



**HAL**  
open science

## Strategic updating of threshold response in an agent-based market model.

Francois Ghoulmie

► **To cite this version:**

Francois Ghoulmie. Strategic updating of threshold response in an agent-based market model.. 2004.  
hal-00002141v1

**HAL Id: hal-00002141**

**<https://hal.science/hal-00002141v1>**

Preprint submitted on 21 Jun 2004 (v1), last revised 1 Dec 2004 (v4)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Strategic updating of threshold response in an agent-based market model.

François GHOULMIÉ<sup>a,b,1</sup>

(a)Centre de Mathématiques Appliquées  
Ecole Polytechnique, F-91128 Palaiseau, France.

(b)Laboratoire de Physique Statistique  
Ecole Normale Supérieure, F-75321 Paris Cedex 05, France

June 29, 2004

## Abstract

We propose an agent-based model of a single-asset financial market described in terms of few parameters.

We show that the effect of agents adjusting their threshold response to new information is to generate a market price which fluctuates endlessly and a volatility which displays a mean-reverting behavior : the volatility goes neither to zero nor to infinity in the long-run.

Our agent-based model generically leads to absence of autocorrelation in returns, excess volatility, volatility clustering and endogeneous bursts of activity non-attributable to external noise.

This study illustrates a possible link between the famous El Farol bar problem and financial markets.

1

---

<sup>1</sup>Tel.: +33 1 69 33 47 48; fax: +33 1 69 33 30 11.  
*E-mail adress:* ghoulmie@cmapx.polytechnique.fr (F. Ghoulmié).

# Contents

<b>1</b>	<b>El Farol bar problem and financial markets.</b>	<b>3</b>
1.1	Agent-based approach for studying market phenomena. . . . .	3
1.2	Minority game and financial markets. . . . .	4
<b>2</b>	<b>Description of the model.</b>	<b>5</b>
2.1	Trading rules. . . . .	5
2.2	Price response to aggregate demand. . . . .	6
2.3	Updating scheme. . . . .	6
<b>3</b>	<b>Numerical simulations</b>	<b>7</b>
3.1	Simulation procedure. . . . .	7
3.2	Simulation results. . . . .	8
3.3	Emergence of coordination due to adaptation. . . . .	8
<b>4</b>	<b>Conclusion</b>	<b>10</b>

# 1 El Farol bar problem and financial markets.

## 1.1 Agent-based approach for studying market phenomena.

There is a growing body of research on agent-based models of financial markets that explores the relation between market participants and the statistical properties of aggregate market variables such as prices and trading volume.

Indeed, financial time series exhibit non-trivial and intriguing statistical features [4] which are not easy to model and even less to explain : volatility clustering and heavy tailed increments for example.

In a previous paper [9], we considered some methodological issues related to this approach. We argued that two main ingredients in agent-based models (and in real speculative markets) which lead to realistic behavior of prices and trading volume are heterogeneity in behavioral rules, resources or beliefs among market participants and feedback effects, the agents behavior determining the state of market variables which in turn influences these behaviors.

Let us recall the crucial ingredients of this model:

- Threshold behavior of agents that leads to investor inertia.
- There is no exogeneous “fundamental price” process: prices move through market fluctuations of supply and demand. In particular, we do not distinguish between “fundamentalist” and “chartist” traders.
- No information asymetry: the same information is available to all agents. Agents differ in the way they process the information.
- Absence of “social interaction”: agents interacts indirectly via the price, as in standard Walrasian markets. We do not introduce any “social interaction” among agents. In particular, no notion of locality, lattice or graph structure.
- Endogeneous heterogeneity: heterogeneity of agents behavioral rules appears endogeneously due to the asynchronous updating scheme.

The structure of the model is similar to the previous one. We refer to the previous study for a theoretical analysis in order to understand the origin of the properties of the model. The previous model is more general: the random nature of updating can be seen as a parsimonious way to differentiate agents through their updating “frequencies”. In the present work, we introduce a strategic updating scheme making a link between El Farol bar problem and financial markets. Indeed, it uses inductive reasoning inspired from the famous Brian Arthur’s problem and the related works on the minority game in the physics community that we are going to briefly review in the next subsection.

## 1.2 Minority game and financial markets.

The El Farol Bar problem was posed as an example of inductive reasoning in scenarios of bounded rationality [1]. In situations where ignorance about other agents ability and willingness to apply perfect rationality, agents adjust their behavior based on what they think other agents are going to do, and these expectations are generated endogeneously by information about what other agents have done in the past. In this problem,  $N$  agents decide independently each week whether to go to a bar or not. They go if they expect the attendance to be less than a certain threshold and stay at home if they expect it will be overcrowded. There is no communication among the agents; the only information available is the numbers who came in the past weeks. Agents are given strategies that translate the idea that when agents face complex situations like this they tend to look for patterns that occurred in the past and predict next move from past experience. In the original Minority Game, the mathematical formulation of this problem considered by physicists [3], each player has a small number of randomly picked decision tables that prescribe an action for each possible history. Those tables receive points according to how well they have predicted the best action in the course of the game, and the best table is used to actually make the decisions. Computer simulations of this model shows that the attendance, or the aggregate variable, fluctuates around the threshold value illustrating how the strategies self-organize, and how coordination emerge due to adaptation.

This problem deals with many agents interacting through an aggregate variable, the bar attendance. In the language of statistical physics, it is a mean field type problem. The system is frustrated since there is not a unique winning strategy in the problem and have quenched disorder since agents have a set of fixed strategies. These properties are very appealing for statistical physicists, and earlier studies offered elegant solutions [11] using techniques borrowed from disordered systems [14] detecting for example a phase transition between an efficient regime towards information and an inefficient one.

Several attempts to use this model as a market model have been made [2, 10]. Markets are then considered as adaptive complex systems driven by a minority rule. The analogy was understood as agents having the choice of buying or selling an asset, and the minority group always win. This behavioral assumption may be questionable when applied to financial markets as opposed to cases in ecology or vehicular traffic. In this paper, we also consider a market as an adaptive complex system driven by a minority rule but we propose a different approach.

This approach is inspired from a variant of the original minority game [12] where agents do not change their decisions when they are successful, otherwise they change their decision with a probability  $p$ , being reluctant to give up their positions. In this variant, agents do not have a memory and learn by their actions; the model illustrates thus a case of reinforcement learning [8]. Another difference with the original minority game is that disorder is not quenched anymore.

In the next section, we describe the agent-based market model. We discuss the results of the numerical simulations in Section 3.

## 2 Description of the model.

Our model describes a market where a single asset, whose price is denoted by  $p_t$ , is traded by  $N$  agents. Trading takes place at discrete dates  $t = 0, 1, 2, \dots$ <sup>2</sup>. At each period, every agent receives public news about the asset's performance and, using a subjective criterion, judge whether this news is significant. If the news is found to be significant, the agent places a buy or sell order, depending on whether the news received is pessimistic or optimistic. Prices then moved up or down according to excess demand. We now describe these ingredients in more precise terms.

### 2.1 Trading rules.

At each period, agents have the possibility to send an order to the market for buying or selling a unit of asset: denoting by  $\phi_i(t)$  the demand of the agent, we have  $\phi_i(t) = 1$  for a buy order and  $\phi_i(t) = -1$  for a sell order. We allow the value  $\phi_i(t)$  to be zero; the agent is then inactive at period  $t$ . The inflow of new information is modeled by a sequence of IID Gaussian random variables  $(\epsilon_t, t = 0, 1, 2, \dots)$  with  $\epsilon_t \sim N(0, D^2)$ .  $\epsilon_t$  represents the value of a common signal received by all agents at date  $t$ . The signal  $\epsilon_t$  is a forecast of the future return  $r_t$  and each agent has to decide whether the information conveyed by  $\epsilon_t$  is significant, in which case she will place a buy or sell order according to the sign of  $\epsilon_t$ .

The trading rule of each agent  $i = 1, \dots, N$  is represented by a (time-varying) decision threshold  $\theta_i(t)$ . By comparing the signal to her threshold, the agent decides whether the news is significant enough to generate a trade ( $|\epsilon_t| > \theta_i(t)$ ):

$$\begin{aligned} & \text{if } \epsilon_t > \theta_i^+, \phi_i = 1 \\ & \text{if } \epsilon_t < \theta_i^-, \phi_i = -1 \\ & \text{otherwise } \phi_i = 0. \end{aligned} \tag{1}$$

This trading rule may be seen as a stylized example of threshold behavior: without sufficient external stimulus, an agent remains inactive and if the external signal is above a certain threshold, the agent will act. The corresponding demand generated by the agent is therefore given by:

$$\phi_i(t) = 1_{\epsilon_t > \theta_i} - 1_{\epsilon_t < -\theta_i}. \tag{2}$$

---

<sup>2</sup>Provided the parameters of the model are chosen in a certain range, we will be able to interpret these periods as "trading days".

## 2.2 Price response to aggregate demand.

The aggregate excess demand is then given by:

$$Z_t = \sum_i \phi_i(t). \quad (3)$$

A non zero value of  $Z_t$  produces a change in the price, and the resulting log return is given by :

$$r_t = \ln \frac{p_t}{p_{t-1}} = g\left(\frac{Z_t}{N}\right). \quad (4)$$

where the price impact function  $g : \Re \rightarrow \Re$  is an increasing function with  $g(0)=0$ . We define the (normalized) market depth  $\lambda$  by:

$$g'(0) = 1/\lambda. \quad (5)$$

While most of the analysis below holds for a general price impact function  $g$ , in some cases it will be useful to consider a linear price impact:  $g(z) = z/\lambda$ .

## 2.3 Updating scheme.

The model is based on the following *assumption*:

- Agents believe that they do not benefit from trading if the majority of agents act too.

Initially, we start from a population distribution  $F_0$  of thresholds:  $\theta_i(0), i = 1..N$  are positive IID variables drawn from  $F_0$ .

When market activity is higher than a certain threshold,  $A(t) > \alpha N$  with  $0 < \alpha < 1$ , active agents change with a probability  $p$  ( $p \in [0, 1]$ ) their threshold response. The parameter  $p$  expressed their reluctance to change. She sets it to be proportional to the recently observed market activity:

$$\theta_i(t) = \mu A(t) \quad (6)$$

Note that when choosing  $\mu = \frac{1}{\lambda N}$ , we have the same updating rule as in the previous work [9] (we will choose a close numerical value in order to get realistic daily returns).

When market activity is lower than this threshold,  $A(t) < \alpha N$ , inactive agents update their threshold response according to (6). This updating scheme means that only the losers of the minority game change their threshold response.

Introducing IID random variables  $u_i(t), i = 1..N, t \geq 0$  uniformly distributed on  $[0, 1]$ , which indicate whether agent  $i$  updates her threshold or not, we can write the updating rule as:

$$\theta_i(t) = 1_{u_i(t) < p} \mu A(t) + 1_{u_i(t) \geq p} \theta_i(t-1). \quad (7)$$

Here  $\epsilon_t$  represents randomness due to public news arrivals whereas the random variables  $u_i(t)$  represent idiosyncratic sources of randomness. Note that, even if we start from an initially homogeneous population  $\theta_i(0) = \theta_0$ , heterogeneity creeps into the population through the updating process. In this sense, the heterogeneity of agents strategies is endogenous in this model and evolves in a random manner as opposed to quenched disorder in the initial minority game.

### 3 Numerical simulations

#### 3.1 Simulation procedure.

After simulating a sample path of the price  $p_t$  for  $T = 10^4$  periods, we compute the following quantities:

- the time series of returns  $r_t = \ln(p_t/p_{t-1}), t = 1..T$ .
- the histogram of returns, which is an estimator of its unconditional distribution.
- a moving average estimator of the standard deviation of returns:

$$\hat{\sigma}^2(t) = 250\left[\frac{1}{T'} \sum_{t'=t-T'+1}^t |r_{t'}|^2 - \left(\frac{1}{T'} \sum_{t'=t-T'+1}^{T'} r_{t'}\right)^2\right]. \quad (8)$$

This quantity is a frequently used indicator for “volatility”. We “annualize” it by multiplying the “daily” estimate by 250.

- the sample autocorrelation function of returns:

$$C_r(\tau) = \frac{T}{\sum_{t=1}^T |r_t|^2} \left[ \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} r_t r_{t+\tau} - \left( \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} r_t \right) \left( \frac{1}{T-\tau} \sum_{t=\tau+1}^T r_t \right) \right] \quad (9)$$

- the sample autocorrelation function of absolute returns:

$$C_{|r|}(\tau) = \frac{T}{\sum_{t=1}^T |r_t|^2} \left[ \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} |r_t| |r_{t+\tau}| - \left( \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} |r_t| \right) \left( \frac{1}{T-\tau} \sum_{t=\tau+1}^T |r_t| \right) \right] \quad (10)$$

These quantities can then be used to compare with the empirical stylized facts outlined by Cont [4]. Finally, in order to decrease the sensitivity of results to initial conditions, we allow for an initial transitory regime and discard the first  $10^3$  periods before averaging.

The rationale behind the choice of the parameters is similar to that of the previous study [9]. The results discussed in the next section are generic within a wide range of parameters.



### 3.2 Simulation results.

Figure 1 illustrate a typical sample path of price behavior obtained with the following parameters:  $\lambda = 10$ ,  $N = 1500$ ,  $D = 0.001$ ,  $p = 0.015$ ,  $\alpha = 2.10^{-2}$ ,  $\mu = \frac{1}{\lambda N}$ .

The figure represents also the histogram of returns both in linear and log scales, the ACF of returns  $C_r$ , the ACF of absolute returns  $C_{|r|}$ .

We get series of returns with realistic ranges and realistic values of annualized volatility and with some regularities which match some empirical properties:

- **Excess volatility**[13, 5]: the sample standard deviation of returns can be much larger than the standard deviation of the input noise representing news arrivals  $\hat{\sigma}(t) \gg D$ .

- **Mean-reverting volatility**: the market price fluctuates endlessly and displays “stochastic volatility”: the volatility, as measured by the moving average estimator  $\hat{\sigma}(t)$ , does neither go to zero nor to infinity and displays a mean-reverting behavior. This behavior is attested by many empirical studies and GARCH models [6] on one hand and stochastic volatility models on the other hand aim at reproducing this mean-reverting stochastic behavior of volatility.

- The simulated process generates a **leptokurtic distribution of returns with (semi-)heavy tails**, with an excess kurtosis around  $\kappa \simeq 5.5$ . As shown in the logarithmic histogram plot in figures 1, the tail exhibit an approximately exponential decay, as observed in various studies of daily returns [7]. Note that  $\kappa = 6$  for a (two-sided) exponential distribution.

- **The returns are uncorrelated**: the sample autocorrelation function of the return exhibits an insignificant value (very similar to that of asset returns) at all lags, indicate absence of linear serial dependence in the returns.

- **Volatility clustering**: the autocorrelation function of absolute returns remains positive, and significantly above the autocorrelation of the returns, over many time lags, corresponding to persistence of the amplitude of returns a time scale ranging from a few weeks to several months. This is an indication of non-linear dependence in the returns.

### 3.3 Emergence of coordination due to adaptation.

At each time step, in the case where  $A(t) > \alpha N$ , there are  $A(t)$  losers and  $N - A(t)$  winners, so the total gain (or loss) is then  $N - 2A(t)$ . The total gain is also  $N - 2A(t)$  in the other case.

It is still not clear if the system reaches a steady state and fluctuates around it, however we can say that coordination due to adaptation occurs when looking at the numerical simulations because we have  $A(t) > N/2$  most of the time.

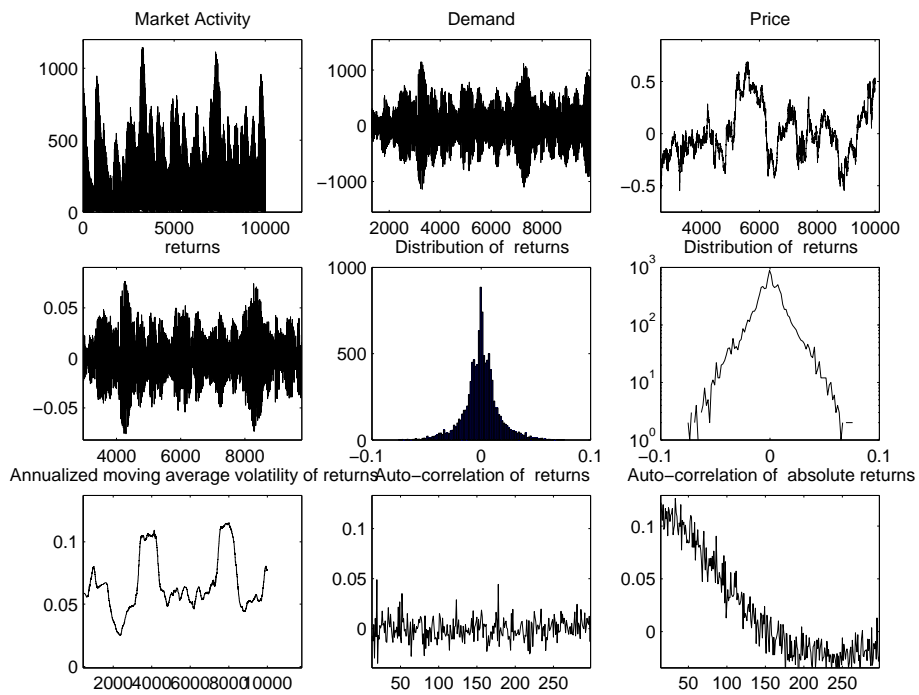


Figure 1: Numerical simulation of the model with updating frequency  $p = 0.015$  (updating period: 67 “days”) and  $N = 1500$  agents.

The updating scheme is responsible for investor inertia.

Now, let us understand qualitatively how the learning process is responsible for the dynamics of heterogeneity and to the emergence of the stylized facts. When a majority of agents have a low value for their threshold, they will very probably act. Because only a small fraction of agents, due to their reluctance to change their belief (expressed by the parameter  $p$ ), do higher their threshold response, when the market activity is high, it is very probable to have a high activity at next time step. This slow feedback mechanism causes persistence in the amplitudes of the fluctuations of the returns.

## 4 Conclusion

We presented a very minimalist agent-based market model : only one type of agents interact indirectly via the market activity. The numerical simulation generates generic results that capture the essential statistical features of financial markets. These stylized facts are the consequence of the presence of coordination and learning in the market. Indeed, we considered the market as a complex adaptive system driven by a minority rule, where agents with inductive rationality adjust their behavior. A full exploration of the parameter phase space and a full theoretical analysis of the complex dynamics of heterogeneity would be worth other works. This study illustrates the link between bounded rationality and heterogeneity with its dynamics in economic systems.

## References

- [1] Arthur, W.B., 1994. *Inductive Reasoning and Bounded Rationality*, American Economic Review 84, 406-411.
- [2] Challet, D., Chessa, A., Marsili, M., Zhang, Y.-C., 2001. *From minority games to real markets*, Quantitative Finance 1, 168-176.
- [3] Challet, D., Zhang, Y.-C., 1997. *Emergence of Cooperation and Organization in an Evolutionary Game*, Physica A 246, 407.
- [4] Cont, R., 2001. *Empirical properties of asset returns: stylized facts and statistical issues*, Quantitative Finance 1, 223-236.
- [5] Cutler, D., Poterba, J., Summers, L., 1989. *What moves stock prices?*, Journal of Portofolio Management, 4-12.

- [6] Ding, Z., Granger, C., Engle, R., 1983. *A long memory property of stock market returns and a new model*, Journal of empirical finance 1, 83-106.
- [7] Engle, R., 1995. *ARCH models*, Oxford University press, Oxford.
- [8] Fudenberg, D., Levine, D., 1998. *Theory of learning in Games*, MIT Press.
- [9] Ghoulmíé, F., Cont, R., Nadal, J.-P., 2004. *Heterogeneity and feedback in an agent-based market model*, paper prepared for the 10th International Conference of the Society for Computational Economics, July 2004, Amsterdam, The Netherlands.
- [10] Giardina, I., Bouchaud, J.-P., Mézard, M., 2001. *Microscopic Models for Long Ranged Volatility Correlations*, cond-mat/0105076.
- [11] Marsili, M., Challet, D., Zecchina, R., 2000. *Exact Solution of a Modified El Farol's Bar Problem: Efficiency and the Role of Market Impact*, Physica A 280, 522.
- [12] Reents, G., Metzler, R., Kinzel, W., 2000. *A New Stochastic Strategy for the Minority Game*, cond-mat/0007351.
- [13] Shiller, R., 2000. *Irrational Exuberance*, Princeton University Press.
- [14] Young, A.P, 1998. *Spin glasses and random fields*, World Scientific, Singapore.