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Francois Ghoulmie. Agents switching between two trading behaviors and the stylized facts of financial markets.. 2004. hal-00002231v1

HAL Id: hal-00002231

<https://hal.science/hal-00002231v1>

Preprint submitted on 20 Jul 2004 (v1), last revised 1 Dec 2004 (v4)

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Agents switching between two trading behaviors and the stylized facts of financial markets.

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July 20, 2004

Abstract

We propose an agent-based model of a single-asset financial market, described in terms of a small number of parameters. We show that agents switching between two trading behaviors (informed vs. liquidity traders) leads to a market price which fluctuates endlessly and a volatility which displays a mean-reverting behavior : the volatility goes neither to zero nor to infinity in the long-run.

Our agent-based model generically leads to price returns with statistical properties similar to the stylized facts observed in financial time series: absence of autocorrelation in returns, stochastic volatility, excess volatility, volatility clustering non-attributable to the external signal. The parsimonious structure of the model allows to identify the mechanism leading to these effects. We investigate theoretically some properties of this model and present analytical results.

Keywords: agent-based model, financial markets, stylized facts, asynchronous switching of trading rules, informed vs. liquidity traders.

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Agent-based models studied in the literature have pointed to various possible origins for empirical stylized facts common to a wide variety of markets, instruments and periods: herd behavior, social interaction and mimetism, heterogeneity, investor inertia and switching between “chartist” and “fundamentalist” behavior have been invoked as possible mechanisms. However, an appeal for simplicity is necessary in order to enhance the explanatory power of such models.

We propose here a parsimoniously parametrized agent-based model of a single-asset financial market, which generates returns with statistical properties similar to the stylized facts observed in financial time series. Our agent-based model generically leads to absence of autocorrelation in returns, mean-reverting stochastic volatility, excess volatility and volatility clustering. The structure of the model allows to identify a simple stochastic process of agents switching between two trading behaviors (informed versus liquidity traders) as the key mechanism leading to these effects.

The article is structured as follows. Section 1 recalls some stylized empirical facts about returns of financial assets and reviews some agent-based models presented in the literature to explain the stylized facts. Section 2 presents our model; simulation results are presented in section 3 and a theoretical analysis is given in section 4.

1 Agent-based approach for market phenomena.

Time series of asset returns exhibit non trivial and intriguing statistical features which seem to be common to a wide range of markets and time-periods as reviewed by Cont (2001):

- **Excess volatility:** observation that the level of variability in market prices is much higher than what can be expected based on the variability of fundamental economic variables and is unexplained by news arrivals as noted by Cutler et al. (1989), and Shiller (2000).
- **Heavy tails:** the (unconditional) distribution of returns displays a heavy tail with positive excess kurtosis.
- **Absence of autocorrelation in returns:** (linear) autocorrelations of asset returns are often insignificant, except for very small intraday time scales ($\simeq 20$ minutes) for which microstructure effects come into play.
- **Volatility clustering:** while returns themselves are uncorrelated, absolute returns $|r_t(\Delta)|$ or their squares display a positive, significant and slowly decaying autocorrelation function: $\text{corr}(|r_t|, |r_{t+\Delta}|) > 0$ for Δ ranging from a few minutes to several weeks (e.g., Mandelbrot, 1963).
- **Volume/volatility correlation:** trading volume is positively correlated with market volatility.

Agent-based market models, which are based on a stylized description for the behavior of agents attempt to explain the origins of the observed behavior of market prices as emerging from simple behavioral rules of a large number of heterogeneous market participants and largely independent of markets “microstructure”. Stauffer (2001) proposed for example percolation models that generate price behavior with statistical properties similar to those observed in real markets. Giardina & Bouchaud (2003) proposed also an agent-based model inspired from the Santa Fe artificial market and the Minority Game and review several mechanisms leading to long-range correlations such as switching between two trading styles.

Indeed, various possible explanations for empirical stylized facts have been suggested in the literature:

- **mimetism** (e.g., Orléan, 1995).
- **herd behavior** (e.g., Cont & Bouchaud, 2000 and Stauffer et al., 1999).
- **switching between “chartist” and “fundamentalist” behavior** as studied by Lux & Marchesi (2000), Kirman & Teyssiere (2002), Brock & Hommes (1998) and Farmer & Joshi (2002).

•**heterogeneity** in expectations concerning future prices (e.g., Gaunersdorfer, 2000; Arthur et al., 1997; Hommes et al., 2003), towards risk (e.g., Chiarella et al, 2002) and in agent’s time scale (e.g., LeBaron, 2001).

•**investor inertia** introduced in Cont & Bouchaud (2000) and Horst et al. (2003) studies.

Numerical simulations of many of the models above lead to time series of “returns” which have properties consistent with (some of) the empirical stylized facts observed above. However, due to the complexity of such models it is often not clear *which* aspect of these models is responsible for generating the stylized facts and whether all the ingredients of the model are indeed required for explaining empirical observations. Thus, an appeal for simplicity is necessary and we will propose an agent-based market model described with few parameters.

An important issue in microstructure studies is the competition between informed traders and liquidity provider or uninformed traders. Information-based microstructure models from Kyle (1984), Glosten & Milgrom (1985), Easley & O’Hara (1987) typically assume that uninformed traders do not act strategically.

Based on this remark, we will describe the dynamics of agents switching asynchronously from an informed trader group to a liquidity provider group. This mechanism will be a crucial ingredient in formulating our market model.

2 Description of the model.

Our model describes a market where a single asset, whose price is denoted by p_t , is traded by N agents. Trading takes place at discrete dates $t = 0, 1, 2, \dots$ ². At each period, informed agents receive common news about the asset’s performance and places a buy or sell order, depending on whether the news received is pessimistic or optimistic. Liquidity provider agents place systematically the opposite order. Prices then moved up or down according to excess demand. Agents switch asynchronously their trading behavior. We now describe these ingredients in more precise terms.

2.1 Trading rules.

At each period, agents have the possibility to send an order to the market for buying or selling a unit of asset: denoting by $\phi_i(t)$ the demand of the agent, we have $\phi_i(t) = 1$ for a buy order and $\phi_i(t) = -1$ for a sell order. The inflow of new information is modeled by a sequence of IID Gaussian random variables

²We will see that, provided the parameters of the model are chosen in a certain range, we will be able to interpret these periods as “trading days”.

$(\epsilon_t, t = 0, 1, 2, \dots)$ with $\epsilon_t \sim N(0, D^2)$. ϵ_t represents the value of a common signal received by informed agents at date t .

The trading rule of an informed agent is to follow the signal when he considers it significant. N_A is the number of informed traders, they place the same order $\phi_A(t)$:

$$\begin{aligned} & \text{if } \epsilon_t > \theta, \phi_A = 1 \\ & \text{if } \epsilon_t < \theta, \phi_A = -1 \\ & \text{otherwise } \phi_A = 0. \end{aligned} \tag{1}$$

This threshold response leads to investor inertia, however we will see that it has no effects on the dependence properties of the returns. The trading rule of liquidity provider agents or uninformed traders is to place an order opposite to the informed traders. N_B is the number of agents of this type, they place the same order $\phi_B(t)$:

$$\phi_B(t) = -\phi_A(t). \tag{2}$$

2.2 Price response to aggregate demand.

The aggregate excess demand is then given by:

$$Z_t = \sum_i \phi_i(t). \tag{3}$$

A non zero value of Z_t produces a change in the price, and the resulting log return is given by :

$$r_t = \ln \frac{p_t}{p_{t-1}} = g\left(\frac{Z_t}{N}\right). \tag{4}$$

where the price impact function $g : \mathfrak{R} \rightarrow \mathfrak{R}$ is an increasing function with $g(0)=0$. We define the (normalized) market depth λ by:

$$g'(0) = 1/\lambda. \tag{5}$$

While most of the analysis below holds for a general price impact function g , in some cases it will be useful to consider a linear price impact: $g(z) = z/\lambda$.

We do not describe the details of the mechanism of price formation, one can assume for example that prices are set period by period via a market maker mechanism.

2.3 Switching of trading rule.

Switching of trading rule is *asynchronous*: at each time step, any agent i has a probability s ($s \in [0, 1]$) of switching from a trading rule to the other one, for example from an informed trader behavior to a liquidity provider behavior.

Thus, in a large population, s represents the fraction of agents changing their behavior at any period; $1/s$ represents the typical time period during which an agent will hold a given trading rule. A possible explanation for this switching is that information is costly and agents do not seek it at each time step.

2.4 Extreme events.

The parameter θ describes an inability of agents to access information. When it is the case, a crowd dynamics occurs and all agents seek information at next time step. There is no more liquidity provider agents and this will be considered as an extreme event causing a large fluctuation in the prices.

2.5 Summary.

Let us recall the main ingredients of the model described above. At each time period:

- informed agents follow a common signal ϵ_t .
- liquidity provider agents submit an opposite order.
- agents switch from one type of trading rule to the other one at each time period with a probability s .
- The market price is impacted by the excess demand and moves according to (4).

With regard to some of the agent-based models considered in the literature, some important aspects are the following:

- There is no exogeneous “fundamental price” process: prices move through market fluctuations of supply and demand. In particular, we do not distinguish between “fundamentalist” and “chartist” traders.
- We do not introduce any “social interaction” among agents. In particular, no notion of locality, lattice or graph structure is introduced.

The model has very few parameters: s describes the average updating frequency, D the standard deviation of the noise representing the news arrival process, λ the market depth and θ the level of threshold response. We will observe nevertheless that this simple model generates time series of returns with interesting dynamics and properties similar to empirically observed properties of asset returns.

Note that we are not interested in describing the details of the mechanism of transactions and the problem of optimization of each agent’s portfolio. Indeed,

we describe two types of agent assuming that they are the main responsible of the price fluctuations of a single asset, but we do not describe the whole market. However, it would be interesting to check the effects of adding constraints on the agent's behavior (budget constraint for example), and this will be investigated in an other work.

3 Numerical simulations

The model described above is straightforward to simulate. We describe in this section the simulation procedure, describe the quantities of interest and presents some typical results. The simulation procedure allows us to identify generic properties of the model and calibrate the range of parameters in accordance with empirical data on asset returns.

3.1 Simulation procedure.

Simulation is done through an iterative procedure, each iteration repeating the steps described in Section 2.4. Although the model setting accomodates for more general price impact functions, in absence of an empirically motivated parametric form, we have chosen a linear function $g(x) = x/\lambda$. This choice can be viewed as a linearization of a more general g , valid for small values of excess demand or in a market with large market depth.

In a usual Monte Carlo simulation approach, expectations, moments and distributions of quantities of interest can be computed by running independent simulations and averaging the quantities of interest over the simulation runs. Note however that, in order for a direct comparison with empirical stylized facts to be meaningful, we have to consider that in the case of empirical data only a single sample path of the price is available and (unconditional) moments are computed by averaging over the (single) sample path. We therefore adopt a similar approach here: after simulating a sample path of the price p_t for $T = 10^4$ periods, we compute the following quantities:

- the time series of returns $r_t = \ln(p_t/p_{t-1}), t = 1..T$.
- the histogram of returns, which is an estimator of its unconditional distribution.
- a moving average estimator of the standard deviation of returns:

$$\hat{\sigma}^2(t) = 250\left[\frac{1}{T'} \sum_{t'=t-T'+1}^t |r_{t'}|^2 - \left(\frac{1}{T'} \sum_{t'=t-T'+1}^{T'} r_{t'}\right)^2\right]. \quad (6)$$

This quantity is a frequently used indicator for “volatility”. We “annualize” it by multiplying the “daily” estimate by 250.

- the sample autocorrelation function of returns:

$$C_r(\tau) = \frac{T}{\sum_{t=1}^T |r_t|^2} \left[\frac{1}{T-\tau} \sum_{t=1}^{T-\tau} r_t r_{t+\tau} - \left(\frac{1}{T-\tau} \sum_{t=1}^{T-\tau} r_t \right) \left(\frac{1}{T-\tau} \sum_{t=\tau+1}^T r_t \right) \right] \quad (7)$$

- the sample autocorrelation function of absolute returns:

$$C_{|r|}(\tau) = \frac{T}{\sum_{t=1}^T |r_t|^2} \left[\frac{1}{T-\tau} \sum_{t=1}^{T-\tau} |r_t| |r_{t+\tau}| - \left(\frac{1}{T-\tau} \sum_{t=1}^{T-\tau} |r_t| \right) \left(\frac{1}{T-\tau} \sum_{t=\tau+1}^T |r_t| \right) \right] \quad (8)$$

These quantities can then be used to compare with the empirical stylized facts described in Section 1.1. Finally, in order to decrease the sensitivity of results to initial conditions, we allow for an initial transitory regime and discard the first 10^3 periods before averaging.

3.2 Choosing the range of parameters

Simulation of the model requires the specification of the parameters s , λ , θ and the number of agents N .

In order to interpret the trading periods as “days” and compare the results obtained to properties of daily returns, some restrictions must be imposed on parameter values. First, note that the one-period returns are bounded by $\max\{|g(x)|, x \in [-1, 1]\}$. In the case where g is linear $|r_t| \leq \frac{1}{\lambda}$.

This suggest that the (normalized) market depth λ should not be too large in order to allow for realistic range of daily returns. We have chosen here $1 \leq \lambda \leq 10$. In practice, varying λ within this range does not affect the qualitative properties of the return process.

As noted above, $1/s$ represents the average number of periods an agent takes to update her trading rule. We get realistic long-range correlations in the amplitude of the returns for s close to 1 and for $s \ll 1$.

We choose $N = 100$ in order to get a realistic number of investors in a market. We took $D = 10^{-3}$ in order to get excess volatility and to have a realistic variability of fundamental economic variables.

The choice of θ determines the frequency of extreme events and the excess kurtosis of the distribution of returns, we choose $\theta = 10^{-6}$ in the simulations presented. Note that when $\theta = 0$, the kurtosis is close to 3 (gaussian distribution).

Let us emphasize that we are discussing the calibration of the *order of magnitude* of parameters, not fine-tuning them to a set of critical values. These results discussed in the following subsection are generic within this range of parame-

ters, which simply make a comparison with daily returns possible.

3.3 Simulation results.

Using the parameter ranges above, we have performed an extensive simulation study of price behavior in this model. Figures 1 and 2 illustrate typical sample paths obtained with different parameter values: they all generate series of returns with realistic ranges and realistic values of annualized volatility. For each series, the figures represent also the histogram of returns both in linear and log scales, the ACF of returns C_r , the ACF of absolute returns $C_{|r|}$.

More interestingly, we note that all the return series obtained possess some regularities which match some empirical properties outlined in Section 1.1:

- Excess volatility: the sample standard deviation of returns can be much larger than the standard deviation of the input noise representing news arrivals

$$\hat{\sigma}(t) \gg D. \tag{9}$$

In the simulation example shown in figure 1, $D = 10^{-3}$ which corresponds to an annualized volatility of 1.6%, while the annualized volatility of returns is in the range of 30%, an order of magnitude larger. This is a generic phenomenon also observed in other simulations: the order of magnitude of the volatility of returns may be totally different from that of the input noise.

- Mean-reverting volatility: the market price fluctuates endlessly and displays “stochastic volatility”: the volatility, as measured by the moving average estimator $\hat{\sigma}(t)$, does neither go to zero nor to infinity and displays a mean-reverting behavior. Many microstructure models-especially those with learning or evolution-when observed over large time intervals, converge to an equilibrium where prices and other aggregate quantities cease to fluctuate randomly. Of course, this is not observed in financial markets: prices fluctuate endlessly and the volatility exhibits mean-reverting behavior. Indeed, this behavior is attested by many empirical studies and GARCH models (e.g., Engle, 1995) on one hand and stochastic volatility models on the other hand aim at reproducing this mean-reverting stochastic behavior of volatility.

- The simulated process generates a leptokurtic distribution of returns with heavy tails, with an excess kurtosis around $\kappa \simeq 11$.

- The returns are uncorrelated: the sample autocorrelation function of the return exhibits an insignificant value (very similar to that of asset returns) at all lags, indicate absence of linear serial dependence in the returns.

- Volatility clustering: the autocorrelation function of absolute returns remains positive, and significantly above the autocorrelation of the returns, over many

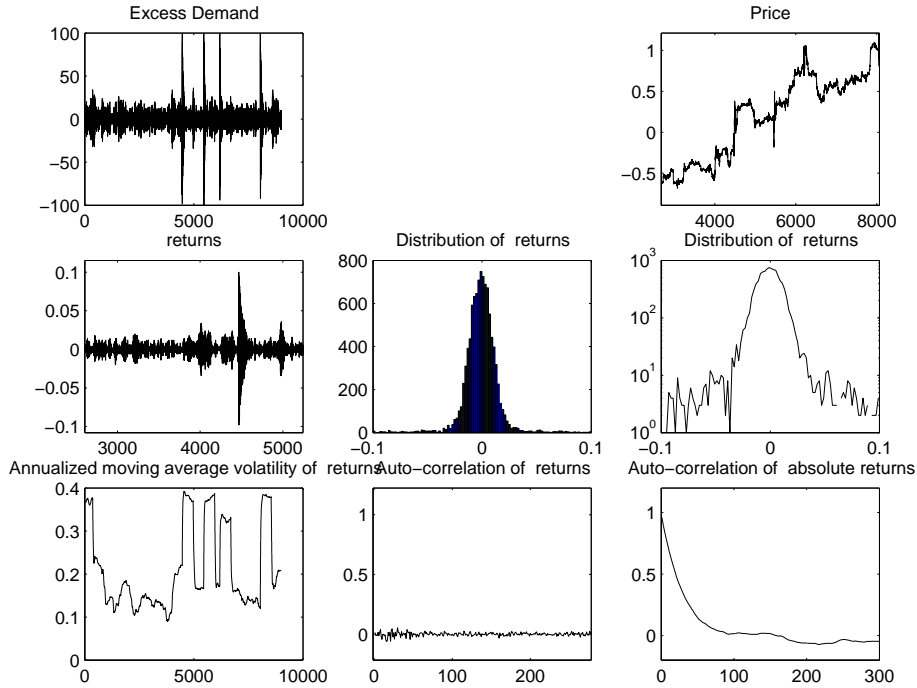


Figure 1: Numerical simulation of the model with updating frequency $s = 0.01$ (updating period: 100 “days”), $N = 100$ agents, $\lambda = 10$, $D = 0.001$, $\theta = 10^{-6}$.

time lags, corresponding to persistence of the amplitude of returns a time scale ranging from a few weeks to several months. This is an indication of nonlinear dependence in the returns. Note that this dependence is not function of the threshold level θ .

We will now analyze the dependence properties of the returns in the model.

4 Dependence properties of returns.

One deduces from the two-groups dynamics the following properties:

Property 1 (Uncorrelated returns) *Assume that g is an odd function. Asset returns $(r_t)_{t \geq 0}$ are uncorrelated: $\text{cov}(r_t, r_{t+1}) = 0$.*

This can be understood easily when noting that the sign of the returns depends directly on the sign of the signal ϵ_t .

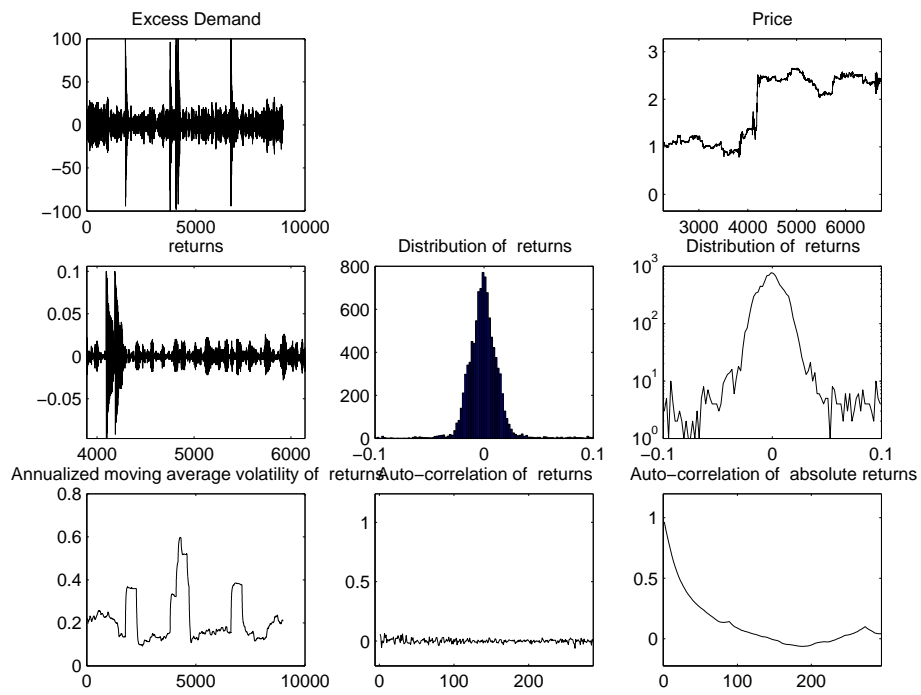


Figure 2: Numerical simulation of the model with updating frequency $s = 0.99$, $N = 100$ agents, $\lambda = 10$, $D = 0.001$, $\theta = 10^{-6}$.

Property 2 (Volatility clustering) *Amplitudes of consecutive returns are positively correlated: $cov(|r_t|, |r_{t+1}|) > 0$.*

Of course, the volatility clustering property is observed to hold well beyond the first lag in the simulations shown above. In fact, defining the cluster length τ_c as the first lag for which the autocorrelation of absolute returns becomes zero, one can inquire into the dependence of this cluster length with respect to the updating frequency s . We will prove that the duration τ_c of volatility clusters is linked with the average updating time $1/s$: as a first approximation $\tau \simeq 1/\ln|1-2s|$. This interpretation is interesting since it related an observable quantity, τ_c , to the parameter s which describes the updating behavior of the agents. In most markets, the length of volatility cluster is roughly of the order of months, indicating that the range $s \sim 10^{-2}$ and $s \sim 0.99$ is in fact quite consistent with this behavior.

Let us note $N_A(t)$ the number of informed agents and $N_B(t)$ the number of agents in the other group at time t . We have the following relation:

$$|r_t| = \frac{|Z_t|}{N\lambda} = \frac{|N_A(t) - N_B(t)|}{N\lambda}. \quad (10)$$

We first have:

$$E[N_A(t+1)|N_A(t)] = (1-s)N_A(t) + sN_B(t) = sN + (1-2s)N_A(t). \quad (11)$$

$$E[N_A(t+\tau)|N_A(t)] = sN \sum_{l=0}^{\tau-1} (1-2s)^l + (1-2s)^\tau N_A(t) = \frac{N}{2} [1 - (1-2s)^\tau] + (1-2s)^\tau N_A(t). \quad (12)$$

From this relation, we get:

$$E[N_A(t+\tau)N_A(t)] - E[N_A(t+\tau)]E[N_A(t)] = \text{var}(N_A(t))(1-2s)^\tau. \quad (13)$$

We deduce the dependence property of returns:

$$C_{|r|}(\tau) = |1-2s|^\tau. \quad (14)$$

5 Conclusion.

We have presented a minimal agent-based model which is capable of reproducing the main empirical stylized facts observed in returns of financial assets. Our model is based on the main ingredient:

- switching between two trading rules.

The first trader type is the informed trader following an external signal, and the second one is the liquidity provider trader placing the opposite order of the informed traders. Extreme events occurs after periods of lack of information. Numerical simulations of the model generically produce time series that capture the stylized facts observed in asset returns. Due to the simple structure of the model, these simulation results can be explained by a full analytical solution of the price process in the model. The challenge would be in an other work to endogenize this switching.

These ingredients suffice for reproducing several empirical stylized facts as heavy tails, absence of autocorrelation in returns and volatility clustering, with realistic values in the time scales involved. We hope that the present work will contribute to the understanding of market behavior.

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