_c$  versus  $L_{imp}$  is accurate for the samples studied, we extracted an effective distance  $L_{imp} \simeq 0.17 - 0.23 \mu\text{m}$  between pinning

centers, for  $\xi_0 = 115 \text{ \AA}$ , coherent with the large amplitudes seen. Our device is small enough and clean enough that there are a small number of effective pinning centers. More generally we can argue that in micron and, by extension, submicron superconducting devices, strong oscillations in the transport are to be expected as generic properties.

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Table 1

Parameters	Experimental Values
$L_x \times L_y \times L_z$	$50 \mu\text{m} \times 5 \mu\text{m} \times 1000 \text{ \AA}$
$T_c$	8.4K
$n_e$	$8.5 \times 10^{22} \text{ cm}^{-3}$
$\lambda_0$	180 \AA
$\xi_0$	115 \AA
$\kappa$	1.6
$B_{c2}$	1.3T (4K)
$R_{xx}$ (normal state)	1.7\Omega (4K)
$\sigma_n$ (normal state)	$1.2 \times 10^7 \Omega^{-1} \text{ m}^{-1}$ (4K)
$l_e$	76 \AA (4K)

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