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LOCAL INDEX OF POTENTIAL OPERATORS OF MONOTONE TYPE

SATYANAD KICHENASSAMY

Abstract. We prove the following result: if $f = \varphi'$ is the Gâteaux derivative of a functional on a reflexive Banach space, and is demicontinuous of class $(S)_+$, then it has local index +1 at every isolated zero which is also a local minimum point for φ . An application to the existence of multiple solutions of nonlinear partial differential equations is given.

1. INTRODUCTION AND STATEMENT OF RESULTS.

1.1 A topological result. We consider in this paper a reflexive Banach space X and an open bounded set $G \subset X$, containing the origin. We may, by Trojanski's theorem, renorm X so that it be locally uniformly convex as well as X^* (with the dual norm). Let J be the associated duality map from X to X^* . F. E. Browder has constructed in [3] a degree function for maps of class $(S)_+$, demicontinuous from $cl(G)$ to X^* , which is normalized by J and reduces to the Leray-Schauder degree when X is a Hilbert space and the map considered is a compact perturbation of the identity (we recall this construction in §2.1).

Our main result (proved in §2.2) is the following:

THEOREM 1. *Assume that $f : cl(G) \rightarrow X^*$ satisfies*

- (i) $f \in (S)_+$ and is demicontinuous
- (ii) f is in G the Gâteaux derivative of $\varphi : G \rightarrow \mathbf{R}$
- (iii) If $x \in cl(G)$ and $x \neq 0$, then $f(x) \neq 0$
- (iv) If $x \in G$, then $\varphi(x) \geq \varphi(0)$.

Then,

$$\deg(f, G, 0) = +1.$$

In case X is a Hilbert space (identified with X^*) and $f = Id +$ compact, we recover results of P. H. Rabinowitz [11], K. Thews [15] and H. Amann [1] ([11] assumes that $\varphi \in C^2$ and [15], [1] that $\varphi \in C^1(G; \mathbf{R})$). For the finite dimensional case (Poincaré-Hopf) see also e.g., J. Milnor [10] and E. H. Rothe [12].

1.2 An illustration. We now illustrate Theorem 1 by following multiplicity result: Let us first take $g \in C^1(\mathbf{R}; \mathbf{R})$ be such that

- (i) g has m zeros $0 < a_1 < \dots < a_m$,
- (ii) (“Area condition”) For $k \geq 2$ and $F(s) := \int_0^s g(t)dt$

$$\sup \{F(s) : 0 \leq s \leq a_{k-1}\} < F(a_k),$$

- (iii) $g(0) > 0$.

Let also Ω be a bounded smooth domain in \mathbf{R}^N ($N \geq 1$), $1 < p < \infty$. We then have:

THEOREM 2. *If g satisfies (i), (ii) and (iii) then the problem (P):*

$$\begin{aligned} Au := -div \left((1 + |\nabla u|^2)^{(p-2)/2} \nabla u \right) &= \lambda g(u) && \text{on } \Omega \\ u &= 0 && \text{on } \partial\Omega \end{aligned}$$

has at least $2m - 1$ weak solutions for all λ large enough.

This result contains theorems of K. J. Brown and H. Budin [5], P. Hess [6], J. Smoller and A. Wasserman [14] for semi-linear equations ($p = 2$). The case of radial solutions (in a more general setting including the case $p \neq 2$) has been solved by J. Smoller and the author [7] by O.D.E. methods.

REMARKS: (1) The very classical “area condition” (ii) is necessary for radial solutions with maximum “close” to each a_k to exist (see e.g., [7]).

(2) Note that no growth condition on g is assumed; we may even take $g = 0$ far out. The problem is “sublinear.”

This theorem will be proved in §3.

2. PROOF OF THEOREM 1.

2.1 Degree of a demicontinuous map of class $(S)_+$. Let G be an open bounded subset of a reflexive Banach space X and let $f : cl(G) \rightarrow X^*$ be a mapping. We say that f is *demicontinuous* if it is continuous from the strong topology of X to the weak topology of X^* . We say that f is of *class $(S)_+$* and write $f \in (S)_+$ whenever for any sequence (x_j) in $cl(G)$ converging weakly to x in X , and such that

$$(1) \quad \limsup \langle f(x_j), x_j - x \rangle \leq 0,$$

one has $x_j \rightarrow x$ (strongly). Note that J (duality mapping) is $(S)_+$ and demicontinuous.

We also define for $X_0 \subset X$ finite dimensional, the corresponding Galerkin approximation

$$(2) \quad f_0 = i_0^* f i_0$$

where $i_0 : X_0 \rightarrow X$ is the inclusion, i_0^* its transpose. J_0 is defined in a similar way for the duality map J .

We define the degree of a mapping $X_0 \rightarrow X_0^*$ as the Brouwer degree normalized by J_0 .

Assume now that $0 \notin f(\partial G)$, $f \in (S)_+$ and is demicontinuous. One can then show [3] that there exists a finite dimensional X_f such that for any finite dimensional $X_0 \supset X_f$

- (a) $\deg(f_0, G \cap X_0, 0)$ is well defined
- (b) It does not depend on X_0 .

We call the integer thus obtained $\deg(f, G, 0)$. For details and examples see [3], [4].

REMARKS: (1) The class of mappings we consider is stable under compact perturbations. Thus if $X =$ Hilbert space, $J = Id$, we obtain indeed an extension of Leray-Schauder degree theory.

(2) In case f is the Gâteaux derivative of φ and X_0 is large enough, we might try to define $\deg(f_0, G \cap X_0, 0)$ in another way, namely by choosing an identification of X_0 with \mathbf{R}^n , and by using Brouwer degree in \mathbf{R}^n . That amounts to a normalization by the duality mapping I_0 for the Euclidean norm on \mathbf{R}^n (while J_0 is the duality map for X_0 equipped with $\|\cdot\|_X$ restricted to X_0). Both viewpoints are equivalent because

- (a) for every $x \neq 0$, $(J_0 x, x)$ and $(I_0 x, x)$ are > 0 and by homotopy, for every R ,

$$\deg_{\mathbf{R}^n}(J_0, B_R, 0) = +1;$$

- (b) I_0 and J_0 have degree 0 on every set which does not contain 0 in its closure.

This remark justifies our use of customary degree in \mathbf{R}^n for Galerkin approximations.

2.2 Proof of Theorem 1. We proceed in 3 lemmas. We write $f = \varphi'$, $\varphi(0) = 0$, and fix $\epsilon > 0$ such that $B_{2\epsilon} \subset G$.

LEMMA 1. *There are $\alpha, \alpha', c, c' > 0$ such that*

$$(3) \quad B_\alpha \subset (\varphi < c)_\epsilon \subset B_{\alpha'} \subset (\varphi < c')_\epsilon \subset B_{\epsilon/2},$$

where $(\varphi < c)_\epsilon$ stands for $B_\epsilon \cap (\varphi < c)$ and similarly for c' .

LEMMA 2. *There is a finite dimensional subspace X_0 of X such that*

$$(4) \quad \deg(f, G, 0) = \deg(f_0, X_0 \cap B_\epsilon, 0)$$

and

$$(5) \quad f_0 \neq 0 \text{ on } X_0 \cap [cl(B_\epsilon) \setminus B_{\alpha/2}].$$

LEMMA 3. $\deg(f_0, X_0 \cap (\varphi < c')_\epsilon, 0) = +1$.

This will end the proof. Indeed, by (5), f_0 has no zero on $cl(X_0 \cap (B_\epsilon \setminus (\varphi < c')_\epsilon))$ and therefore

$$\begin{aligned} \deg(f, G, 0) &= \deg(f_0, X_0 \cap B_\epsilon, 0) \\ &= \deg(f_0, X_0 \cap (\varphi < c')_\epsilon, 0) \\ &= +1. \blacksquare \end{aligned}$$

It remains to prove the lemmas.

PROOF OF LEMMA 1: For $0 < a < \epsilon$ let

$$m_a = \inf \{ \varphi(x) : a \leq \|x\| \leq \epsilon \}.$$

We claim that $m_a > 0$. Indeed $m_a \geq 0$ and if $m_a = 0$, there is a sequence (x_n) in $cl(B_\epsilon) \setminus B_a$ such that

$$(6) \quad \varphi(x_n) \leq n^{-2}.$$

By a result due to Ekeland, as $cl(B_{2\epsilon})$ is a complete metric space, one can find another sequence (y_n) such that for each n

$$(7) \quad \|x_n - y_n\| \leq \frac{1}{n}; \quad \|y_n\| \leq 2\epsilon$$

$$(8) \quad \varphi(y_n) \leq \varphi(x_n)$$

$$(9) \quad \text{For } y \neq y_n \text{ and } \|y\| \leq 2\epsilon, \varphi(y) > \varphi(y_n) - \frac{\|y - y_n\|}{n}.$$

These properties ensure that for $n^{-1} < a/2$,

$$(10) \quad \|y_n\| \geq \frac{a}{2}$$

$$(11) \quad \|f(y_n)\| \leq \frac{1}{n}.$$

(Indeed (9) implies that for every $z \in X$, for t small, as $t \rightarrow 0$ one has

$$\varphi(y_n + tz) - \varphi(y_n) + \frac{\|tz\|}{n} = t \left[(f(y_n), z) + \frac{\|z\|}{n} \right] + o(t) \geq 0.$$

Now $\|y_n\| \leq 2\epsilon$. Therefore one may (after extraction of a subsequence) assume that

$$\exists y \in X; \quad y_n \rightarrow y \text{ and } f(y_n) \rightarrow 0.$$

f being $(S)_+$, this implies that $y_n \rightarrow y$, and $f(y) = 0$. Now $y \neq 0$ because of (10). This is a contradiction.

We have proved that $m_a > 0$. Now take $c' < m_{\epsilon/2}$ and (as $(\varphi < c')$ is a neighborhood of 0) α' such that $B_{\alpha'} \subset (\varphi < c') \cap B_\epsilon \subset B_{\epsilon/2}$. Repeat the operation: take $c < m_{\alpha'/2}$ and $B_\alpha \subset (\varphi < c)_{\alpha'/2}$. (1) follows.

PROOF OF LEMMA 2: As f has no zeros on $G \setminus B_\epsilon$ we may restrict our attention to $\text{deg}(f, B_\epsilon, 0)$. We know that (4) holds for $X_0 \supset X_f$. Let $\{X_\lambda\}_{\lambda \in \Lambda}$ be the set of all finite dimensional subspaces of X containing X_f . If (5) were false the following sets would be nonempty for every λ :

$$V_\lambda = \{x \in \text{cl}(B_\epsilon) \setminus B_{\alpha/2} : (f(x), x) = 0 \text{ and } v \in X_\lambda\}$$

(indeed zeros of the Galerkin approximation of f in X_λ belong to V_λ).

V_λ being bounded, its weak closure is weakly compact. The sets $wk - \text{cl}(V_\lambda)$ enjoy the finite intersection property. Therefore there is an x in

$$\bigcap_{\lambda \in \Lambda} wk - \text{cl}(V_\lambda).$$

Now let $w \in X$. We wish to prove that $(f(x), w) = 0$. For that purpose take X_λ containing both x and w . Now (Eberlein-Šmulian) we may find a *sequence* $x_j \rightarrow x$ with x_j belonging to X_λ . We have

$$(13) \quad (f(x_j), x_j - x) = 0 \quad (f(x_j), w) = 0$$

for all j . As $f \in (S)_+$, $x_j \rightarrow x$ and as f is demicontinuous, (11) gives

$$(14) \quad (f(x), w) = 0.$$

This holds for every $w : f(x) = 0$. On the other hand $\|x_j\| \rightarrow \|x\|$, so that

$$(15) \quad \frac{\alpha}{2} \leq \|x\| \leq \epsilon.$$

But f has no zero on $cl(B_\epsilon) \setminus B_{\alpha/2}$. (5) is therefore true.

PROOF OF LEMMA 3: We recall that i_0 is the inclusion $X_0 \rightarrow X$. Let $\varphi_0 = \varphi i_0$. Notice that φ_0 is of class C^1 on X_0 .

We are now dealing with maps defined on X_0 . Identify X_0 with \mathbf{R}^n . One may clearly find, as in Lemma 1, $\alpha, \alpha', c, c', \theta > 0$ such that

$$(16) \quad B_\alpha \subset (\varphi_0 < c - \theta); \quad (\varphi_0 < c + \theta) \subset B_{\alpha'},$$

$$(17) \quad B_{\alpha'} \subset (\varphi_0 < c' - \theta); \quad (\varphi_0 < c' + \theta) \subset B_{\epsilon/2}.$$

Let us approximate φ_0 in C^1 norm by a Morse function ψ in $B_{3\epsilon/2}$ (generic situation). If $\|\varphi_0 - \psi\|_{C^1} < \theta$ then, by (16) — (17), ψ satisfies (3). As $\|f(x)\|$ is bounded below for $\alpha/2 \leq \|x\| \leq \epsilon$ we may assume that all critical points of ψ are of norm $< \alpha$. Now the gradient flow η defined on $(\psi < c')_\epsilon$ by

$$\begin{aligned} \eta(0, x) &= x \\ \frac{d\eta(t, x)}{dt} &= -\psi'(\eta(t, x)) \end{aligned}$$

deforms $(\psi < c')_\epsilon$ into $(\psi < c)$ in some finite time T . But this deformation stays within B_ϵ because ψ decreases along trajectories and there is no continuous curve from $(\psi < c')_\epsilon$ to $\{\|x\| \geq \epsilon\}$ (because of (3)). As $(\psi < c) \subset B_\alpha$, the map ζ defined by

$$\begin{aligned} \zeta(t, x) &= \eta(t, x) \quad \text{for } 0 \leq t \leq T \\ \zeta(t, x) &= \frac{(2T - t)\eta(T, x)}{T} \quad \text{for } T \leq t \leq 2T \end{aligned}$$

is a deformation of $(\psi < c')_\epsilon$ to a point. By the Poincaré-Hopf theorem, Lemma 3 follows.

3. PROOF OF THEOREM 2.

The problem is to find $2m - 1$ critical points of

$$(18) \quad \varphi(\lambda, u) = \int_{\Omega} \left[\frac{(1 + |\nabla u|^2)^{p/2}}{p} - \lambda F(u) \right] dx$$

defined on $W_0^{1,p}(\Omega)$. This functional is Gâteaux differentiable and its gradient is demicontinuous of class $(S)_+$ by classical arguments of the theory of monotone operators.

We assume that all solutions are isolated — otherwise there is nothing to prove. We first obtain m variational solutions u_1, \dots, u_m which minimize truncated functions $\varphi_1, \dots, \varphi_m$. The point is then to find $m - 1$ more solutions, which is achieved thanks to Theorem 1.

We proceed in 3 steps:

STEP 1: m variational solutions: Let, for every $k \geq 1$, $g_k = g$ on $[0, a_k]$, extended so that it be C^0 , ≥ 0 on $] - \infty, 0]$, ≤ 0 on $[a_k, + \infty[$; we may assume that g_k and g_{k+1} differ only in $[a_k, a_{k+1}]$. Let also

$$F_k(s) := \int_0^s g_k(t) dt.$$

We minimize

$$(19) \quad \varphi_k(\lambda, u) = \int_{\Omega} \left[\frac{(1 + |\nabla u|^2)^{p/2}}{p} - \lambda F_k(u) \right] dx.$$

The minimum is attained at some u_k (φ_k is w.l.s.c. coercive). We show that this function is a solution of (P):

LEMMA 4. *All critical points u of φ_k satisfy $u < a_k$ and are positive solutions of (P).*

PROOF: (For $p = 2$ see [2] and [6].) It suffices to show $u < a_k$. We have $u \leq a_k$ by the maximum principle. Moreover by standard regularity results, u is bounded in C^1 [9] and therefore we may assume that (P) reads:

$$A'u = - \operatorname{div} (\rho(|\nabla u|^2) \nabla u) = g_k(u)$$

with ρ bounded. Now as $g_k(a_k) = 0$ if μ is large enough, $u \rightarrow g_k(u) + \mu u$ is increasing and

$$(A' + \mu)(a_k - u) \geq 0,$$

and the strong maximum principle (which follows from Harnack's inequality [9]) shows that $u < a_k$ everywhere.

We now show that the u_k 's are distinct.

LEMMA 5. For $k < m$ and λ large enough,

$$\varphi_{k+1}(\lambda, u_{k+1}) < \varphi_k(\lambda, u_k).$$

PROOF: We exhibit u such that $0 \leq u \leq a_{k+1}$ and

$$\text{if } 0 \leq v \leq a_k \text{ then } \varphi(\lambda, u) < \varphi(\lambda, v).$$

Let $\delta > 0$, $\Omega_\delta = \{x : d(x, \partial\Omega) < \delta\}$, so that $|\Omega_\delta| \rightarrow 0$, and $w_\delta \in C_0^\infty(\Omega)$ such that $0 \leq w_\delta \leq a_k$ and $w_\delta = a_k$ for $x \in \Omega \setminus \Omega_\delta$ (e.g., regularize $a_k \chi_{(d(x, \partial\Omega) \geq \delta/2) \cap \Omega}$).

We have by (ii)

$$F(a_{k+1}) - \max_{0 \leq s \leq a_k} F(s) = \tau > 0$$

so that if $0 \leq v \leq a_k$,

$$\begin{aligned} \int_\Omega F(w_\delta) dx &\geq \int_{\Omega \setminus \Omega_\delta} (F(v) + \tau) dx - \int_{\Omega_\delta} \max_{[0, a_{k+1}]} |F| dx \\ &\geq \int_\Omega F(v) dx + \tau |\Omega| - 2 \max_{[0, a_{k+1}]} |F| |\Omega_\delta| \\ &\geq \int_\Omega F(v) dx + \tau |\Omega| / 2 \end{aligned}$$

for δ small enough. Let $u = w_\delta$. Then

$$\varphi(\lambda, u) - \varphi(\lambda, v) \leq \int_\Omega \left[\frac{(1 + |\nabla u|^2)^{p/2}}{p} - \frac{\lambda \tau}{2} \right] dx < 0$$

for λ large enough. Lemma 5 is proved.

STEP 2: Degree evaluation: Consider $f_k = \varphi'_k$. Let $S_k = \{u \in W_0^{1,p}(\Omega) : f_k(u) = 0\}$.

LEMMA 6. Let $1 \leq k \leq m$.

(i) For any R large enough,

$$\deg(f, B_R(0), 0) = +1$$

(ii) For $k \geq 2$, there is an $\epsilon_k > 0$ such that if $0 < \epsilon \leq \epsilon_k$,

$$\deg(f_k, S_{k-1} + B_\epsilon, 0) = +1$$

(iii) For $k \geq 1$,

$$\deg(f_k, S_k + B_\epsilon, 0) = +1.$$

PROOF: The theory of §2 applies.

First note that $S_k + B_\epsilon$ is open and bounded because if $v \in S_k$, then Av and $\|v\|$ are bounded. We omit in (i) the dependence of φ and f on λ .

(i) We have for $v \in X = W_0^{1,p}(\Omega)$

$$(Av, v) \geq \|v\|^p.$$

Therefore if J is the normalizing duality map and if we define A_t by

$$A_t v = t f(v) + (1 - t) J(v)$$

we have for $\|v\| = R$

$$(A_t v, v) \geq t R^p - C R + (1 - t) R^2 > 0$$

for R large enough. This proves (by homotopy invariance) that

$$\deg(f, B_R, 0) = \deg(A_t, B_R, 0) = \deg(J, B_R, 0) = +1.$$

(ii) (a) We consider the following homotopy: let for $v \in X$ and $0 \leq t \leq 1$

$$A'_t v := t f_{k-1}(v) + (1 - t) f_k(v).$$

Let $v \in S_{k-1} + B_\epsilon$. We claim that if ϵ is small enough then $\deg(A'_t, S_{k-1} + B_\epsilon, 0)$ is defined for all t . Indeed if this were false

$$\forall n, \exists v_n \in X, \exists t_n \in [0, 1]$$

such that

$$0 < d(v_n, S_{k-1}) < \frac{1}{n}$$

and

$$t_n f_{k-1}(v_n) + (1 - t_n) f_k(v_n) = 0.$$

Now $A'_t v_n = \lambda h(v_n)$ with $h \equiv 0$ outside $[a_{k-1}, a_k]$. We have, by the maximum H principle, $v_n \leq a_k$, and v_n must take values $> a_{k-1}$ (because $v_n \notin S_{k-1}$). Thus there is an x_n such that

$$a_{k-1} < v_n(x_n) \leq a_k$$

and (modulo a subsequence),

$$\begin{aligned} v_n &\rightarrow v \in S_{k-1} \\ v_n &\rightarrow v \text{ uniformly (by regularity [9])} \\ x_n &\rightarrow x \in cl(\Omega). \end{aligned}$$

Now $a_{k-1} \leq v(x) < a_k$ by Lemma 5. This is a contradiction. Therefore

$$\deg(f_k, S_{k-1} + B_\epsilon, 0) = \deg(f_{k-1}, S_{k-1} + B_\epsilon, 0).$$

We are thus reduced to the proof of (iii).

(iii) By excision,

$$\deg(f_k, S_k + B_\epsilon, 0) = \deg(f_k, B_R, 0)$$

for R large enough. By (i), this quantity equals +1.

The Lemma is proved.

STEP 3: Non variational solutions: Let $k \geq 2$. u_k is by assumption isolated. Assume that $S_k = S_{k-1} \cup \{u_k\}$. For ϵ small enough (so that $B_\epsilon(u_k) \cap (S_{k-1} + B_\epsilon) = \emptyset$) and R large enough, by the additivity property of the degree and Theorem 1 on $B_\epsilon(u_k)$

$$\begin{aligned} &\deg(f_k, B_R \setminus [(S_{k-1} + cl(B_\epsilon)) \cup (\{u_k\} + cl(B_\epsilon))], 0) \\ &= \deg(f_k, B_R, 0) - \deg(f_k, S_{k-1} + B_\epsilon, 0) - \deg(f_k, \{u_k\} + B_\epsilon, 0) \\ &= -1. \end{aligned}$$

It follows that $S_k \neq S_{k-1} \cup \{u_k\}$. ■

The theorem is proved.

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