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# Switching by agents between two trading behaviors and the stylized facts of financial markets.

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## Abstract

I propose an agent-based model of a single-asset financial market, described in terms of a small number of parameters. I show that switching by agents between two trading behaviors (informed vs. liquidity traders) leads to a market price that fluctuates endlessly and a volatility that displays a mean-reverting behavior.

This agent-based model generically leads to price returns having statistical properties similar to the stylized facts observed in financial time series: an absence of autocorrelation in returns, stochastic volatility, excess volatility, volatility clus-

tering that is not attributable to the external signal. The model's parsimonious structure allows the identification of the mechanism leading to these effects. I investigate some properties of this model theoretically and present analytical results.

*Keywords:* agent-based model, financial markets, stylized facts, asynchronous switching of trading rules, informed vs. liquidity traders.

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Agent-based models studied in the literature have pointed to various possible origins for empirical stylized facts common to a wide variety of markets, instruments, and periods: herd behavior, social interaction and mimetism, heterogeneity, investor inertia, and switching between “chartist” and “fundamentalist” behavior have been suggested as possible mechanisms. However, an appeal for simplicity is necessary to enhance the explanatory power of such models.

I propose here a parsimoniously parametrized agent-based model of a single-asset financial market that generates returns with statistical properties similar to the stylized facts observed in financial time series. This model generically leads to an absence of autocorrelation in returns, mean-reverting stochastic volatility, excess volatility, and volatility clustering. The structure of the model allows identifying a simple stochastic process of switching by agents between two trading behaviors (informed versus liquidity traders) as the key mechanism leading to these effects.

The article is structured as follows. Section 1 recalls some stylized empirical facts about returns of financial assets and reviews some agent-based models presented used to explain the stylized facts. Section 2 presents my model. Simulation results are presented in section 3, and a theoretical analysis is given in

section 4.

# 1 Agent-based approach for market phenomena.

Time series of asset returns exhibit non trivial and intriguing statistical features that seem common to a wide range of markets and time-periods, Cont (2001):

- **Excess volatility:** the level of variability in market prices is much higher than can be expected from the variability of the fundamental economic variables and is unexplained by the arrival of news as noted by Cutler et al. (1989) and Shiller (2000).

- **Heavy tails:** the (unconditional) distribution of returns displays a heavy tail with positive excess kurtosis.

- **Absence of autocorrelation in returns:** (linear) autocorrelations of asset returns are often insignificant, except for very small intraday time scales ( $\simeq 20$  minutes) for which microstructure effects come into play.

- **Volatility clustering:** while returns themselves are uncorrelated, absolute

or squared returns  $|r_t(\Delta)|$  display a positive, significant, and slowly decaying autocorrelation function:  $\text{corr}(|r_t|, |r_{t+\Delta}|) > 0$  for  $\Delta$  ranging from a few minutes to several weeks (e.g., Mandelbrot, 1963).

- **Volume/volatility correlation:** trading volume is positively correlated with market volatility.

Agent-based market models, which are based on a stylized description for the behavior of agents, attempt to explain the origins of the observed behavior of market prices that emerges from simple behavioral rules for a large number of heterogeneous market participants. Stauffer (2001) proposes, for example, percolation models that generate a price behavior having statistical properties similar to those observed in real markets. Giardina & Bouchaud (2003) proposes an agent-based model inspired from the Santa Fe artificial market and the Minority Game and reviews several mechanisms, that lead to long-range correlations, such as switching between two trading styles.

Other possible explanations for empirical stylized facts suggested in the literature include:

● **mimetism** (Orléan, 1995).

● **herd behavior** (Cont & Bouchaud, 2000 and Stauffer et al., 1999).

● **switching between “chartist” and “fundamentalist” behavior** (Lux & Marchesi, 2000, Kirman & Teyssiere, 2002, Brock & Hommes, 1998 and Farmer & Joshi, 2002).

● **heterogeneity in expectations concerning future prices** (Gauernsdorfer, 2000; Arthur et al., 1997; Hommes et al., 2003), **towards risk** (Chiarella et al, 2002) **and in agent’s time scale** (LeBaron, 2001).

● **investor inertia** (Cont & Bouchaud, 2000 and Horst et al., 2003).

Numerical simulations of many of these models lead to time series of “returns” that have properties consistent with some of the empirical stylized facts noted above. However, due to the complexity of such models it is often not clear



which aspect of these models is responsible for producing the stylized facts and if all the elements of the model are required. Thus, an appeal for simplicity is in order, and hence I will propose an agent-based market model able to be described with only a few parameters.

An important issue in microstructure studies is the competition between informed traders and liquidity providers-or uninformed traders. The information-based microstructure models in Kyle (1984), Glosten & Milgrom (1985), Easley & O'Hara (1987) typically assume that uninformed traders do not act strategically. With this in mind, I will describe here the dynamics of asynchronous switching by agents from an informed trader group to a liquidity provider group. This mechanism is a crucial ingredient in formulating my market model.

## **2 Description of the model.**

My model describes a market where a single asset, whose price is denoted by  $p_t$ , is traded by  $N$  agents. Trading takes place at discrete dates  $t = 0, 1, 2, ..$ . Provided the model parameters are chosen in a certain range, these periods may be

interpreted as “trading days”. At each period, informed agents receive common news about the asset’s performance and place buy or sell orders depending on whether the news received is pessimistic or optimistic. Liquidity-provider agents systematically place the opposite order. Prices then move up or down according to excess demand. Agents switch their trading behavior asynchronously. These ingredients are now described in more precise terms.

## 2.1 Trading rules.

At each period, agents may send buy or sell order to the market for a unit of asset. Let us denote by  $\phi_i(t)$  the demand of the agent, where  $\phi_i(t) = 1$  for a buy order and  $\phi_i(t) = -1$  for a sell order. The inflow of new information is modeled by a sequence of IID Gaussian random variables  $(\epsilon_t, t = 0, 1, 2, \dots)$  with  $\epsilon_t \sim N(0, D^2)$ .  $\epsilon_t$  represents the value of a common signal received by informed agents at date  $t$ .

The trading rule for an informed agent is to follow the signal when he considers it significant. The  $N_A$  informed traders place the same order  $\phi_A(t)$  according to:

$$\phi_A = 1 \quad \text{if} \quad \epsilon_t > \theta \quad (1)$$

$$\phi_A = -1 \quad \text{if} \quad \epsilon_t < -\theta \quad (2)$$

$$\text{otherwise} \quad \phi_A = 0. \quad (3)$$

This threshold response leads to investor inertia. That, however, will be seen to have no effects on the dependence properties of the returns. The trading rule for liquidity-provider agents, or uninformed traders, is to place an order opposite to the informed traders. The  $N_B$  agents of this type place the same order  $\phi_B(t)$ :

$$\phi_B(t) = -\phi_A(t). \quad (4)$$

## 2.2 Price response to aggregate demand.

Aggregate excess demand is:

$$Z_t = \sum_i \phi_i(t). \quad (5)$$

When  $Z_t$  is non zero, there is a change in the price, and the resulting log return is given by

$$r_t = \ln \frac{p_t}{p_{t-1}} = g\left(\frac{Z_t}{N}\right), \quad (6)$$

where the price impact function  $g : \mathfrak{R} \rightarrow \mathfrak{R}$  is increasing in its argument with  $g(0)=0$ . We define the (normalized) market depth  $\lambda$  by

$$g'(0) = 1/\lambda. \quad (7)$$

While most of the analysis below holds for a general price impact function  $g$ , in some cases it will be useful to consider the linear case:  $g(z) = z/\lambda$ .

I do not describe the details of the mechanism of price formation. One could assume, for example, that prices are set period-by-period via a market maker mechanism.

### 2.3 Switching of trading rule.

The switching of trading rules is *asynchronous*: at each time step, any agent has a probability  $s$  of switching from one trading rule to the other, for exam-

ple, from being informed trader behavior to liquidity provider. Thus, in a large population,  $s$  represents the fraction of agents changing their behavior at any period;  $1/s$  represents the typical time period over which an agent will retain a given trading rule. A possible explanation for switching is that information is costly and not worth obtaining at each time step.

## 2.4 Extreme events.

This part of the model is based on the intuition that when information is not available in the market, agents seek it at next time step. A parameter  $\theta$  is introduced to describe the inability of agents to access information. If the amplitude of the news is below this threshold, no trade occurs. When this happens, all agents seek information at next time step. There is no more liquidity-provider agents and this will be considered an extreme event causing a large fluctuation in the prices. The value of the threshold  $\theta$  will determine the frequency of extreme events, in other words, it will determine how rare are these large fluctuations.

## 2.5 Summary.

Let us recall the main ingredients of the model described above. At each time period, informed agents follow a common signal  $\epsilon_t$ , liquidity-provider agents submit an opposite order; agents switch from one type of trading rule to the other with a probability  $s$ ; the market price is affected by the excess demand and moves according to (4).

With regard to some of the agent-based models considered in the literature, some important aspects are the following: there is no exogeneous “fundamental price” process: prices move through market fluctuations of supply and demand. In particular, I do not distinguish between “fundamentalist” and “chartist” traders, also, I do not introduce any “social interaction” among agents. In particular, no notion of locality, lattice, or graph structure is introduced.

The model has very few parameters:  $s$  describes the average updating frequency,  $D$  is the standard deviation of the news arrival process,  $\lambda$  is the market depth and  $\theta$  is the level of threshold response. Nevertheless, this simple model generates time series of returns with interesting dynamics and with properties similar

to those observed empirically.

I am not concerned here with describing the details of the transactions mechanism or any problems of optimization in each agent's portfolio. I simply describe two types of agents, assuming them to be the main source of price fluctuations for a single asset; I do not describe the whole market. However, it would be interesting to examine the effects of adding constraints on the agents' behavior (budget constraints, for example), and this will be investigated in an other work.

### **3 Numerical simulations**

The model described above is straightforward to simulate. I describe the simulation procedure here and present some typical results. The simulations identify generic properties of the model and indicate the range of parameters that accord with empirical data on asset returns.

### 3.1 Simulation procedure.

Simulation is done through an iterative procedure, each iteration repeating the steps described in Section 2.4. Although the model allows for more general price impact functions, in the absence of an empirically motivated parametric form, I have chosen a linear function  $g(x) = x/\lambda$ . This choice can be viewed as a linearization of a more general  $g$ , valid for small values of excess demand or for markets with large market depth.

Usually in Monte Carlo simulations, expectations, moments, and distributions of quantities of interest are computed from averages over many independent simulation runs. However, for direct comparison with empirical stylized facts to be meaningful, we should consider that only a single sample path of the price is available and compute the (unconditional) moments by averaging over the (single) sample path. I adopt such an approach here: after simulating a sample path of the price  $p_t$  for  $T = 10^4$  periods, I compute the following quantities:

- the time series of returns  $r_t = \ln(p_t/p_{t-1}), t = 1..T$ .
- the histogram of returns, which is an estimator of its unconditional distri-



bution.

- a moving average estimator of the standard deviation of returns:

$$\hat{\sigma}^2(t) = 250 \left[ \frac{1}{T'} \sum_{t'=t-T'+1}^t |r_{t'}|^2 - \left( \frac{1}{T'} \sum_{t'=t-T'+1}^t r_{t'} \right)^2 \right]. \quad (8)$$

This quantity is a frequently used indicator for “volatility”. We “annualize” it by multiplying the “daily” estimate by 250.

- the sample autocorrelation function of returns:

$$C_r(\tau) = \frac{T}{\sum_{t=1}^T |r_t|^2} \left[ \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} r_t r_{t+\tau} - \left( \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} r_t \right) \left( \frac{1}{T-\tau} \sum_{t=\tau+1}^T r_t \right) \right] \quad (9)$$

- the sample autocorrelation function of absolute returns:

$$C_{|r|}(\tau) = \frac{T}{\sum_{t=1}^T |r_t|^2} \left[ \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} |r_t| |r_{t+\tau}| - \left( \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} |r_t| \right) \left( \frac{1}{T-\tau} \sum_{t=\tau+1}^T |r_t| \right) \right] \quad (10)$$

These quantities can then be compared to those of the empirical stylized facts described in Section 1.1. Finally, to decrease the sensitivity of results to initial conditions, I allow for an initial transitory regime and discard the first  $10^3$  periods before averaging.

### 3.2 Choosing the range of parameters

Simulation of the model requires the specification of the parameters  $s$ ,  $\lambda$ ,  $\theta$  and the number of agents  $N$ .

To be able to interpret the trading periods as “days” in order to compare the results to properties of daily returns, some restrictions must be imposed on the parameter values. First, note that the one-period returns are bounded by  $\max\{|g(x)|, x \in [-1, 1]\}$ . In the case where  $g$  is linear  $|r_t| \leq \frac{1}{\lambda}$ .

This suggest that the (normalized) market depth  $\lambda$  should not be too large in order to allow for a realistic range of daily returns. I choose  $1 \leq \lambda \leq 10$ . In practice, varying  $\lambda$  within this range does not affect the qualitative properties of the return process.

As noted above,  $1/s$  represents the average number of periods an agent takes to update his trading rule. I get realistic long-range correlations in the amplitude of the returns when  $s$  is close to 1 and for  $s \ll 1$ .

I choose  $N = 100$  to get a realistic number of investors in a market.

I take  $D = 10^{-3}$  to get excess volatility and to have a realistic variability in the fundamental economic variables.

The choice of  $\theta$  determines the frequency of extreme events and the excess kurtosis of the distribution of returns. I choose  $\theta = 10^{-6}$  in the simulations presented.

Note that when  $\theta = 0$ , the kurtosis is close to 3 (gaussian distribution).

It should be emphasized that I am discussing the calibration of the *order of magnitude* of parameters, and am not fine-tuning them to a set of critical values. These results given in the following subsection are characteristic of those obtained within this range of parameters, that allow for a comparison with daily returns.

### 3.3 Simulation results.

Using the parameter ranges above, I perform an extensive simulation study of price behavior in this model. Figures 1 and 2 illustrate typical sample paths obtained with different parameter values: they all generate series of returns with realistic ranges and realistic values of annualized volatility. For each series, the figures also give the histogram of returns both in linear and log scales, and the ACF of returns  $C_r$ , the ACF of absolute returns  $C_{|r|}$ .

We note that all the return series possess some regularities that match some of the empirical properties given in Section 1.1:

- Excess volatility: the sample standard deviation of returns can be much larger than the standard deviation of the input noise representing news arrivals

$$\hat{\sigma}(t) \gg D. \tag{11}$$

In the simulation shown in figure 1,  $D = 10^{-3}$ , which corresponds to an annualized volatility of 1.6%, while the annualized volatility of returns is in the range of 30%, an order of magnitude larger. This is a basic phenomenon also observed

in other simulations: the order of magnitude of the volatility of returns may be totally different from that of the input noise.

- Mean-reverting volatility: the market price fluctuates endlessly and displays “stochastic volatility”: the volatility, as measured by the moving average estimator  $\hat{\sigma}(t)$ , goes neither to zero nor to infinity and displays a mean-reverting behavior. Many microstructure models, especially those with learning or evolution, when observed over large time intervals, converge to an equilibrium where prices and other aggregate quantities cease to fluctuate randomly. Of course, this is not observed in financial markets: prices fluctuate endlessly and the volatility exhibits mean-reverting behavior. Indeed, this behavior is found in many empirical studies. GARCH models (e.g., Engle, 1995), on one hand, and stochastic volatility models, on the other, aim at reproducing this mean-reverting stochastic behavior of volatility.

- The simulated process generates a leptokurtic distribution of returns with heavy tails, with an excess kurtosis around  $\kappa \simeq 11$ .

- The returns are uncorrelated: the sample autocorrelation function of the return exhibits an insignificant value (very similar to that of asset returns) at all lags, indicating the absence of linear serial dependence in the returns.

- Volatility clustering: the autocorrelation function of absolute returns remains positive, and significantly above the autocorrelation of the returns, over many time lags, corresponding to persistence in the amplitude of returns over periods ranging from a few weeks to several months. This is an indication of nonlinear dependence in the returns. Note that this dependence is not function of the threshold level  $\theta$ .

I will now analyze the dependence properties of the returns in the model.

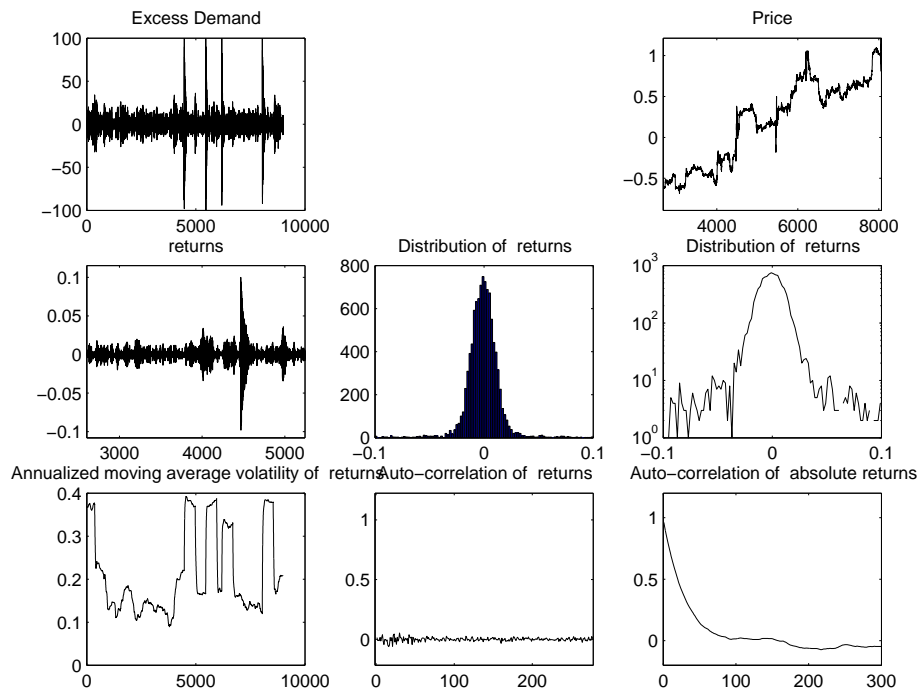


Figure 1: Numerical simulation of the model with updating frequency  $s = 0.01$  (updating period: 100 “days”),  $N = 100$  agents,  $\lambda = 10$ ,  $D = 0.001$ ,  $\theta = 10^{-6}$ .

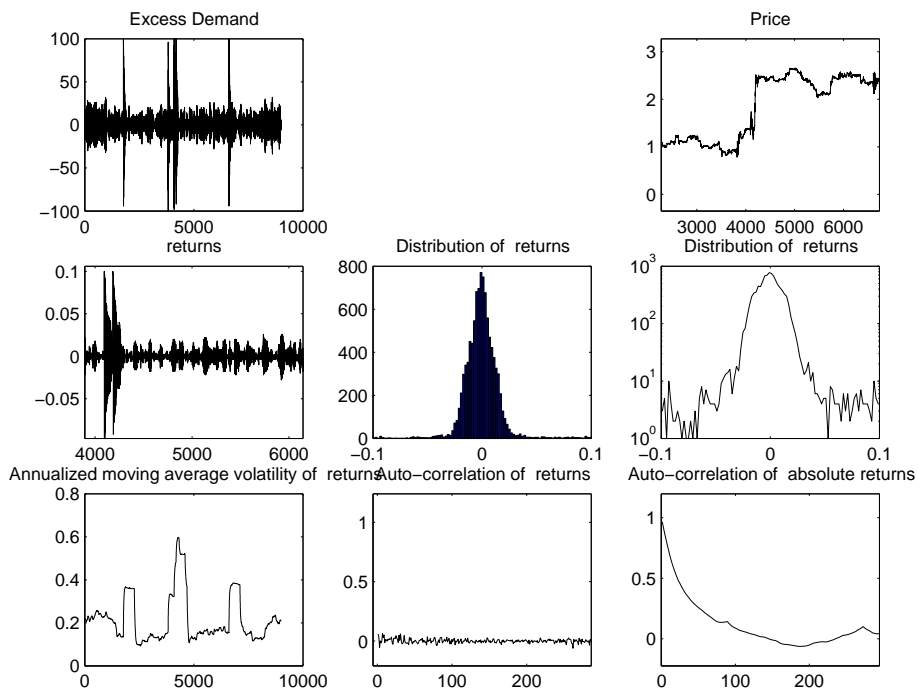


Figure 2: Numerical simulation of the model with updating frequency  $s = 0.99$ ,  $N = 100$  agents,  $\lambda = 10$ ,  $D = 0.001$ ,  $\theta = 10^{-6}$ .



## 4 Dependence properties of returns.

One deduces from the two-groups dynamics the following properties:

**Property 1 (Uncorrelated returns)** *Assume that  $g$  is an odd function. As-*  
*set returns  $(r_t)_{t \geq 0}$  are uncorrelated:  $\text{cov}(r_t, r_{t+1}) = 0$ .*

This can be understood easily when noting that the sign of the returns depends directly on the sign of the signal  $\epsilon_t$ .

**Property 2 (Volatility clustering)** *Amplitudes of consecutive returns are*  
*positively correlated:  $\text{cov}(|r_t|, |r_{t+1}|) > 0$ .*

Of course, the volatility clustering property is observed to hold well beyond the first lag in the simulations shown above. In fact, defining the cluster length  $\tau_c$  as the first lag for which the autocorrelation of absolute returns becomes zero, one can inquire into the dependence of this cluster length with respect to the updating frequency  $s$ . I show below that the duration  $\tau_c$  of volatility

clusters is linked with the average updating time  $1/s$ : as a first approximation  $\tau \simeq 1/\ln|1-2s|$ . This interpretation is interesting since it relates an observable quantity,  $\tau_c$ , to the parameter  $s$ , which describes the updating behavior of the agents. In most markets, the length of volatility cluster is roughly of the order of months, indicating that the range  $s \sim 10^{-2}$  and  $s \sim 0.99$  is in fact quite consistent with this behavior.

Letting  $N_A(t)$  be the number of informed agents and  $N_B(t)$  the number of agents in the other group at time  $t$ , one obtains:

$$|r_t| = \frac{|Z_t|}{N\lambda} = \frac{|N_A(t) - N_B(t)|}{N\lambda}. \quad (12)$$

Further,

$$E[N_A(t+1)|N_A(t)] = (1-s)N_A(t) + sN_B(t) = sN + (1-2s)N_A(t). \quad (13)$$

$$E[N_A(t+\tau)|N_A(t)] = sN \sum_{l=0}^{\tau-1} (1-2s)^l + (1-2s)^\tau N_A(t) = \frac{N}{2} [1 - (1-2s)^\tau] + (1-2s)^\tau N_A(t). \quad (14)$$

Thus,

$$E[N_A(t + \tau)N_A(t)] - E[N_A(t + \tau)]E[N_A(t)] = \text{var}(N_A(t))(1 - 2s)^\tau. \quad (15)$$

This entails the dependence property of returns

$$C_{|r|}(\tau) = |1 - 2s|^\tau. \quad (16)$$

## 5 Conclusion.

I have presented a minimal agent-based model capable of reproducing the main empirical stylized facts observed in returns of financial assets.

The model's main ingredient is switching between two trading rules. The first trader type is the informed trader following an external signal, and the second one is the liquidity-provider trader placing the opposite order of the informed traders. Extreme events occurs after periods having a lack of information.

Numerical simulations of the model generically produce time series that capture the stylized facts observed in asset returns. Due to the simple structure of the

model, these simulation results can be explained by a full analytical solution of the price process in the model.

These ingredients suffice for reproducing several empirical stylized facts such as heavy tails, absence of autocorrelation in returns, and volatility clustering, with realistic values in the time scales involved. A challenge reserved for future work would be to endogenize this switching.

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