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# Control of chaos in parametric decay instability

V.I.Arhipenko<sup>1</sup>, E.Z. Gusakov, L.V.Simonchik<sup>1</sup>, B.O.Yakovlev, V.A. Pisarev<sup>1</sup>

*Ioffe Institute, St.Petersburg, Polytechnicheskaya 26, Russia*

<sup>1</sup>*Institute of Molecular and Atomic Physics, Minsk, Belarus*

## I. INTRODUCTION

The presence of chaos in physical systems has been widely demonstrated and is very common in nature. In practice it is often desired to have a mechanism, which allows controlling or avoiding chaos state. In particular during the past decade, the problem of chaos controlling in plasmas attracted significant attention due to the importance of this topic for many processes [1-3].

Thus in addition to the general interest of this subject the possibility of reduction of the anomalous transport, associated with turbulent phenomena, also motivated the studies of chaos in plasma. Practically all systems reach turbulent state via chaos development, therefore, chaos control investigation may contribute to control and understanding of the turbulence phenomena in plasma processes. Up to now, despite the occurrence of numerous chaotic situations often deeply studied in plasma physics during the past years, a few results have been reported on the control of chaos in the plasma physics. The main idea in chaos controlling is to use the high sensitivity of the physical system to small perturbations, which ordinary leads to chaos development and to change the dynamical state. This approach has been successively used in several effective methods of chaos control proposed by Ott, Grebogi, and Yorke (OGY) [4] and Paragas (TDAS) [5].

In this paper we demonstrate experimentally the possibility of three-wave system transition from chaotic to regular behavior under influence of the pump frequency modulation without directly using the well-known OGY and TDAS methods. Studies has been performed using the wave system in magnetized inhomogeneous plasma in which absolute decay parametric instabilities of stimulated back scattering and forward scattering  $l \rightarrow l' + s$  has been excited [6,7].

## II. EXPERIMENTAL SITUATION

Experiments were carried out at the linear plasma device with magnetic field of 0.35 T, in which inhomogeneous plasma ( $n_e=n_e(z,r)$ ) was produced by ECR discharge in argon at pressure 1-2 Pa [6]. The electron plasma pump wave at frequency  $f_0 = 2.5-2.7$  GHz – (EPW) – was excited in this plasma using a waveguide system. In vicinity of resonant (focal) point, where  $n_e(z,0)=n_c$  (i.e.,  $2\pi f_0=\omega p=(2\pi n_e e^2/m_e)^{1/2}$ ), the electric field of the electron plasma wave increases.

The growth of electric field in the vicinity of focal point is so significant, that parametric decay instabilities of stimulated back scattering and forward scattering  $l \rightarrow l' + s$  are excited at the

relatively small pump wave power  $P_0$  exceeding 20 mW. These instabilities and their competition were studied previously in experiments with monochromatic pump wave [6,7].

The back scattering instability excitation mechanism, according to [6], is related to the complicated spatial structure of pump wave, namely to the small fraction of the first radial mode present in the pump along with the dominant fundamental radial mode ( $P_1 \leq 0.1P_0$ ). The first radial mode leads to appearance of the second resonance region of the three-wave interaction and to the formation of the feedback loop. An instability growth rate and an unstable spectrum structure are determined by velocity of the ion-sound wave and the feedback loop dimension. At the linear stage the absolute decay instability is a coherent process with the limited number of oscillatory modes excited, which should possess a narrow frequency spectrum.

However in reality both the spectrum consisting of several narrow lines and broad spectrum are observed depending on the plasma parameters connected with each other (density and temperature of plasma components, pump power and its frequency width, external magnetic and etc). High sensitivity of the absolute instability to the incident pump power is observed at small excess of its threshold ( $P_0 \geq 20$  mW), when the induced scattering emission is increased by 2-3 orders of magnitude at the rise of the pump power by a few percents.

The induced forward scattering parametric decay instability, studied in [7] is excited in accordance with M.Rosenbluth mechanism [8]. This intensive wave process appears possible because the decay condition for forward scattering is fulfilled not in a single, but in a couple of close points. It manifests itself in intensive anomalous absorption of forward scattering wave and in generation of back scattering frequency up-shifted wave.

As it was shown in [9], it is possible to influence the absolute parametric decay instability of stimulated back scattering using a harmonic pump frequency modulation. It was demonstrated in this paper that the stimulated back scattering can be suppressed by several orders of magnitude by modulation at frequency  $f_m=1$  MHz with deviation not exceeding  $\pm 1-2\%$  of pump wave frequency ( $f_0=2350$  MHz). It should be mentioned that the stimulation of absolute parametric instability was also observed in this experiment at modulation frequency  $f_m \sim 0.2$  MHz [7]. In both cited papers only the back scattering power spectra were under investigation, whereas in the present paper we consider the temporal variation of back scattering signal and acoustic wave, paying special attention to proper characterization of  $l \rightarrow l'+s$  three-wave dynamic system.

In the pump frequency modulation experiment the decay ion acoustic wave was studied by enhanced scattering technique [9,10]. For this purpose the probing wave at frequency  $f_p=2.35$  GHz  $< f_0$  and small power ( $P_p < 5$  mW) was launched into plasma by the same waveguide system. In vicinity of the own resonance point the probing wave is effectively scattered off the parametrically driven ion-sound wave. The scattered wave amplitude, proportional to the ion-sound

one, was used after homodyne detection for studying the dynamics of the three-wave system. The homodyne detector signal was digitized by means of an 8 bit A/D converter with time quantization  $\Delta t=0.025 \mu s$ . The obtained scalar time series were used for creation of the system phase space and investigation of its attractor.

### III. THEORETICAL BACKGROUND

Before the 1980s it was assumed that to study dynamics of systems with many degrees of freedom, time-series measurements of all the variables and derivatives were necessary to represent dynamics of studied system. However in paper [11] it has been demonstrated that a phase-space of the dynamics system could be reconstructed from a single variable time series using the time-delay coordinates. Considering time interval  $\tau$  and integer number  $m$  one can construct  $m$ -dimensional vector, with components provided by experimentally measured values  $x(t)$  taken at different time ( $t, t + \tau, t + 2\tau, \dots, t + (m - 1)\tau \dots$ ):

$$\vec{X} = [x(t), x(t + \tau), \dots, x(t + (m - 1)\tau)] \quad (1)$$

This vector  $\vec{X}$  defines a point in  $m$ -dimensional space, which moves in time along a trajectory. In experimental case a discrete raw of data can be obtained and as a result one can obtain a number of points along the  $m$ -dimensional point trajectories:

$$\vec{X}_i = [x(t_i), x(t_i + \tau), \dots, x(t_i + (m - 1)\tau)]$$

This delay-coordinate reconstruction would be then topologically equivalent to the dynamics of the true system. This allows to make a qualitative conclusion relating on the graphics view for  $m$  equal 2 and 3. It can be interpreted as a projection of the trajectory and its attractor if any from real phase space to reconstructed  $m$ -dimensional space. If the trajectory projection looks like an area chaotically filled with points it means that real dimension of the system is incomparable with  $m$  or that signal  $x(t)$  has a noise nature, whereas if the signal is generated by the dynamical system with small dimension a picture with a fine internal structure can be observed.

Numerical criterion for the chaotic dynamical system characterization has been suggested in [11]. As it was shown all the system attractors can be characterized by the correlation dimension. This dimension is defined on the basis of a long time-series of points on the attractor by considering the correlation integral:

$$C(\varepsilon) = \lim_{N \rightarrow \infty} \frac{1}{N(N-1)} \sum_{i,j=1}^N \Theta\left(\varepsilon - \|\bar{X}_i - \bar{X}_j\|\right) \quad (2)$$

where  $\Theta$  is the Heaviside function. This correlation integral counts the number of point pairs separated by distance  $\|\bar{X}_i - \bar{X}_j\|$  smaller than  $\varepsilon$ . In the case  $C(\varepsilon)$  for small  $\varepsilon$  scales as:

$$C(\varepsilon) \sim \varepsilon^D \quad (3)$$

the exponent  $D$  can serve as a satisfactory measure of the strange attractor structure. Thus a plot of  $\log_2 C(\varepsilon)$  as a function of  $\log_2 \varepsilon$  will have a slope of  $D$  down to small length scale, which can be used for the attractor dimension determination. This procedure is however limited in small  $\varepsilon$  region by noise [12] and a lack of experimental points, which play a crucial role and significantly vary dependence (3). This procedure should be continuously repeated for growing dimension  $m=1,2,3\dots$ . Presence or absence of saturation on the dependence  $D(m)$  on  $m$  is treated as a criteria for distinguishing the phase space generated by dynamic or noise like system. The level of  $D$  saturation is taken as the correlation dimension estimation for the dynamic system attractor. In order to have a longer line dependence suitable interval for  $D = d \log_2 C(\varepsilon) / d \log_2 \varepsilon$  determination a longer time realization should be taken into account. The minimal number of phase space points necessary for reliable correlation dimension estimation was obtained in [14] as:

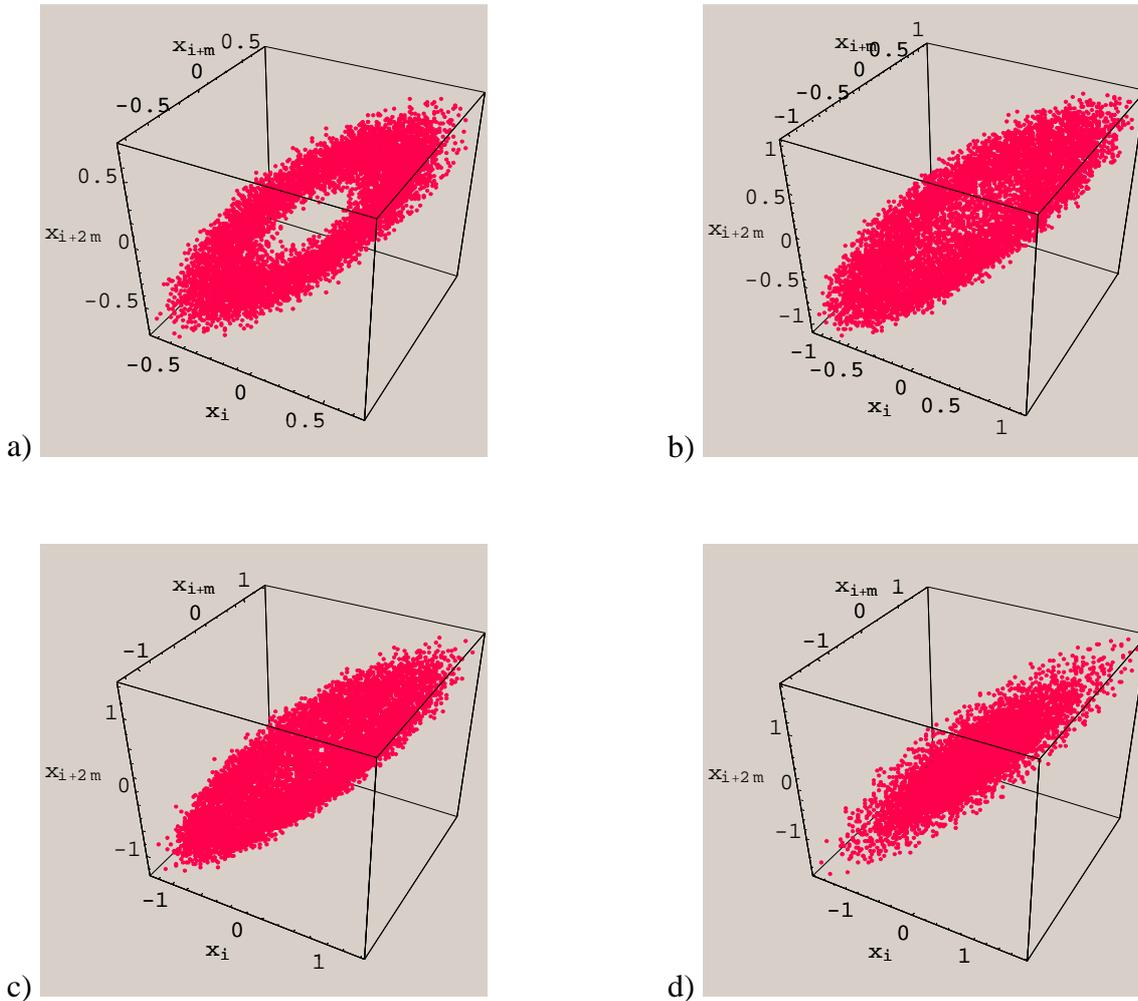
$$D_{\max} \cong 2 \log_{10} N \rightarrow N_{\min} \cong 10^{D_{\max}/2}$$

but it has to be taken into account that the longer realizations can play negative role fading the result due to existence of the inevitable errors of measurements [13].

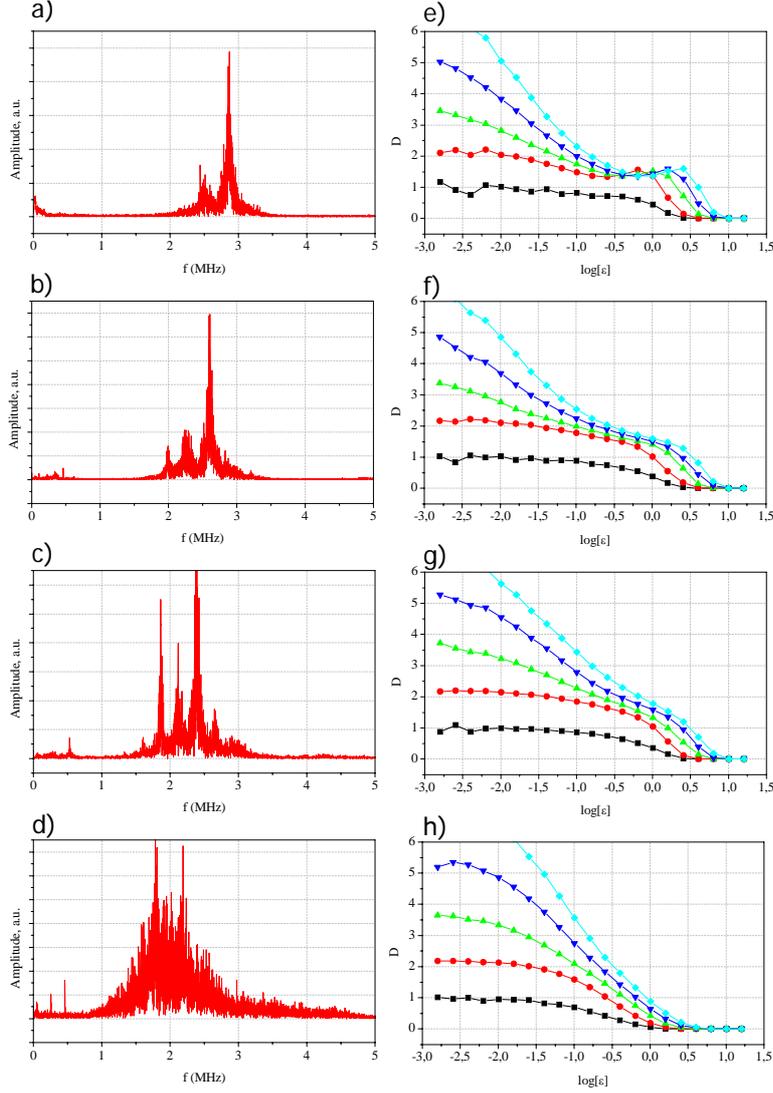
## IV. EXPERIMENTAL RESULTS AND DISCUSSION

### A. Chaos control in induced back scattering

The following pump wave modulation parameters were used in the induced back scattering decay instability control experiments: the central frequency  $f_0 = 2600$  MHz, modulation frequencies  $f_m = 0.2$  MHz and 1 MHz, maximal frequency deviation  $\delta f = \pm 35$  MHz. The Poincaré section of the phase space reconstructed from the homodyne detector signals for four consequently growing pump powers are presented in fig. 1. The first section, shown in Fig1a corresponds to power  $P=25$  mW, close to the induced back scattering absolute instability threshold  $P_{th}=20$  mW. The Poincaré section possesses evident structure, corresponding to the quasiperiodic behavior. The back scattering



**Fig. 1** System trajectory in the 3D phase space corresponding to the time-series for induced back scattering a)-d) correspondingly for growing pump power  $P_a < P_b < P_c < P_d$  spectrum of the probing wave, corresponding to this power is shown in fig2a. As it is seen the single line shifted by 3MHz dominates in the spectrum. The correlation dimension is reliably determined at the level  $D \simeq 1.5$  for this close to threshold state of the induced back scattering three-wave system, as it is seen in Fig2e. At higher power  $P=30$  mW the structure is less evident in the Poincaré section (Fig.1b). The probing wave back scattering spectrum become more complicated and consists of three pronounced lines (Fig2b), whereas the correlation dimension

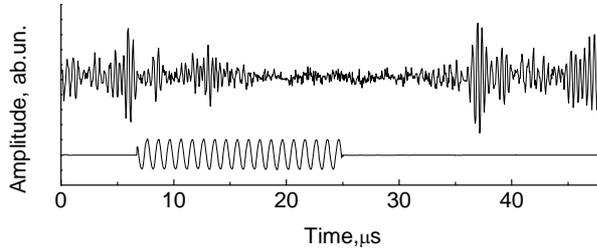


**Fig. 2** The homodyne signal correlation dimensions of attractors (a,b,c,d), power spectra (e,f,g,h), in case of monochromatic pump wave. —■—  $m=1$ ; —●—  $m=2$ ; —▲—  $m=3$ ; —▼—  $m=4$ ; —◆—  $m=5$

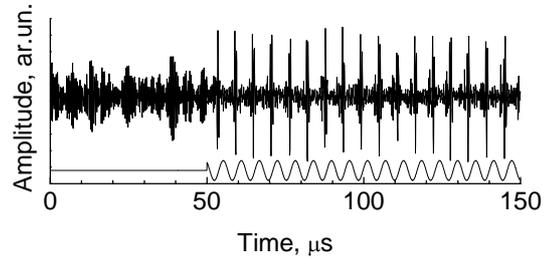
grows to the value  $D \simeq 1.7$  (Fig2f). It should be mentioned that it is determined less reliably here. At further growth of the pump power the tendency mentioned above become even more pronounced. The Poincaré section structure become less evident in Fig.1c and disappear in Fig.1d. Correspondingly the dominance of the single line disappears in Fig2c and especially in Fig2d. The probing wave back scattering spectrum become broad. At these high pump powers no tendency to fluttering of dependence  $D$  on  $\log \varepsilon$  is observed in Fig.2g and in Fig.2h. The correlation dimension is increased continuously with growing phase space dimension  $m$ .

The homodyne detector signals at harmonic pump frequency modulation are presented in fig. 3,4

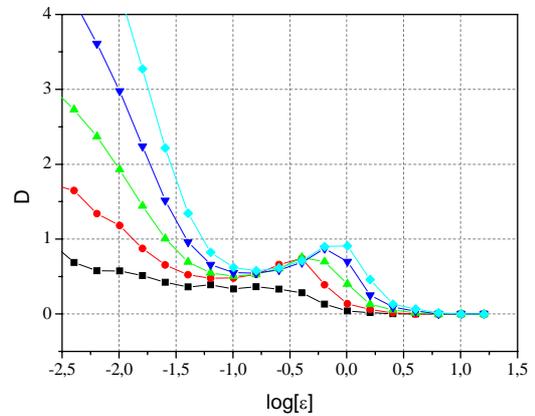
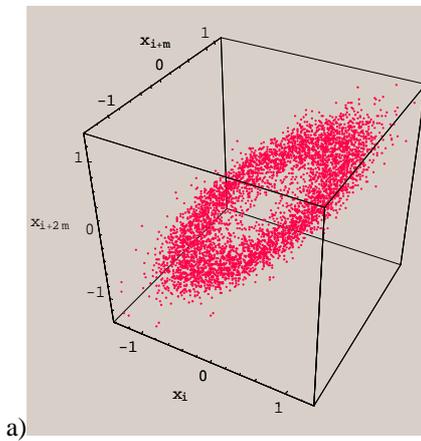
Poincaré section of the attractor and the procedure of its correlation dimension estimation after switching on of the modulation in the last case fig.4 are shown in fig. 5. It is clearly seen that the system changes from chaotic to quasi-periodic state when the pump wave frequency modulation is applied.



**Fig. 3** Oscilloscope of the homodyne and control signals at modulation frequency 1 MHz



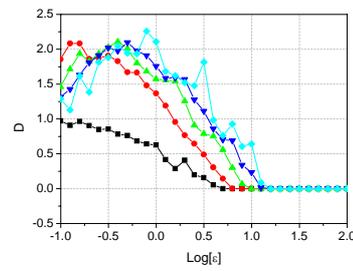
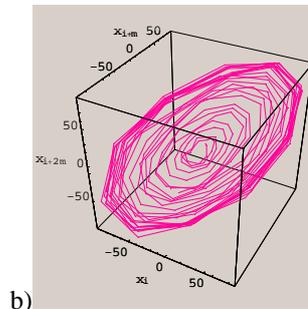
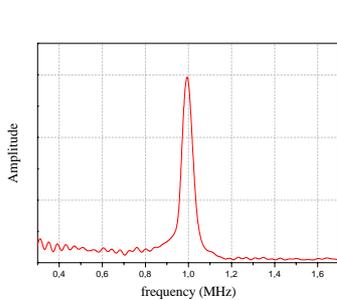
**Fig. 4** Oscilloscope of the homodyne (a) and control (b) signals at modulation frequency 0.2 MHz



**Fig. 5** The system trajectory in the 3D phase space (a) and correlation dimensions of the attractor determination (b) in case of frequency modulated pump wave.

**B. Chaos control in induced forward scattering**

In the case of induced forward scattering parametric decay instability all secondary lines observed in the back scattering spectra are very narrow (see Fig.6a). The clear structure, corresponding to the quasiperiodic motion is observed in the Poincaré section in this case (see Fig.6b). Determination of correlation dimension in this case is shown in Fig.6c.



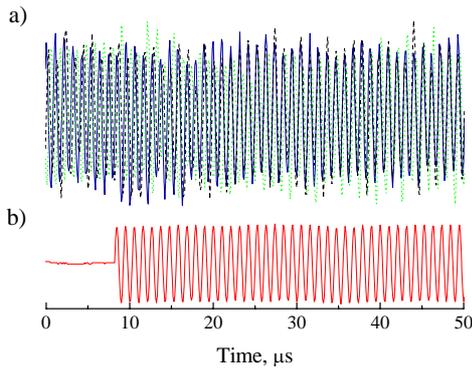
a)

b)

c)

**Fig. 6** The homodyne signal power spectra a), system trajectory in the 3D phase space b) and c) correlation dimensions:  
 -■- m=1; -●- m=2; -▲- m=3; -▼- m=4; -◆- m=5

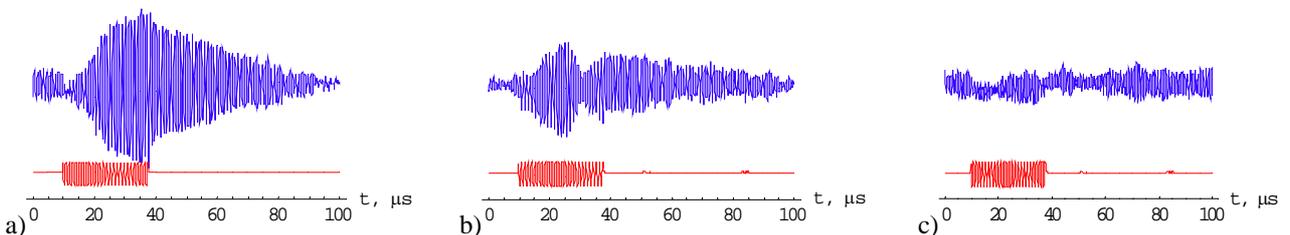
The fluttering of dimension  $D$  dependence on  $\log \varepsilon$  is clear in this figure, allowing reliable determination of  $D \simeq 2.0$ . In spite of the quasiperiodic behavior of the signal associated with induced forward scattering process is not completely regular. As it is seen in Fig.7a, where different realizations of this signal are shown, its phase changes randomly from realization to realization. To synchronize the phase of the signal we switched on harmonic pump frequency modulation at the frequency  $f_m$  close to the frequency shift of the backscattered probing wave.



**Fig. 7** Different realization of forward scattering signal a) and control signal

The output of the homodyne detector, shown in Fig.7a is synchronized to the control signal pulse, shown in Fig.7b. As it is seen in Fig.7a after onset of the control signal, with delay of  $\Delta t \simeq 10 \div 15 \mu s$  synchronization of back scattering takes place. This synchronization is demonstrated especially clear in Fig.8a, where a sum of 32 consequent outputs of homodyne detector are shown, where pronounced growth of oscillations at frequency  $f_s$

is observed after onset of the control signal, in the case when the modulation frequency  $f_m$  and frequency dominant in the back scattering signal  $f_s$  coincide ( $f_m=f_s=1\text{MHz}$ ). These oscillations are suppressed by averaging due to their random phase. After the onset of the pump phase modulation the phase locking and synchronization takes place manifesting itself in the growth of averaged oscillation amplitude. In the case the phase modulation frequency is slightly different from the back scattering frequency shift  $f_m-f_s=0,01\text{MHz}$  or  $f_m-f_s=0,1\text{MHz}$  averaged amplitude oscillations is decreased significantly (see Fig.8b,c)



**Fig. 8** Averaged output of homodyne detector for different modulation frequency a)  $f_m-f_s=0$ , b)  $f_m-f_s=-0,5\text{MHz}$ , c)  $f_m-f_s=-0,4\text{MHz}$

## V. CONCLUSION

Summarizing the presented results the conclusion that the harmonic pump frequency modulation can provide an effective way for control of chaos in a nonlinear dynamic three-wave system  $l \rightarrow l' + s$  can be made.

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