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Mechanism of the Atmospheric Ball Lightning Using the Triple Beltrami Equation

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Abstract

Ball lightning, also known as fire ball, is a luminous globe which occurs in the course of a thunderstorm. It has been the object of investigation by figures in science since the early nineteenth century. The difficult and long-standing problem of ball lightning has attracted few meteorologists or atmospheric scientists. Rather, physicists constitute the larger part of the company studying the atmospheric fire balls.

Taking as model, the two fluid plasma consisting of electrons and one species ions, for the fire balls physics and considering that the plasma flow is a finite quantity, we can derive the equation of relaxed energy state, maintaining the helicity constant, in the form of triple Beltrami equations for the magnetic field:

$$s\nabla \times \nabla \times \nabla \times \vec{B} + p\nabla \times \nabla \times \vec{B} + q\nabla \times \vec{B} + r\vec{B} = 0$$

where $\vec{B}$ is the magnetic field, s, p, q, and r are constants to be determined through boundary conditions. This equation is coupled with an equation which describes the hydrodynamic vortex.

When $s = p = 0$, we have the Taylor relaxed state of plasma [1], without the fluid flow. In the case of electron-ion plasma, both s and p are non-zero quantities. In particular, when electron mass is neglected the results is $s = 0$ and it describes the Double Beltrami relaxed minimum energy state as derived by S. M. Mahajan and Z. Yoshida [2]. Its solution is a spheromak type solution. The problem of the formation of an isolated luminous mass in the sky and the moderate persistence of the resulting form combined with observations of

ball lightning describing hollow globes, surface coronas and rapid rotation led to theories depicting ball lightning as a vortex. We explore the solutions of this equation using the method of Chandrasekhar-Kendall [3] eigenfunctions and appropriate boundary conditions similar, but more extensive, to the description given by Shukla, Dasgupta and Sakanaka [4]. We will show solutions which might explain the fire ball configurations.

1. Introduction

Ball lightning is a luminous globe which occurs in the course of a thunderstorm. It has been the object of investigation by well known scientists since the early nineteenth century. Although its appearance is very haphazard and far less frequent than ordinary linear lightning, it is a source of astonishment when the fire ball is observed in the lower atmosphere, often entering dwellings while floating at a leisurely pace. By its random appearances, it has eluded the measurements with the scientific instruments. Even photographic captures of fire balls are rare. However, from the qualitative reports of eye-witnesses reports (now it numbers in the thousands, particularly from collections from the Soviet Union) the general properties can be deduced despite the wide variability found in the reports.

Ball lightning makes appearance as flame globe, typically in orange, red-orange or intense white, and less often in blue, green or yellow. Its dimensions is usually 25-30 cm in diameter, but much smaller or much larger dimensions are also notified in the reports (from 1 cm to 10 m and larger). The color of the ball lightning would depend on the current in the channel as in the experiments in which a weak current gives a bluish glow while increasingly stronger currents gives dark red, brick red, orange red, and finally white. Occasionally, the balls move against the direction of prevailing winds and penetrate window panes without making a hole in the glass; it is these properties that Ohtsuki and Ofuruton report from their experimental discharges.

The difficult and long-standing problem of ball lightning has attracted only few meteorologists or atmospheric scientists. Rather, physicists constitute the larger part of the company studying the atmospheric fire balls. Numerous theories have been put forward, covering chemical, electrical, nuclear and relativistic models. New states of matter have been proposed to explain the unusual properties reported for ball lightning.

The problem of the formation of an isolated luminous mass in the sky and the moderate persistence of the resulting form combined with observations of ball lightning describing hollow globes, surface coronas, and rapid rotation led to theories depicting ball lightning as a vortex.

The generation of small vortexes constituting ball lightning in the atmosphere by whirl winds, cyclones, or tornadoes was suggested following numerous observations of fire balls in connection with a tornado which appeared at night in France in 1890.

The possibility of a close relationship between the formation of ball lightning and tornadoes has been suggested, The combined action of electrical and hydrodynamic forces in generating ball lightning as a highly ionized vortex was specifically adduced in general terms in 1905.

The magnetic field generated by rotation of charged particles in the ring, evidently in helical paths in its cross section as well as in circular paths around the ring, was suggested to assist in confinement. The discharge of a large current through a fine wire ring was presented as a method of generating such a vortex.

The vortex theories provide a very direct explanation of the numerous observations indicating rotation of ball lightning. The formation of luminous globes of this type can be readily ascribed to either the role of a preliminary linear lightning flash or the hydrodynamic action of a whirl wind.

Some vortex theories may be classified equally well as electrical discharge theories and, especially in the most recent examples, as plasma theories. None of the studies of ball lightning as a vortex, however, has conclusively dispelled by detailed consideration the serious difficulties on which other theories have founded, such as the continued luminosity of the balls for long periods while they travel inside structures. If the vortex is to be an isolated, self-contained sphere, its energy would be expended in viscous drag and turbulence in a very short time.

We explore the possibilities of relaxed states of plasma for atmospheric plasma with positive charged particles, and electrons in the form of Triple Beltrami equations. It is shown that the associated velocity field has the similar vortex structure as in the magnetic field. Solutions of this equation is obtained using the method of Chandrasekhar-Kendall eigenfunctions and appropriate boundary conditions similar, but more extensive, to the description given by Shukla, Dasgupta and Sakanaka [2] . We will show solutions which might explain the ball lightning configurations.

Theory

We consider a warm, homogeneous, electron-ion plasma. The dynamics of low phase velocity e-i plasma are governed by the electron and ion momentum equations, which are, respectively,

$$\frac{\partial \mathbf{v_e}}{\partial t} + \frac{1}{2}\nabla {\mathbf{v_e}}^2 - \mathbf{v_e} \times \nabla \times \mathbf{v_e} = -\frac{e}{m_e}\left[\mathbf{E} + \frac{1}{c}\mathbf{v_e} \times \mathbf{B}\right] - \frac{1}{m_e n_e}\nabla p_e \tag{1}$$

and

$$\frac{\partial \mathbf{v_i}}{\partial t} + \frac{1}{2}\nabla {\mathbf{v_i}}^2 - \mathbf{v_i} \times \nabla \times \mathbf{v_i} = \frac{e}{m_i}\left[\mathbf{E} + \frac{1}{c}\mathbf{v_i} \times \mathbf{B}\right] - \frac{1}{m_i n_i}\nabla p_i \tag{2}$$

supplemented by Faraday's law

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}, \tag{3}$$

and Ampere's law

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} = \frac{4\pi}{c}ne(\mathbf{v_i} - \mathbf{v_e}) \equiv \frac{8\pi}{c}ne\mathbf{U} \tag{4}$$

where $\mathbf{v_e}$ ($\mathbf{v_i}$) is the electron (ion) fluid velocity, $\mathbf{U} = \frac{1}{2}(\mathbf{v_i} - \mathbf{v_e})$, and, $\mathbf{E}$ ($\mathbf{B}$) is the electric (magnetic) field, n_e (n_i) is the uniform electron (ion) number density (given by the continuity equation), e is the magnitude of the electron charge, m_e (m_i) is the electron (ion) mass, and p_e (p_i) is the scalar electron (ion) pressure. The fluid velocity $\mathbf{V}$ for e-i plasma is defined by,

$$\mathbf{V} = \frac{m_i\mathbf{v_i} + m_e\mathbf{v_e}}{m_i + m_e} \simeq \mathbf{v_i} + \mu\ \mathbf{v_e}, \quad \mu = \frac{m_e}{m_i} \tag{5}$$

one gets

$$\mathbf{v_i} = \mathbf{V} + 2\mu\mathbf{U}, \qquad \mathbf{v_e} = \mathbf{V} - 2\mathbf{U} \tag{6}$$

From equation (4), we can write,

$$\mathbf{U} = \frac{c}{8\pi ne}\nabla \times \mathbf{B}$$

Using this, the electron and ion velocities can be expressed in the normalized form

$$\mathbf{v_i} = \mathbf{V} + \sqrt{\mu}\ \nabla \times \mathbf{B}, \quad \mathbf{v_e} = \mathbf{V} - \frac{1}{\sqrt{\mu}} \nabla \times \mathbf{B} \tag{7}$$

We normalize the variables and to express the momentum equations, we start with (1) and (2) and after taking "curl" of both sides, in a dimensionless form; the magnetic field $\mathbf{B}$ is normalized to some arbitrary B_0, the velocity to $V_0 = B_0/\sqrt{4\pi n_e m_i}$, length and time by skin depth for e-i plasma $\lambda = c/\omega_{p_e} = c/\sqrt{4\pi n_e e^2/m_e}$ and inverse of cyclotron frequency, $\tau_c = (m_e c/eB_0)$ respectively. Taking "curl" of both sides of equations (1) and (2), and using equation (7) and the Faraday's law, equation (3), we get the momentum equations for electron and ion in normalized variables. Introducing a pair of generalized vortices, $\mathbf{\Omega_i}$, $\mathbf{\Omega_e}$

$$\mathbf{\Omega_i} = \nabla \times \mathbf{v_i} + \sqrt{\mu}\ \mathbf{B}; \quad \mathbf{\Omega_e} = \nabla \times \mathbf{v_e} - \frac{1}{\sqrt{\mu}}\ \mathbf{B} \tag{9}$$

and effective velocities, $\mathbf{U_i}$, $\mathbf{U_e}$ (these velocities are the normalized electron and ion velocities), where

$$\mathbf{U_e} = \mathbf{V} - \frac{1}{\sqrt{\mu}}\ \nabla \times \mathbf{B}, \quad \mathbf{U_i} = \mathbf{V} + \sqrt{\mu}\ \nabla \times \mathbf{B} \tag{10}$$

the electron and ion momentum equations can be put in a symmetric form,

$$\frac{\partial \mathbf{\Omega_j}}{\partial t} - \nabla \times (\mathbf{U_j} \times \mathbf{\Omega_j}) = 0, \qquad (j = i, e) \tag{11}$$

The above equations show the effects of the coupling of magnetic field and flow in an exact form. We can look for a simple equilibrium solution of the above equation, the simplest equilibrium is obtained as $\mathbf{U_j} \parallel \mathbf{\Omega_j}$, $(j = 1, 2)$. Thus we get,

$$\mathbf{U_e} = \mathbf{V} - \frac{1}{\sqrt{\mu}}\ \nabla \times \mathbf{B} = a_1(\nabla \times (\mathbf{V} - \frac{\mathbf{1}}{\sqrt{\mu}}\ \nabla \times \mathbf{B}) - \frac{\mathbf{1}}{\sqrt{\mu}\ \mathbf{B}}) \tag{12}$$

and,

$$\mathbf{U_i} = \mathbf{V} + \sqrt{\mu}\ \nabla \times \mathbf{B} = a_2(\nabla \times (\mathbf{V} + \sqrt{\mu}\ \nabla \times \mathbf{B}) + \sqrt{\mu}\ \mathbf{B}) \tag{13}$$

where, a_1 and a_2 are two arbitrary constants, to be determined by the physics of the problem (such as boundary conditions, etc). The above two equations can be combined, after eliminating either $\mathbf{V}$ or $\mathbf{B}$.

The equation for $\mathbf{B}$ can be written,

$$\nabla \times \nabla \times \nabla \times \mathbf{B} + p\nabla \times \nabla \times \mathbf{B} + q\nabla \times \mathbf{B} + r\mathbf{B} = 0 \tag{14}$$

where p, q, r are constants, given in terms of a_1 and a_2: $p = -\frac{a_1+a_2}{a_1 a_2}$, $q = \frac{1+a_1 a_2}{a_1 a_2}$, and $r = \frac{-1}{a_2}$

It may be pointed out that the solution spectrum of equation (14) is much wider and richer that those obtained from the solution of the corresponding equation for electron ion fluid. The general solution of eqn.(14) can be obtained as a linear superposition of the Chandrasekhar-Kendall eigenfunctions, which are the eigenfunctions of the "curl" operator, i.e.

$$\nabla \times \mathbf{B} = \lambda \mathbf{B}$$

This simple solution shows some remarkable properties of e-i plasma. The pressure profile, peaking at the axis, shows the self-confining properties of this equilibrium state. Also, the axial magnetic field reverses at the edge, showing the field-reversal feature, like that seen in the Reversed Field Pinch (RFP). Thus, this work demonstrates that e-i plasma, under appropriate circumstance can behave like a RFP, but with a confining pressure. We believe that this could open a new direction in activities of e-i plasma, like investigation of vortex like structures (i.e., spheromak type of closed field lines) and many other new and interesting physics.

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