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Arbitrary amplitude inertial Alfvén waves in homogeneous magnetized electron-positron-ion plasmas

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Abstract

Nonlinear set of equations for inertial or slow shear Alfvén wave (SSAW) in ideal electron-positron-ion (e-p-i) plasmas are presented. The analytical solution for arbitrary amplitude SSAW in such multi-component plasmas is obtained using Sagdeev potential approach. The numerical solutions for several different cases have also been presented for illustrative purpose. It is found that the electron density dips of SSAW are formed in the super Alfvénic region. The amplitude and the width of the nonlinear shear Alfvén wave reduces with the increase in the concentration of positrons in electron-ion (e-i) plasmas. The width of the soliton also depends upon the direction of propagation of the perturbation in both e-i and e-p-i plasmas.

I. INTRODUCTION

The solitary kinetic Alfvén wave (KAW) was studied in electron-ion (e-i) plasmas by Hasegawa and Mima [1] long ago. These waves can propagate in moderate- β plasmas for $\frac{m_e}{m_i} < \beta (= \frac{4\pi n T}{B_0^2}) < 1$. The phase velocity of the KAW is less than the thermal speed of the electrons and hot electrons are assumed to be inertialess and follow the Boltzmann distribution. Later on the nonlinear coupling of KAW and ion acoustic wave (IAW) in e-i plasma was studied by Yu and Shukla [2]

In low- β ($< \frac{m_e}{m_i}$) plasma, the electron inertia cannot be ignored and the wave propagation is slowed down. The phase velocity of Inertial or slow shear Alfvén wave (SSAW) is larger than the thermal speed of the electrons. The nonlinear (SSAW) was studied in e-i plasma with $\beta < \frac{m_e}{m_i}$ about two decades ago [3] using Sagdeev potential approach. The conditions for the existence of localized solitary solutions were discussed. The linear dispersion relation of SSAW in e-i plasmas is $\omega^2 = \frac{v_{Ai}^2 k_z^2}{(1 + \lambda_e^2 k_\perp^2)}$ where the external magnetic field is $\mathbf{B}_0 = B_0 \hat{z}$, Alfvén speed is $v_{Ai} = (\frac{B_0^2}{4\pi n_{i0} m_i})^{\frac{1}{2}}$ (here n_{i0} and m_i are the ion unperturbed density and mass, respectively) and $\lambda_e = \frac{c}{\omega_{pe}}$ (here c is the speed of light and $\omega_{pe} = (\frac{4\pi n_0 e^2}{m_e})^{\frac{1}{2}}$ is the electron plasma frequency) is the electron collisionless skin depth in e-i plasmas. Furthermore k_z and k_\perp are the parallel and perpendicular wave vectors, respectively, with respect to \mathbf{B}_0 .

However, the KAW has faster phase velocity than SSAW and its linear dispersion relation in e-i plasmas is $\omega^2 = v_{Ai}^2 k_z^2 (1 + \rho_s^2 k_\perp^2)$, where $\rho_s = \frac{c_s}{\Omega_i}$ is the ion Larmor radius at electron temperature and $c_s = (\frac{T_e}{m_i})^{\frac{1}{2}}$ is the ion sound speed). The ion inertia plays an important role in shear Alfvén wave dynamics due to ion polarization drift under low frequency assumption i.e., $|\partial_t| \ll \Omega_i$ (where $\Omega_i = \frac{e B_0}{m_i c}$ is the ion-gyrofrequency).

During the last decade, there has been a great deal of interest in electron-positron-ion (e-p-i) plasmas and a number of authors [4, 5, 6, 7, 8, 9, 10] have studied different linear and nonlinear wave propagation phenomenon in such systems. The e-p-i plasmas are supposed to exist in active galactic nuclei (AGN), pulsar magnetospheres and early universe etc.[5, 6, 7, 8, 9, 10]. The positrons are introduced in e-i plasmas for the purpose of diagnostics and to model the pulsar magnetosphere in laboratory experiments [11, 12, 13] In this paper, we show that the presence of positrons in e-i plasmas can change the nonlinear dynamics of low frequency slow shear Alfvén wave significantly. Or we may say the other way round, that presence of ions in e-p plasmas can change both the spatial and temporal scales. Therefore it seems important to study the linear as well as nonlinear phenomenon in e-p-i plasmas to explain some aspects of laboratory or astrophysical e-p-i plasmas.

Recently kinetic Alfvén wave (KAW) has been studied in electron-positron-ion (e-p-i) plasmas by Saleem and Mahmood [10]. In this case, the electrons and

positrons have been assumed to be inertialess and they follow the Boltzmann distribution. The ions are considered to be inertial and cold. The phase velocity of the wave is less than the thermal speed of both the electrons and positrons. The polarization drifts of electrons and positrons can be ignored in e-p-i plasmas and the ion polarization drift is necessary to maintain quasineutrality conditions.

The paper has been presented in this manner. In Sec. II nonlinear equations for Inertial or SSAW in e-p-i have been defined. The localized stationary solution in the form of energy integral equation has been obtained in Sec. III. Some of the possible numerical solution are presented in Sec. IV. In Sec. V discussion on the obtained results is presented.

II. SET OF EQUATIONS

Let us consider a cold electron-positron-ion (e-p-i) plasma in the presence of external magnetic field $\mathbf{B}_0 = B_0 \hat{z}$ along z-axis. The governing equations for the nonlinear Alfvén wave dynamics in x-z plane in a low β (i.e., $\beta \ll \frac{m_e}{m_i}$) e-p-i plasma with $v_{tj} < \frac{\omega}{k_z}$ (where $v_{tj}^2 = \frac{T_j}{m_j}$ and $j = e, p$) are as follows:

The continuity equations for electrons and positrons can be written as,

$$\partial_t n_j + \partial_z (n_j v_{jz}) = 0 \quad (1)$$

whereas the equations of motion for electrons and positrons along \hat{z} -axis are,

$$\partial_t v_{jz} + v_{jz} \partial_z v_{jz} = \frac{q_j}{m_j} E_z \quad (2)$$

where $j = e, p$ and $q_j = -e, +e$ for electrons and positrons, respectively.

The ion equation of motion in the limit $|\partial_t| \ll \Omega_i$ can be expressed as,

$$v_{ix} = \frac{c}{B_0 \Omega_i} \partial_t E_x \quad (3)$$

We are ignoring the ion parallel motion along the magnetic field, so the ion continuity equation gives,

$$\partial_t n_i + \frac{c}{B_0 \Omega_i} \partial_x (n_i \partial_t E_x) = 0 \quad (4)$$

The Faradays law $\nabla \times \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{B}$ can be written as,

$$\partial_z E_x - \partial_x E_z = -\frac{1}{c} \partial_t B_y \quad (5)$$

and Ampere's law yields,

$$\partial_x B_y = \frac{4\pi e}{c} (n_p v_{pz} - n_e v_{ez}) \quad (6)$$

where the displacement current has been ignored.

The quasi-neutrality condition implies,

$$n_i \simeq n_e - n_p \quad (7)$$

where n_α (where $\alpha = e, p, i$) are the densities, $v_{\alpha z}$ and $v_{\alpha x}$ are the parallel and perpendicular velocities of α -species with respect to the external magnetic field, respectively.

Using two potential approach, one can express the parallel and perpendicular electric fields as $E_x = -\partial_x \phi$ and $E_z = -\partial_z \psi$, where ' ϕ ' and ' ψ ' are electrostatic and electromagnetic potentials, respectively.

The linear dispersion relation of inertial Alfvén wave in low- β e-p-i plasmas turns out to be,

$$\omega^2 = \frac{(1+p)v_{Ai}^2 k_z^2}{[(1+p) + \lambda_e^2 k_x^2]} \quad (8)$$

where $p = \frac{n_{p0}}{n_{e0}}$ (here n_{p0} and n_{e0} are the unperturbed densities of positrons and electrons, respectively).

The limiting case of two component e-i plasmas can be obtained by putting $n_p = 0$ (or $p = 0$) in above equation to obtain,

$$\omega^2 = \frac{v_{Ai}^2 k_z^2}{(1 + \lambda_e^2 k_x^2)} \quad (9)$$

which is the same linear dispersion relation of Inertial or slow shear Alfvén wave (SSAW) in e-i plasmas which has already been studied in Ref. [2]. It can be seen from Eq.(8) that the wave dispersion due to electron inertial length in the presence of positrons is modified because of the factor p . So the positron density can have significant effect on the wave dynamics in nonlinear regime as well.

It may be mentioned here that Eq.(8) cannot reduce to the linear dispersion relation of shear Alfvén waves of e-p plasmas because the polarization drifts of electrons and positrons have been ignored in the limit $|\partial_t| \ll \Omega_j$ (where $\Omega_j = \frac{q_j B_0}{m_j c}$ is the gyrofrequency of j^{th} species and $j = e, p$).

III. NONLINEAR SOLUTION

We are interested in the stationary localized planar solution of the non-linear set of equations. So we transform the set of Eqs. (1)-(8) in moving frame ξ defined as ,

$$\xi = K_x x + K_z z - \Omega t \quad (10)$$

where ' Ω ' is the velocity of the non-linear structure in the moving frame, K_x and K_z are the direction cosines in the x and z directions, respectively and $K_x^2 + K_z^2 = 1$.

Now the electron continuity equation can be written as,

$$v_{ez} = u (1 - n_e^{-1}) \quad (11)$$

the electron momentum equation can be written by using above equation,

$$\frac{1}{K_z} \frac{\partial \varepsilon_z}{\partial \xi} = -u^2 \frac{\partial^2}{\partial \xi^2} n_e^{-1} - \frac{1}{2} u^2 \frac{\partial^2}{\partial \xi^2} (1 - n_e^{-1})^2 \quad (12)$$

The normalized electron density is $n_e = \frac{n_e}{n_{e0}}$ and $\varepsilon_z = \frac{eE_z}{m}$ (where $m_e = m_p = m$). In order to obtain Eq.(11), we have used the boundary conditions i.e., as $\xi \rightarrow |\pm \infty|$, $n_e \rightarrow 1$ and $v_{ez} \rightarrow 0$.

Similarly from positron continuity equation we have,

$$v_{pz} = u (1 - n_p^{-1}) \quad (13)$$

and the positron momentum equation and the above relation give,

$$\frac{1}{K_z} \frac{\partial \varepsilon_z}{\partial \xi} = u^2 \frac{\partial^2}{\partial \xi^2} n_p^{-1} + \frac{1}{2} u^2 \frac{\partial^2}{\partial \xi^2} (1 - n_p^{-1})^2 \quad (14)$$

where $u = \frac{\Omega}{K_z}$ and normalized positron density is $n_p = \frac{n_p}{n_{p0}}$. We have again used the boundary conditions i.e., as $\xi \rightarrow |\pm \infty|$, $n_p \rightarrow 1$ and $v_{pz} \rightarrow 0$ to obtain Eq.(13).

The ion continuity equation yields,

$$-K_x \frac{\partial \varepsilon_x}{\partial \xi} = \Omega_e \Omega_i (1 - n_i^{-1}) \quad (15)$$

where normalized ion density $n_i = \frac{n_i}{n_{i0}}$, $\varepsilon_x = \frac{eE_x}{m_i}$ and $\Omega_e = \frac{eB_0}{mc}$ (electron gyrofrequency).

Using Eqs.(11) and (13) in Eq.(6), one can write Ampere's law in ξ co-ordinate as,

$$K_x \frac{\partial B_y}{\partial \xi} = \frac{4\pi e n_{e0}}{c} u [p (1 - n_p^{-1}) n_p - n_e (1 - n_e^{-1})] \quad (16)$$

Now after transforming Eq.(5) in ξ co-ordinate and using the above relation we have,

$$K_x K_z^2 \frac{\partial \varepsilon_x}{\partial \xi} - K_x^2 K_z \frac{\partial \varepsilon_z}{\partial \xi} = \Omega^2 \lambda_e^{-2} [p (1 - n_p^{-1}) n_p - n_e (1 - n_e^{-1})] \quad (17)$$

Equating left hand side (L.H.S) of Eqs.(12) and (14) and then integrating the resulting equations twice w.r.t. ξ we obtain,

$$(v_{ez} - u)^2 + (v_{pz} - u)^2 = 2u^2 \quad (18)$$

Using Eqs. (11) and (13) in the above relations, we obtain,

$$n_p^{-2} = 2 - n_e^{-2} \quad (19)$$

The quasi-neutrality yields,

$$n_i = \frac{1}{(1 - p)} (n_e - p n_p) \quad (20)$$

Note that the above equations hold for $0 \leq p < 1$ in three component e-p-i plasmas.

Eq. (19) and Eq. (20) yields,

$$n_i^{-1} = \frac{(1-p)}{\left[n_e - \frac{p}{\sqrt{2-n_p^{-2}}} \right]} \quad (21)$$

Then Eq.(19) along with Eqs.(12) and (14) gives,

$$\begin{aligned} & -K_z^2 [\Omega_e \Omega_i (1 - n_i^{-1})] + u^2 K_z^2 K_x^2 \left[\frac{\partial^2}{\partial \xi^2} n_e^{-1} + \frac{1}{2} \frac{\partial^2}{\partial \xi^2} (1 - n_e^{-1})^2 \right] \\ & = \Omega^2 \lambda_e^{-2} [p(n_p - 1) - (n_e - 1)] \end{aligned} \quad (22)$$

Differentiating w.r.t ξ twice the above equation and then after simplification we obtain,

$$K_x^2 \lambda_e^2 \frac{\partial^2}{\partial \xi^2} \left[3n_e^{-4} \left(\frac{\partial n_e}{\partial \xi} \right)^2 - n_e^{-3} \frac{\partial^2 n_e}{\partial \xi^2} \right] - \frac{(1-p)}{M^2} \frac{\partial^2}{\partial \xi^2} (1 - n_i^{-1}) = \frac{\partial^2}{\partial \xi^2} (pn_p - n_e) \quad (23)$$

where $M = \frac{u}{v_A}$ is defined as Mach number.

Integrating Eq.(23) twice w.r.t. ξ , we have,

$$K_x^2 \lambda_e^2 \left[3n_e^{-4} \left(\frac{\partial n_e}{\partial \xi} \right)^2 - n_e^{-3} \frac{\partial^2 n_e}{\partial \xi^2} \right] - \frac{(1-p)}{M^2} (1 - n_i^{-1}) = (n_p p - n_e) + (1-p) \quad (24)$$

In order to obtain above equation, we have used the boundary conditions i.e., $n_p \rightarrow 1$, $n_i \rightarrow 1$ and $n_e \rightarrow 1$ as $\xi \rightarrow |\pm \infty|$.

Let us define $R = n_e^{-3} \frac{\partial n_e}{\partial \xi}$, then multiplying Eq.(24) by 'R' both sides and after integrating once w.r.t ξ we obtain,

$$\frac{1}{2} \left(\frac{\partial n_e}{\partial \xi} \right)^2 + V(n_e) = 0 \quad (25)$$

The Sagdeev potential is defined as,

$$\begin{aligned} V(n_e) = \frac{n_e^6}{K_x^2} & \left[\frac{1}{n_e} (1 + p \sqrt{2n_e^2 - 1}) - \frac{(1-p)}{2n_e^2} \left(\frac{1}{M^2} + 1 \right) - \frac{(1-p)^2}{M^2} \int_1^{n_e} \frac{1}{n_e^4 \left[1 - \frac{p}{\sqrt{2n_e^2 - 1}} \right]} dn_e \right. \\ & \left. - \frac{1}{2} (1 + 3p) + \frac{1}{2} \frac{(1-p)}{M^2} \right] \end{aligned} \quad (26)$$

where $\xi = \frac{\xi}{\lambda_e}$ has been normalized. We have used the boundary conditions i.e., as $\xi \rightarrow |\pm \infty|$ then $\left(\frac{\partial n_e}{\partial \xi} \right) \rightarrow 0$ and $n_e \rightarrow 1$ to obtain Eq.(25).

Equation(25) is a well known equation in the form of "energy integral" of an oscillating particle of a unit mass, with velocity $(\frac{\partial n_e}{\partial \xi})$ and position n_e in a potential $V(n_e)$. The conditions for the existence of localized solution of Eq.(25) require that i) $V(1) = V(N_0) = \frac{\partial V}{\partial n_e} |_{n_e=1} = 0$, (N_0 is the point where the curve crosses the n_e axis and it can have values $>$ or < 1) and it represents the maximum amplitude of the soliton ii) $\frac{\partial^2 V}{\partial n_e^2} |_{n_e=1} < 0$ (where $n_e = 1$ is the unstable point) and from second condition it is seen that solitary structures are formed only in the super Alfvénic region.

The limiting case of two component e-i plasma can be obtained by putting $p = 0$ in Eq.(25) and the Sagdeev potential in this case turn out to be,

$$V(n) = \frac{n^6}{b} \left[\frac{1}{2} \left(\frac{1}{M^2} + 1 \right) \left(1 - \frac{1}{n^2} \right) + \frac{1}{n} - 1 + \frac{1}{3M^2} \left(\frac{1}{n^3} - 1 \right) \right] \quad (27)$$

which is the same as obtained by Shukla et. al., [2] and $b = \lambda_e^2 K_x^2$ has been defined.

IV. NUMERICAL SOLUTIONS

The numerical solutions of Eq.(25) are obtained for solitary structures in the absence as well as in the presence of positrons in e-i plasmas. The Sagdeev potential in the presence of positron (as a third species) in two component e-i plasmas becomes complicated and the third integral term does not have simple analytical solution. However, the numerical solutions exist and the plots of the Sagdeev potential 'V' vs normalized electron densities ' n_e ' corresponding to $p = 0.2$ (dashed curve) i.e., for e-p-i plasmas as well as corresponding to $p = 0$ (solid curve) i.e., e-i plasmas for $M = 1.2$, $K_x = 0.1$ have been shown in Fig.1. The corresponding electron density dips have been shown in Fig.2 for the same parameters as given in Fig.1. It can be seen from the figures that the amplitude and the width of solitary structures reduces with increase in percentage of positrons in e-i plasmas. The normalized density profiles of three species electron-positron-ion plasmas are shown in Fig. 3.

The effects on the direction of propagation on the solitons for both the cases e-i and e-p-i plasmas are shown in figures 4 and 5, respectively. The plots for different directions of propagation i.e., for $K_x = 0.1$ (solid curve) and $K_x = 0.3$ (dotted curve) for two component e-i ($p = 0$) plasmas and for $K_x = 0.1$ (dotted curve) and $K_x = 0.3$ (solid curve) in three component e-p-i ($p = 0.2$) plasmas for the same value of $M = 1.2$ have been shown in figures 3 and 4. It can be seen from the figures that propagation direction effects on the width of the solitary structures. The width of the solitary structure increases with the increase in the obliqueness of the wave.

The dependence of Mach number on soliton corresponding to $M = 1.6$ (solid curve) and $M = 1.2$ (dotted curve) for same $p = 0.2$, $K_x = 0.1$ has been shown in Fig.6. It can be seen from the figure that the Mach number has significant effect on the amplitude as well as on the width of the solitary structure. The wave amplitude increases and the width decreases with the increase in Mach number.

V. DISCUSSION ON RESULTS

We have studied the solitary pulse formation of inertial or slow shear Alfvén waves in electron-positron-ion plasmas. The conditions on the Mach number for the formation of such nonlinear structures in e-i plasmas were presented long ago [2]. It was predicted that super Alfvénic density depletion regions can be formed in e-i plasmas corresponding to an arbitrary amplitude perturbation. However the density profiles were not plotted and the dependence of the nonlinear structures on the propagation direction was not investigated. We have noticed that the pulse width increases with the increase in obliqueness (with respect to the external magnetic field) of the propagation direction in e-i plasmas as shown in Fig.4.

It is pointed out that the conditions for the formation of solitary pulses in e-p-i plasmas due to large amplitude SSW perturbation are similar to the e-i case. That is the density depletion regions can be formed in the super Alfvénic region i.e, for $1 < M$. But the amplitude of the soliton decreases with the increase in the number density of positrons. This result is very common in e-p-i plasmas. For example the amplitudes of the solitary ion acoustic [5] and solitary kinetic Alfvén waves[10] also decrease with the increase in the concentration of positrons in e-i plasmas.

In principle, the Alfvén waves can have a wide range of temporal and spatial scales in e-p-i plasmas because the frequency of the linear wave can vary from $\omega = v_{Ai}k_z$ (with $\omega < \Omega_i$) to $\omega = v_{Ape}k_z$ (with $\omega < \Omega_{e,p}$), where $v_{Aep} = (\frac{B_0^2}{8\pi n_0 m})^{\frac{1}{2}}$ (where $m_e = m_p = m$) is the Alfvén wave speed, and correspondingly the spectrum of wavelengths can be broader. These variations in the temporal and spatial scales depend upon the concentration ratios of different species. Therefore it seems interesting to analyze the linear and nonlinear wave propagation in such plasmas.

Here, we have studied only the Inertial or slow shear Alfvén waves in e-p-i plasmas which propagate on ionic time scale. It may be mentioned that the limiting case of e-p plasmas can not be obtained from our equations because we have ignored the polarization drifts of electrons and positrons. Our findings can be useful to explain some aspects of laboratory and astrophysical space e-p-i plasmas situations.

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Figures Captions

- **Fig.1:**The Sagdeev potential ' V ' is plotted against electron density ' n_e ' for $M = 1.2$, $K_x = 0.1$, $p = 0$ (solid curve) and $p = 0.2$ (dotted curve).
- Fig.2:**The normalized electron density dip decreases with the increase in positrons in e-p-i plasmas for the same parameters a given in Fig.1.
- Fig.3:** The normalized density profiles of electron, positron and ion are plotted for $p = 0.2$, while M and K_x corresponds to the same values as given in Fig.1.
- Fig.4:**Effect of propagation direction on the width of the solitary pulse in e-p-i plasmas is shown with $K_x = 0.1$ (solid curve) and $K_x = 0.3$ (dotted curve) for $M = 1.2$ and $p = 0$.
- Fig.5:**Effect of propagation direction on the width of the solitary pulse in e-p-i plasmas is shown with $K_x = 0.1$ (dotted curve) and $K_x = 0.3$ (solid curve) for $M = 1.2$ and $p = 0.2$.
- Fig.6:**The effect of Mach number on the solitons in e-p-i plasmas is shown plotting n_e and ξ for $p = 0.2$, $K_x = 0.1$, $M = 1.6$ (solid curve) and $M = 1.2$ (dotted curve)