Criteria of quantum correlation in the measurement of continuous variables in optics

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The purpose of this short tutorial paper is to review various criteria that have been used to characterize the quantum character of correlations in optical systems, such as "gemellity", QND correlation, intrication, EPR correlation and Bell correlation, to discuss and compare them. This discussion, restricted to the case of measurements of continuous optical variables, includes also an extension of known criteria for "twin beams" to the case of imbalanced correlations.

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I. INTRODUCTION

One of the most striking features of quantum mechanics is the existence of the so-called entangled states, i.e. of quantum states $|\Psi >$ describing a system made of two separable parts which cannot be written as a tensor product of quantum states $|\Psi_1 >$ and $|\Psi_2 >$ describing separately each of the subsystems :

$$|\Psi > \neq |\psi_1 > \otimes |\psi_2 > \tag{1}$$

In such states, there exist strong correlations between measurements performed on the sub-systems. These correlations have been widely studied, almost from the onset of quantum physics, but they still keep a part of their mystery, and therefore of their attraction. The discovery that quantum correlations play an irreplaceable role in information processing gave recently a new impetus to their study.

The existence of correlations between different measurements is obviously not a specific property of quantum physics : it is simply the consequence of a former interaction, whatever its character, between the two parts submitted to the measurement. Consequently, the observation or prediction of a correlation, even perfect, between the measurements of two variables is not at all a proof of the quantum character of the phenomenon under study, in contrast to what can be found sometimes in articles. One can find in the literature a great deal of criteria setting a border between the classical and the quantum effects, differing by the definitions of the quantum character of a given physical situation. The purpose of the present paper is mainly tutorial : it is to give a short overview of the different criteria which are already well-known and extensively used in the literature, to compare them and discuss their domain of relevance. We will also introduce slight original additions to some already known criteria, especially for the criterion of "gemellity" in the case where the two correlated systems do not play symmetrical roles.

Of course, it is impossible to treat the problem of quantum correlations in all its generality in such a short review. We will restrict ourselves here to the domain of the so-called *continuous variables in optics*. More precisely we will consider correlations between measurements performed on light beams in the case where the photons cannot be distinguished individually, and leave aside the correlations between photo-counts and between measurements performed on other observables than the ones arising from photodetection. One of the important application of this work is to be able to precisely assess the quantum aspects of the correlations appearing between two points of the transverse plane of an optical image.

After a short paragraph devoted to the introduction of the problem and of the notations, we will successively focus on the different criteria of quantum correlations, involving either a single correlation measurement, and assessing first the impossibility of a classical description of the phenomenon, then the QND character of the correlation, or involving the measurement of two correlations on non-commuting observables, and assessing first the intrication of the state describing the system, then its EPR character, and finally its impossibility of description in terms of local hidden variables.

II. POSITION OF THE PROBLEM, NOTATIONS

Let us consider two modes of the electromagnetic field, labelled 1 and 2, that can be separated without introducing losses in the system, and that are measured by detectors situated at different locations : the modes can differ either by the frequency, the direction of propagation, the polarization, the transverse shape, or by several of these characteristics. $\hat{a}_1$ and $\hat{a}_2$ are the corresponding annihilation operators, and

$$\hat{X}_i^+ = \hat{a}_i + \hat{a}_i^\dagger \quad \hat{X}_i^- = \frac{\hat{a}_i - \hat{a}_i^\dagger}{i} \quad (i = 1, 2) \tag{2}$$

the quadrature operators. The measurement performed on these modes can be either a direct photodetection, which measures the fluctuations of the amplitude quadrature component, parallel to the mean field $\bar{E}_i$ in the Fresnel representation plane, or a balanced homodyne detection, which measures any quadrature component. We will call in a generic way $\hat{X}_i$ the quadrature component which is measured, and $\delta\hat{X}_i$ its fluctuations.

¿From the fluctuations measured on a single detector, one can deduce the quantity :

$$F_i = \langle \delta \hat{X}_i^2 \rangle \tag{3}$$

equal to 1 when the field is in a coherent state. F_i is the Fano factor [1, 2] of the beam in the case of a direct photodetection, and the quadrature noise normalized to shot noise in a homodyne measurement.

The simultaneous recording of the fluctuations measured by the two detectors allows us to determine the normalized correlation coefficient :

$$C_{12} = \frac{\langle \delta X_1 \delta X_2 \rangle}{(F_1 F_2)^{1/2}} \tag{4}$$

which varies between -1 (perfect anti-correlation) and 1 (perfect correlation).

For the sake of simplicity, we will assume in the following, when it turns out to be necessary, that the system under study has correlations, and not anti-correlations, and therefore that C_{12} is positive. All our following discussion can be readily extended to the case of anti-correlations by exchanging the role of sum and differences between the quantities measured on the two modes. One can also note that the study performed here can be applied to the fluctuations of non-optical physical systems, as soon as a protocol exists to transfer these fluctuations to an optical field, as is done in the case of cold atoms and entanglement between light and cold atoms [3, 4].

Let us consider now the case of an optical experiment which gives an experimental value of C_{12} close to 1. In which respect can one claim the quantum nature of the observed correlations ? That is the question that we will address in the following sections.

III. NON CLASSICAL CHARACTER OF THE CORRELATED BEAMS : "TWIN BEAMS"

One knows that most of the optical phenomena can be explained by using the so-called *semi-classical approach of the light-matter interaction*, in which a quantized matter interacts with the electro-magnetic field treated as a classical quantity, possibly endowed with classical fluctuations. Even the photo-electric effect, for which the photon was introduced by Einstein [5] lies in this category, which also includes all interference effects that are directly measured on the intensity of the field. In this model, the fluctuations which exist in the photo-detection signal are due to the random character of the "quantum jump" occurring in the atom because of its irradiation by the classical field. The minimum noise measured on the photo-detector is the shot noise, or standard quantum noise limit. As explained in many textbooks on Quantum Optics [1, 6, 7], it was realized in the seventies that there existed light states which gave rise to measurements that could not be accounted for within the semi-classical approach. These states are named "non-classical states", and are unveiled by measurements involving either intensity correlations between two photocurrents, or intensity fluctuations of a given photocurrent around the mean. The observation of photon antibunching using single photon states has been historically the first unambiguous experimental situation[8] where a classical description of the field was not able to account for the observed results. Let us note that Herbert Walther has played a major role in the study of these effects, using the light emitted by a single trapped ion [9, 10]. One can therefore define a first level of quantum correlation by the following statement :

Quantum correlation, level 1 : *The correlation measured in the system cannot be described by a semi-classical model involving classical electromagnetic fields having classical fluctuations.*

In the domain of continuous variables and intense beams to which we restrict the present discussion, one shows that the situations where one records quadrature fluctuations above the standard quantum noise limit can be described using the semi-classical approach with classical stochastic electro-magnetic fields [1, 11], and that quantum fluctuations below the this limit are only produced by non-classical states. Squeezed states are one example of non-classical states. The border between the classical and quantum world corresponds to the situation where all the beams used in the experiment are in coherent and vacuum states (Fano factor of 1). Furthermore, it is easy to show that *the classical character of the field is preserved by linear "passive" optical devices*, which involve only linear, energy-preserving, optical elements like beamsplitters and free propagation.

To ascertain whether the correlation between two given beams can be described in a classical frame or not, the simplest way is therefore to process the two beams by all possible linear passive optical devices : if one is able to produce in such a way a beam having fluctuations below the quantum noise limit, the correlation will be termed as "non-classical". This procedure is easy to implement if the two beams have the same frequency. If the two beams have different frequencies ω_1 and ω_2, it is more difficult but still possible, at least in principle : the noise reduction will then be measured by a homodyne detection scheme using a local oscillator at frequency $(\omega_1 + \omega_2)/2$, and will appear at a noise frequency $\Omega = |\omega_1 - \omega_2|/2$.

A. Classical correlation

Let us first consider the simplest way to produce correlated beams by classical means : one inserts a 50% beamsplitter in a given classical beam, which is thus divided into two output beams having a degree of correlation that can be simply calculated. Taking into account the vacuum mode entering through the unused port of the beamsplitter, one obtains the following value for the correlation obtained by splitting an input classical field on a 50% beamsplitter :

$$[C_{12}]_{class} = \frac{F_{in} - 1}{F_{in} + 1} \tag{5}$$

where F_{in} is the Fano factor of the input beam on the beamsplitter, or equivalently by :

$$[C_{12}]_{class} = 1 - \frac{1}{F} \tag{6}$$

where F is the common value of the Fano factor of the two beams at the output of the beamsplitter ($F = (F_{in}+1)/2$).

Let us note that C_{12} tends to 1 when F_{in} goes to infinity, i.e. when the vacuum noise of the second input can be neglected with respect to the proper noise of the input beam. A very strong correlation is therefore not always the sign of a quantum origin : it can be just the reverse, and due to the fact that the quantum fluctuations can be neglected in the problem ! The normalized correlation factor C_{12} is thus not the most unambiguous way to appreciate the quantum character of a correlation.

B. "Gemellity"

Let us now exploit the operational definition of the quantum character of the correlation given at the beginning of this section, which is to use a linear passive operation which transforms the correlation into a sub-shot noise beam. In the case of two beams, this operation simply consists of recombining the beams on a beamsplitter of variable transmission and reflection after variable optical paths. The phases are adjusted so that one mixes the relevant quadrature components X_1 and X_2. One eventually obtains a beam having quadrature fluctuations $\delta\hat{X}_{out}$ given by :

$$\delta\hat{X}_{out} = r\delta\hat{X}_1 - t\delta\hat{X}_2 \tag{7}$$

r and t being adjustable amplitude reflection and transmission coefficients. If the minimum noise on this beam is below the standard quantum limit, we are sure that the initial correlation can only be described in a full quantum frame. We will name by the neologism "gemellity" ("twinship") the minimum variance of this quantity, labelled G, which can be found to be :

$$G = \frac{F_1 + F_2}{2} - \sqrt{C_{12}^2 F_1 F_2 + \left(\frac{F_1 - F_2}{2}\right)^2} \tag{8}$$

and state :

$$G < 1$$
$$\Downarrow$$

Impossibility of a classical description of correlated beams

C. Balanced case

Let us first consider the case of two beams of equal means and noises, so that $F_1 = F_2 = F$. In this case, the gemellity has a very simple expression :

$$G = F(1 - |C_{12}|) \tag{9}$$

The reflection and transmission amplitude coefficients r and t are both equal in this case to $1/\sqrt{2}$, so that G can also be written as :

$$G = \frac{\langle \left(\delta \hat{X}_1 - \delta \hat{X}_2\right)^2 \rangle}{2} \tag{10}$$

It is nothing else than the normalized noise on the difference between the fluctuations of the two measurements, and can be easily monitored by simple electronic means. In the classical case described in the previous paragraph, it is easy to show that G takes the value 1, whatever the initial Fano factor F_{in} of the beam. If the gemellity G has a value smaller than 1, the two beams have identical mean values and almost identical fluctuations (within the quantum noise). Such beams are usually named "twin beams", in a way reminiscent of the "twin photons" studied in the photon counting regime. One can distinguish between "intensity twin beams", where the measured quadrature is the amplitude quadrature (in that case, the measured gemellity is equivalent to the normalized difference of the intensity fluctuations of the two beams), and which are produced by above threshold OPOs [12, 13] or by the mixing on a 50% beamsplitter of a coherent state and a squeezed vacuum [14], and "quadrature twin beams", which are produced by non degenerate OPOs below threshold [15, 16]. The smallest measured value of the gemellity G is to the best of our knowledge $G = 0.11$ [17].

The non-classical region ($G < 1$) corresponds to correlations C_{12} larger than $1 - 1/F$. The correlation likely to produce non-classical twin beams has a lower limit which is more and more close to 1 when the two fields have more and more excess noise. If each field is at shot noise, any non-zero correlation is a proof of gemellity, and therefore of non-classical character.

D. Unbalanced case

Unbalanced beams may have also strong, or even perfect, classical correlations. To produce classically correlated fields of unequal intensities and fluctuations, one can use a non equal beam-splitter with different amplitude transmission and reflection coefficients. In this case, the correlation C_{12} is found to be

$$[C_{12}]_{class} = \sqrt{(1 - \frac{1}{F_1})(1 - \frac{1}{F_2})} \tag{11}$$

which is the generalization of relation (6). This amount of correlation, as expected, gives a value larger than or equal to 1 to the gemellity G, defined by Eq(8).

If F_1 or F_2 is equal to 1, Expression(8) implies that any non-zero correlation C_{12} gives a value of G smaller than 1 : any correlation between a field at shot noise and another field has thus a quantum origin.

In order to experimentally determine the gemellity, one uses the operational definition : it is the minimum noise -normalized to shot noise - obtained when one mixes the two considered beams on a beamsplitter of variable transmission and reflection. The gemellity criterion for a non-classical correlation between unbalanced beams is interesting from an experimental point of view, because in a given experimental situation the two measured beams do not have necessarily the same mean power and noise [18, 19, 20].

IV. NON-CLASSICAL CHARACTER OF THE MEASUREMENT PROVIDED BY THE CORRELATION : "QND-CORRELATED BEAMS"

When two observables M_1 and M_2 are perfectly correlated, the measurement of M_2 gives without uncertainty the value of M_1. The first measurement is thus a *Quantum Non Demolition measurement* (QND) of the observable M_1 performed on the second sub-system.

We can now define a second level in the quantum character of correlations :

Quantum correlation, level 2 : *The information extracted from the measurement on one field provides a Quantum Non Demolition measurement of the other.*

In the last decade, many studies have been devoted to the precise definition of QND criteria[21], that we can use now in our discussion. In the present case, the "Non Demolition" part of the measurement is automatically ensured, as the measurement, performed on beam 2, does not physically affect the measured system, which is beam 1. Its quantum character is effective when the measurement is able to provide enough information on the instantaneous quantum fluctuations of the other beam so that it is possible, using the information acquired on mode 2, to correct mode 1 from its quantum fluctuations and transform it into a non-classical state in the meaning of the previous section by a feed-back or feed-forward electronic device. This criterion is well known in QND studies[22], where it is shown that it is equivalent to state that the *conditional variance* $V_{1|2}$ of beam 1 knowing beam 2 takes a value smaller than 1. The conditional variance has the following expression in terms of the Fano factor of beam 1 and the normalized correlation C_{12} between the two :

$$V_{1|2} = F_1(1 - C_{12}^2) \tag{12}$$

A. Balanced case

Let us first consider the case where the two beams have identical mean values and fluctuations ($F_1 = F_2 = F$). In this case there is only one conditional variance $V_{1|2} = V_{2|1} = V$, and the criterion for "QND-correlated beams" is :

$$V_{1|2} = V_{2|1} = V < 1 \tag{13}$$

The conditional variance and the gemellity are related by :

$$V = G(1 + C_{12}) = 2G - \frac{G^2}{F} \tag{14}$$

so that :

$$G \leq V \leq 2G \tag{15}$$

One notices that the conditional variance is always bigger than the gemellity, so that all the QND-correlated beams are twin beams, whereas the reverse is not true. We see also that a small enough gemellity, namely smaller than 0.5, implies that the beams are QND-correlated.

It is possible to show [22] that the conditional variance can be directly measured by using an adjustable amplification on one of the two photocurrents, i.e. by measuring the quantity :

$$\hat{X}_g = \hat{X}_1 - g\hat{X}_2 \tag{16}$$

The conditional variance is equal to the minimum value of $< \delta\hat{X}_g^2 >$ when g is varied.

B. Unbalanced case

In this case the two conditional variances are different, and there are two possible criteria $V_{1|2} < 1$ and $V_{2|1} < 1$. They are not always simultaneously satisfied : there exist situations where $V_{1|2} < 1$ and $V_{2|1} > 1$ for example. This shows that the QND criterion evaluates the correlation from the point of view of one beam, and the information that one can have on this beam from measurements on another one, and does not intrinsically quantize the quantum correlation between the two fields.

It is easy to show that it is enough to have one of the two conditional variances smaller than 1 to have twin beams. In contrast, there are regions of the parameter space where G is smaller than 0.5 and where one of the two conditional variances is bigger than 1.

We will therefore give an "asymmetrical" criterion to characterize this second level of quantum correlation :

$$V_{1|2} < 1 \quad \textbf{or} \quad C_{12} > \sqrt{1 - \frac{1}{F_1}}$$

$$\Downarrow$$

Possibility of a QND measurement of beam 1 using the correlation between beams 1 and 2

V. IMPOSSIBILITY OF DESCRIPTION BY A STATISTICAL MIXTURE OF FACTORIZABLE STATES : "INSEPARABLE BEAMS"

We now define a new level in the quantum character of correlations, related to the entangled character of the state, as already stated in the introduction :

Quantum correlation, level 3 : *The correlation arises from a system which can be described only by an entangled or non-separable quantum state.*

Let us first consider the case of a pure state, which is described by a state vector $|\Psi>$. If this vector can be written as a tensor product of states belonging to each Hilbert sub-space (i.e it is not entangled nor non-separable), the mean value of a product of observables $\hat{O}_1$ and $\hat{O}_2$ acting separately in the two sub-spaces (1) and (2) will be the product of the mean value of each observable : there will be therefore no correlations in such a system, whatever the two observables. So, if one finds a non-zero correlation on a single couple of observables, even when this correlation is weak, it is a proof that the system is in an entangled state : correlation implies entanglement for pure cases.

Reciprocally, if the system is described by an entangled state $|\Psi>$ of the form (1), what are the conditions to get a non-zero correlation between two observables $\hat{O}_1$ and $\hat{O}_2$? One knows that $|\Psi>$ can be written in the following form (Schmidt decomposition [23]) :

$$|\Psi>=\sum_j |\psi_{1,j}>\otimes|\psi_{2,j}> \tag{17}$$

where the states $|\Psi_{i,j}>$ belong to the Hilbert space of the sub-system labelled (i) ($i=1,2$). Non-zero correlations will happen when, firstly, the measurement on sub-system (1) is performed on an observable $\hat{O}_1$ which has not all the states $|\Psi_{1,j}>$ in the same eigenspace, so that the state reduction due to the measurement changes the total state $|\Psi>$. Secondly, in order to affect the measurement performed on an observable $\hat{O}_2$ on system (2) the states $|\Psi_{2,j}>$ must not be in the same eigenspace of $\hat{O}_2$.

These arguments prove that the presence of entanglement in a pure state does not imply that any couple of observable will be correlated, and, if a correlation between two observables is obtained, it does not imply that the correlation has reached even the level 1 of quantum correlation. The requirement of the quantum description of the correlation (twin beams) is therefore stronger than the requirement of having an entangled state.

The situation is completely different if one allows the system to be in a *statistical mixture of quantum states*, so that it is described by a density matrix instead of a state vector: in this case, the existence of a correlation between two measured quantities does not imply that the system is in an entangled state. A single correlation, even perfect, between a given observable of sub-system (1) and a given observable of sub-system (2) can be obtained with "separable states" in the meaning of [24], i.e. with states that are classical statistical mixtures of factorizable states. They can be written as :

$$\rho=\sum_j p_j|\psi_{1,j}>\otimes|\psi_{2,j}><\psi_{1,j}|\otimes<\psi_{2,j}| \tag{18}$$

with p_j positive real numbers such that $\sum_j p_j=1$. States which cannot be written as (18) will be called non-separable. They are also named "entangled states" in an extended meaning.

Let us consider as an example the system described by the separable density matrix :

$$\rho=\sum_n p_n|n:1,n:2><n:1,n:2| \tag{19}$$

where $|n:1,n:2>$ is a Fock state with the same number n of photons in the two modes (1) and (2). This state yields a perfect intensity correlation between the two modes, and satisfies the two previous criteria : the correlation C_{12} is 1, and therefore the gemellity G is zero, as well as the conditional variances $V_{1|2}$ and $V_{2|1}$.

Note that the state described by (19) is indeed very "quantum", in spite of not being entangled or non-separable, as it is built from Fock states having exactly the same number of photons in the two modes, which cannot be produced classically, but only through cascade processes, such as parametric down-conversion. We see here that quantum correlations and entanglement are different notions, which are of course related, but not in a straightforward and simple way.

Duan et al. [24] have shown that in order to ascertain the separable character of the physical state of a system, one needs to make *two joint correlation measurements on non-commuting observables* on the system, and not only one, as was the case in the two previous sections. They have shown that in the case of Gaussian states, there exists a necessary and sufficient criterion of separability in terms of the quantity S_{12}, that we will call "separability", and is given by :

$$S_{12} = \frac{1}{2}\left(\langle\delta(\hat{X}_{+1} - \hat{X}_{+2})^2\rangle + \langle\delta(\hat{X}_{-1} + \hat{X}_{-2})^2\rangle\right) \tag{20}$$

The separability S_{12} appears as the sum of the gemellity G_+ measuring the correlations between real quadrature components of the two beams, and the (anti)gemellity G_- measuring the anticorrelation between the imaginary quadrature component of the same beams (defined with a + instead of a − in equation (**??**)).

The third level of quantum correlation is evaluated by the well-known Duan criterion, which writes :

$$S_{12} < 2$$

$$\Downarrow$$

Quantum correlation arising from an entangled or non-separable state

This criterion allows us to establish some relations between the different levels of quantum correlations that we have already considered.

For example, classical beams will give values larger than 1 for the gemellities measured on any variables, and in particular on $\hat{X}_+$ and $\hat{X}_-$. In this case, the quantity S_{12} is larger than 2, and the two beams are therefore separable. A contrario, non separable beams imply that at least one of the two gemellities is smaller than 1, and therefore that the beams are at least "twins", in intensity or in phase. For quadrature measurements on statistical mixtures of Gaussian states, the non-separability criterion implies that the criterion 1 is fulfilled and is therefore stronger than this latter one. Note that the beams are not necessarily QND-correlated in one of these variables., so that level 2 is not necessarily reached.

Non separable beams are usually prepared by mixing two non-classical states, such as squeezed states, on a beam-splitter [25], but it has been shown [26] that one can generate an entangled state from a single squeezed beam mixed with a coherent state plus some well adapted linear processing of the two output beams.

VI. POSSIBILITY OF QND MEASUREMENT OF TWO CONJUGATE VARIABLES : "EPR BEAMS"

In their famous paper, Einstein, Podolsky and Rosen [27] have exhibited the following wavefunction for the continuous variables position $\hat{x}_i$ and momentum $\hat{p}_i$ $(i = 1, 2)$ of two particles :

$$\psi(x_1, x_2) = \int_{-\infty}^{+\infty} e^{ip(x_1 - x_2 + x_0)/\hbar} dp \tag{21}$$

where x_0 is a constant, and shown that it provides perfectly correlated position measurements and perfectly anti-correlated momentum measurements of the two particles. This state, which is obviously entangled, can be readily transferred in the domain of quadrature operators of two light modes [28]. In quantum optics terms, it allows us to perform perfect QND measurements of the two quadrature components of mode 1, by measurements performed only on beam 2. This perfect information that one eventually gets on the two quadratures of the field apparently contradicts the fact that these measurements are associated to two non-commuting operators, and therefore obey a Heisenberg inequality.

We now reach a fourth level in the quantum character of the correlations :

Quantum correlation, level 4 : *The information extracted from the measurement of the two quadratures of one field provides values for the quadratures of the other which violate the Heisenberg inequality*

This situation has been extensively considered and discussed by M. Reid and co-workers [28, 29], which have shown that this violation is only apparent, and does not violate the basic postulate of Quantum Mechanics. They have introduced the following criterion to characterize this fourth level of quantum correlation of the so-called "EPR beams" :

$$V_{X_{+1}|X_{+2}} V_{X_{-1}|X_{-2}} < 1$$

$$\Downarrow$$

Possibility of an apparent violation of the Heisenberg inequality
for the quadratures components of beam 1 through measurements performed on beam 2

where $V_{X_{+1}|X_{+2}}$ is the conditional variance of X_{+1} knowing X_{+2}, and $V_{X_{-1}|X_{-2}}$ is the conditional variance of X_{-1} knowing X_{-2}.

This condition is related somehow to the QND-correlated beams of section 4. It can be written in terms of the normalized correlation $C_{X_{+1}X_{+2}}$ and anticorrelation $C_{X_{-1}X_{-2}}$:

$$\left(1 - C^2_{X_{+1}X_{+2}}\right)\left(1 - C^2_{X_{-1}X_{-2}}\right) > \frac{1}{F_+F_-} \tag{22}$$

where F_+ and F_- are the noise variance on quadratures $+$ and $-$ normalized to shot noise (fulfilling $F_+F_- > 1$).

The EPR correlation turns out to be stronger than the non-separability correlation, in the same way as the QND criterion of section 4 is stronger than the non-classical criterion of section 3 : it has been shown [30] that all EPR beams are non-separable, whereas the reverse is not true. In the same article, *Bowen et al.* show that for pure states the two criteria correspond to the same physical states. However, *Duan* criteria is robust with respect to the mixed character of the fields states, whereas the EPR criteria can not be fulfilled in the presence of more than 50% of losses. Let us stress that this behaviour is linked to the *no-cloning* theorem : it has been proved that linear amplification and a 50/50 beam-splitter produces the best possibles two copies (clones) of any input state [31]. Hence, when two beams are EPR correlated, i.e. that less than 50% losses are present, on a quantum information point of view we are sure than no spy has a better copy of the state. This is in the same way relevant for the success of teleportation [32].

VII. IMPOSSIBILITY OF DESCRIPTION OF THE MULTIPLE CORRELATION BY LOCAL CLASSICAL STOCHASTIC VARIABLES : "BELL BEAMS"

Quantum fluctuations can be mimicked in many instances by classically-looking stochastic supplementary variables. This is the case in particular when one uses the approach of "vacuum fluctuations", behaving as classical fluctuations, but with a variance given by quantum mechanics and carrying no energy. This is to be distinguished from the classical stochastic fields, originating from the uncontrolled variations of the classical parameters of the light source. Bell [33] has shown that such a stochastic modelling was not likely to reproduce all the correlations that can be encountered in quantum mechanical systems when these supplementary stochastic variables (usually named "hidden variables") were local, i.e. attached to the sub-system under measurement. He introduced inequalities fulfilled by any local hidden variable models, and violated in some very specific situations of quantum mechanical correlated systems.

We must therefore introduce a new level of quantum correlations :

Quantum correlation, level 5 : *The multiple correlations of the system cannot be described by local hidden variable approaches*

The corresponding criterion for this level of quantum correlation is the celebrated Bell inequality [33]. We will not go into the details of it here for the following reason : one shows in Quantum Optics that all phenomena can be described through the use of quasi-probability distributions [1, 7], such as the Wigner representation or others. For the special case of light beams having a Gaussian statistics, which the case of all the physical situations encountered so far in the regime of continuous variables in optics, the Wigner representation is everywhere positive : the quasi-probability distribution becomes a true probability distribution, the evolution of which can be mapped into stochastic equations for fluctuating fields : these stochastic fields constitute in this case the local "hidden" variables which account for all the observed quantity, including the variances and the correlations between measurements. This means that there is never a violation of the Bell inequality in the continuous variable regime with Gaussian states, and the level 5 of quantum correlations is never reached in this case.

To reach it, one needs to deal with non Gaussian states, with partly negative Wigner functions, such as the Fock states, Schrödinger cat states [34], or states produced through conditional non-Gaussian measurements like photon-counting. The discussion of such situations is beyond the scope of this simple introductory paper on quantum correlations.

VIII. CONCLUSION

The exploration of the quantum world, in which professor Walther has undoubtedly played a major role, has unveiled physical situations which are looking more and more strange for an observer only acquainted to the certainties of classical physics. We have tried in this short review to assess and classify the "degree of oddness" of quantum optical phenomena. In the last decades, theoreticians and experimentalists have gone higher and higher in such a ladder of pure quantum effects. There is no doubt that they are far from reaching the top of the quest, and that new heights of even stranger quantum properties will be attained in the future.

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