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On the effective Liapunov exponent in the tokamak context

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Abstract

Liapunov exponent provides a quantitative measure of the degree of stochasticity for a trajectory in the theory of both Hamiltonian and dissipative dynamical systems. In the context of fusion plasmas, the Liapunov exponent is usually associated with the regime of highly developed magnetic or collisional stochasticity (transport in the highly stochastic magnetic field, ergodic divertor problem, etc.). To treat general case with growing stochasticity, the extension of the standard concept is done introducing the effective Liapunov exponent, $L_e(t)$, as the averaged Liapunov exponent over different trajectories/magnetic field lines. The numerical analysis includes the observation of the Kolmogorov entropy, the number of the trajectories/ magnetic field lines with positive Liapunov exponent and statistics of the Liapunov exponents. Three models relevant for tokamak physics are considered: tokamak, revtokamak and the model of test electron radial diffusion in the magnetic field with irregularities. It is shown that the effective Liapunov exponent is in the correspondence with the degree of stochasticity. Additionally, its behaviour is independent on the origin of stochasticity (deterministic or statistical stochasticity). The dynamical behaviour of the effective Liapunov exponent can be associated with the type of test particle transport in different magnetic stochasticities: strange transport in the magnetic field regions with the mixture of the regular and irregular structures, uniform mixing in the highly irregular regions and standard diffusion in the case with highly frequent collisions and regular or partially destroyed magnetic field regions.

Key words: effective Liapunov exponent, deterministic and collisional stochasticities, sticking effect, escaping particles

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1 Introduction

Generically the concept of the Liapunov exponent as a qualitative measure for growing stochasticity in both Hamiltonian and dissipative dynamical systems is based on the premise that the Liapunov exponent defines unambiguously a sufficient condition for stochastic instability. Therefore, the Liapunov exponent, Liapunov spectra, distribution of finite-time Liapunov exponents and Kolmogorov entropy have been considered in connection with the evolution of chaoticity and ergodicity [1,2], the phase and topological transitions [3], the exchange between localized and extended quantum states [4], the metal-isolator transition in solid state physics [5] etc.

In plasma physics, the Liapunov exponent is studied in the context of transport in the stochastic magnetic field [6,7], ergodic divertor problem [8,9], the radial heat diffusion in the tokamak in the presence of the magnetic field irregularities [10], etc. The last study is done by introducing the concept of the effective radial Liapunov exponent.

In this paper the summary of the effective Liapunov exponent concept is presented. The conclusions are based on the comparative numerical investigation of three tokamak models: tokamak (twist map) [11], revtokamak (nontwist map) [12] and the Monte Carlo model of the test particle radial diffusion. Note that the stochasticity in the first two models is of the deterministic type (magnetic field with irregular domains- configuration space stochasticity) and of both the deterministic and statistical type in the third one (due to the magnetic field irregularities-configuration space stochasticity, and particle collisions- phase space stochasticity, respectively).

In section 2 the tokamak models: tokamak, revtokamak and the model of test particle diffusion are shortly presented. The effective Liapunov exponent and basics of the statistical method are mentioned in section 3. Comparative analysis of the tokamak models is established in section 4. Finally the concluding remarks are given in section 6.

2 Tokamak models

In present paper three tokamak models are considered: maps-tokamak and revtokamak, and the radial diffusion of test particles. Map models are constructed with the aim to model the topology of the magnetic field of tokamaks. Thus, the term 'plasma particle' for maps assumes that the trajectory of a plasma particle is closely tied to the corresponding magnetic field line (structure). In other words, the particle collisions are not considered.

2.1 Maps

In order to understand tokamak physics, the magnetic field equations are constructed [11] as the equations of motion of a $1\frac{1}{2}$ degree of freedom dynamical Hamiltonian system. The nature of orbits is conveniently studied by considering a stroboscopic plot, obtained by recording the values of the coordinates (ψ, θ) at successive equal times:

$$\begin{aligned}\psi_{t+1} &= \frac{1}{2} \left\{ P(\psi_t, \theta_t) + \sqrt{[P(\psi_t, \theta_t)]^2 + 4\psi_t} \right\}, \\ \theta_{t+1} &= \theta_t + W(x_{t+1}) - \frac{K}{(2\pi)^2} \frac{1}{(1 + \psi_{t+1})^2} \cos(2\pi\theta_t) \quad \text{mod } 1,\end{aligned}\quad (1)$$

where the coordinate ψ is dimensionless toroidal flux coordinate, θ is the poloidal angle², $t \equiv \zeta$ is the toroidal angle playing a role of time coordinate, $W(\psi)$ is the winding number of torus, and $K > 0$ the stochasticity parameter. The function $P(\psi, \theta)$ is defined as $P(\psi, \theta) = \psi - 1 - (K/2\pi) \sin(2\pi\theta)$.

The tokamak (Hamiltonian twist map [11]) represents a model of tokamak with monotonously decreasing winding number profile as a function of ψ (hence $q(\psi)$ is a monotonously increasing function - 'classical' experiment circumstances): $W(\psi) = (w/4)(2 - \psi)(2 - 2\psi + \psi^2)$; $w = W(0)(> 0) = \text{const.}$ is the value of the winding number on the polar axis (here $w = 4$). By varying $K(> 0)$, a variety of orbits are possible: cycles (periodic orbits), island chains (encircling the cycles), KAM barriers, and chaotic orbits inside the stochastic layers bounded by island chains and the KAM barriers, $K \approx K_c$, or inside the stochastic sea, $K \gg K_c$, where $K_c \approx \pi$ is the critical value of the stochasticity parameter which is determined by the island overlapping criterion [13]. The tokamak describes a structure that is very robust in the central region, and the global stochasticity starts in the edge region.

The recent experimental and theoretical works have been devoted to tokamak situations with locally reversed shear, i.e. the safety factor profile has a local minimum (especially important in the context of the internal transport barrier [14]). Hence the $W(\psi)$ function possesses a maximum. These reversed shear configurations are modelled by the nontwist $1\frac{1}{2}$ degree of freedom Hamiltonian map - revtokamak [12]. In the present context the profile of W is given as:

$$W(\psi) = w \left[1 - z(c\psi - 1)^2 \right], \quad (2)$$

² In the following the coordinates $r, \psi(\sim r^2)$ and angles θ, ζ are given in units of a (the minor radius of torus), ψ_a and 2π , respectively. Thus the maximum values of r, ψ and θ are unity.

$$z = \frac{w - w_0}{w}, \quad c = 1 + \left(\frac{w - w_1}{w - w_0} \right)^{1/2},$$

where the values of constants are chosen to be: $w_0 = 0.3333$, $w = 0.6667$, $w_1 = 0.1667$ [12].

A particularity of revtokamak is the appearance of a 'transport barrier' separating a robust, central region from a semiglobally chaotic edge region [12] for the stochasticity parameter larger than some threshold: $K > K_T$. According to [12], the value of K_T is defined as the value of K for which the transport barrier enters the physical domain. For actual set of parameters the estimated value of K_T is around 1.4.

In conclusion the stochasticity develops in the configuration space. Strictly speaking it is of interest for $K > K_c(K_T)$ (the global stochasticity regime [13]).

2.2 Model of the radial diffusion

Finally, the radial diffusion of test electrons in the magnetic field with irregular, radially bounded domains is evaluated by the Monte Carlo modelling of the gyro-phase averaged Boltzmann equation with the Coulomb collision part [10].

The stationary magnetic field is formed of the equilibrium magnetic field:

$$\vec{B} = \nabla\psi \times \nabla\theta - \epsilon \nabla\psi \times \nabla\zeta, \quad \epsilon = 1/q = 0.9 - 0.5875 \left(\frac{r}{a} \right)^2, \quad (3)$$

where a is the minor radius, and small perturbation

$$\delta\vec{B} = \nabla \times (b\vec{B}), \quad (4)$$

where $\psi \sim r^2$, θ and ζ are the Boozer flux coordinate, poloidal angle and toroidal angle, respectively [10]. In the small perturbation the function b , which has unit of length, is used to represent the structure of destroyed magnetic field, i.e. the islands and stochastic regions. Its Fourier representation is [10]

$$b(\psi, \theta, \zeta) \approx s_b \sum_{m,n} \exp\left(-\frac{(\psi - \psi_{mn})^2}{\Delta\psi^2}\right) \cos(m\theta - n\zeta), \quad (5)$$

where parameter s_b indicates the strength of perturbation, i.e. the stochasticity parameter and $\Delta\psi$ is the width of perturbation. The critical value is $s_{bc} \approx 7 \times 10^{-5}$ as shown in [10].

The pitch angle scattering which is considered as a Gaussian stochastic process is modelled by the Monte Carlo collision operator [10].

Thus, two types of stochasticity appear: magnetic field stochasticity (deterministic stochasticity in configuration space) and collisional stochasticity (statistical stochasticity in the phase space).

3 The effective Liapunov exponent

Liapunov exponent provides a quantitative measure of the degree of stochasticity for a trajectory in both Hamiltonian and dissipative dynamical systems [13]. The Liapunov exponent of i -th trajectory, for given initial conditions (position, and initial orientation of the infinitesimal displacement) indicates the mean exponential rate of divergence of trajectories surrounding it [6]:

$$l_i(t) \equiv \frac{1}{t} \ln \left(\frac{d_i(t)}{d_i(t=0)} \right). \quad (6)$$

The $d_i(t)$ is the distance at time t between two initially neighboring trajectories, usually evaluated in the tangential space of the trajectory [6]. For maps presented here, the tangential 2D space $(\partial\psi, \partial\theta)$ (two basis vectors), and for the diffusion model the tangential 3D space $(\partial r, \partial\theta, \partial\zeta)$ are of interest.

The positive value of the Liapunov exponent $l_i > 0$ denotes exponential separation of two initially neighboring trajectories. On the other hand, $l_i \leq 0$ corresponds to regular motion (initially neighboring trajectories are stuck to each other). The concept of the Liapunov exponent has generalized to describe the mean rate of exponential growth of a d -dimensional volume in tangential space [13] ($d \leq N_d$, where N_d is the dimensionality of tangential space). For maps: $l_\psi = -l_\theta$ and with respect to the tangential surface 2D Liapunov exponent $L = 0$. Analogously, for test particle diffusion 3D Liapunov exponent is zero. In the present paper the stress is on the non-negative Liapunov exponent for maps, l_ψ , or l_r for the radial diffusion, and their statistics. In the following the mentioned Liapunov exponents are noted by l for both cases.

The effective Liapunov exponent is defined as the averaged 1d Liapunov exponent over different initial conditions (index i)

$$L_e(t) \equiv \langle l(t) \rangle = \frac{1}{N} \sum_{i=1}^N l_i(t), \quad (7)$$

where N is the total number of trajectories (particles). For 2D Hamiltonian maps the Kolmogorov entropy, which is defined as the sum of all positive Liapunov exponents, is equivalent to the L_e . Thus the time behaviour of the effective Liapunov exponents for maps is identical to the time behaviour of the Kolmogorov entropy. To understand generically the development of stochasticity the number of the trajectories with the positive Liapunov exponent N_p is additionally observed.

In order to qualify the distribution of the Liapunov exponents the second, third and fourth cumulant coefficients are defined with respect to the l adopting ensemble average [10]. The second cumulant measures the dispersion around $\langle l(t) \rangle$ [10]. The third cumulant coefficient, γ_3 indicates the degree of asymmetry around $\langle l(t) \rangle$ and the fourth cumulant coefficient, γ_4 the relative peakedness or flatness of a distribution of Liapunov exponents with respect to Gaussian ($\gamma_{n>2} = 0$).

To clarify the long time correlations of the Liapunov exponent distribution, the autocorrelation coefficient $A(t, t')$ (the function of two time instants t and t') is additionally calculated as in [10,15]. The $A(t, t')$ deviation from the corresponding Wiener autocorrelation coefficient is taken as the indicator of finite the long time correlations.

The Liapunov exponents are numerically calculated by the procedure developed in the context of the random matrix theory [16]. Initially, the N particles (trajectories) are started at fixed position $\psi(r) = \text{const.}$ with randomly chosen θ and (θ, ζ) for maps and test diffusion model, respectively. The random variables are numerically generated by the numerical IMSL routine RNSET.

4 Results and discussion

Being the statistical quantity the effective Liapunov exponent is strictly meaningful after the global stochasticity regime is established. In other words, after the island overlapping for tokamap [11] has been started, $K > K_c$; the internal transport barrier has been moved inside the physical domain for revtokamap [12], $K > K_T$; and the island overlapping has been started, $s_b/s_{bc} > 1$, or/and the collision stochasticity has been introduced, $\nu > 0$, for test particle diffusion model [10]. However, the L_e is calculated over the whole parameter regime (parameters K and $s_b/s_{bc}, \nu/\nu_t$ for maps and test diffusion, respectively), due to completeness.

The results shown here are obtained with test particle (trajectory) ensembles of $N = 10000$ constituents for mappings and $N = 1000$ for the diffusion case. Whole ensemble is loaded initially at same flux surface with randomly chosen

angle coordinates. The corresponding values of $\psi(t = 0)$ are noted for each example in the following.

4.1 Magnetic field stochasticity

Increasing the magnetic field stochasticity is nicely illustrated in the phase portraits of revtokamap for different values of K . The initial flux surface is $\psi = 0.37$, which belongs to the plasma core region. As examples the phase portraits for $K = 2.5, 5.5$, and 5.8 are shown in Figure 1. To quantify the stochasticity development, the L_e is calculated for $K = 1.5, 2.5, 3.5, 4.5, 5.5, 5.8$, and 6.28 and results are plotted in Figure 2. Briefly, for $K < K_T$, when the magnetic field structure is regular, or irregular structures are localized around the island boundaries ($K \geq K_T$), the L_e at first decreases rapidly and then slowly saturates to some finite positive value. To the higher value of K corresponds bigger saturation value of L_e (Figure 2). After that the majority of test particles are permanently stuck to regular structures in the magnetic field, i.e. test particles (tied to the magnetic field lines) cannot leave the regular magnetic structures inside the core region of plasma where all particles were initially loaded. The positive value of L_e is a consequence of the sheared magnetic field structure [13].

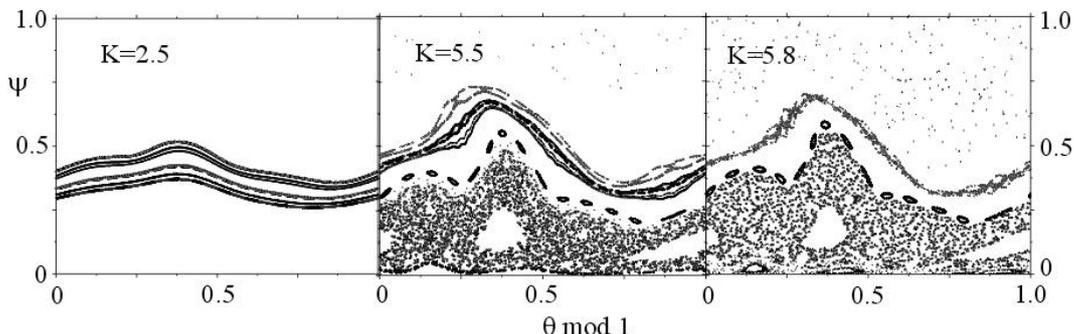


Fig. 1. The phase space trajectories of revtokamap are plotted in long time limit (after 5000 iterations) for $K = 2.5, 5.5$ and 5.8 . The 10 trajectories are started from $\psi = 0.37$ below the noble surface [14]. At the value $K = 5.5 \gg K_T$ the particles start to escape through the barrier into the non-physical domain $\psi > 1$.

Above the threshold, for $K = 5.5 \gg K_T$, L_e continuously increases which can be ascribed to the overcoming of the internal transport barrier by some of the test particles (Figure 1). With further increase of the stochasticity, the irregular (stochastic) domains dominate and the behavior of the L_e qualitatively changes. The sticking effect is more and more suppressed, i.e. test particles wander around wider and wider stochastic region. The effective Liapunov exponent saturates very fast to high positive value, approximately two orders higher than it did around the threshold. The saturation indicates that

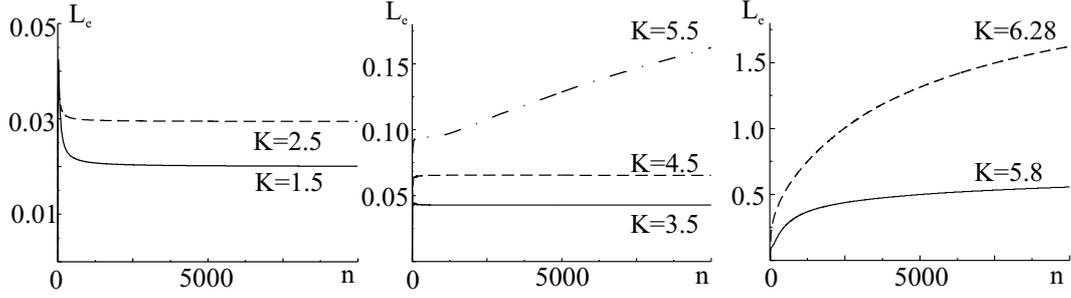


Fig. 2. The L_e vs n is plotted for revtokamap with $K = 1.5, 2.5, 3.5, 4.5, 5.5, 5.8$ and 6.28 . Note that n is the number of iterations (time is $t = n dt$ and dt is the arbitrary time step). The $N = 10000$ test trajectories are started at $\psi_0 = 0.37$. The phase diagrams for 10 trajectories and $K = 2.5, 5.5$ and $K = 5.8$ are shown in figure 1.

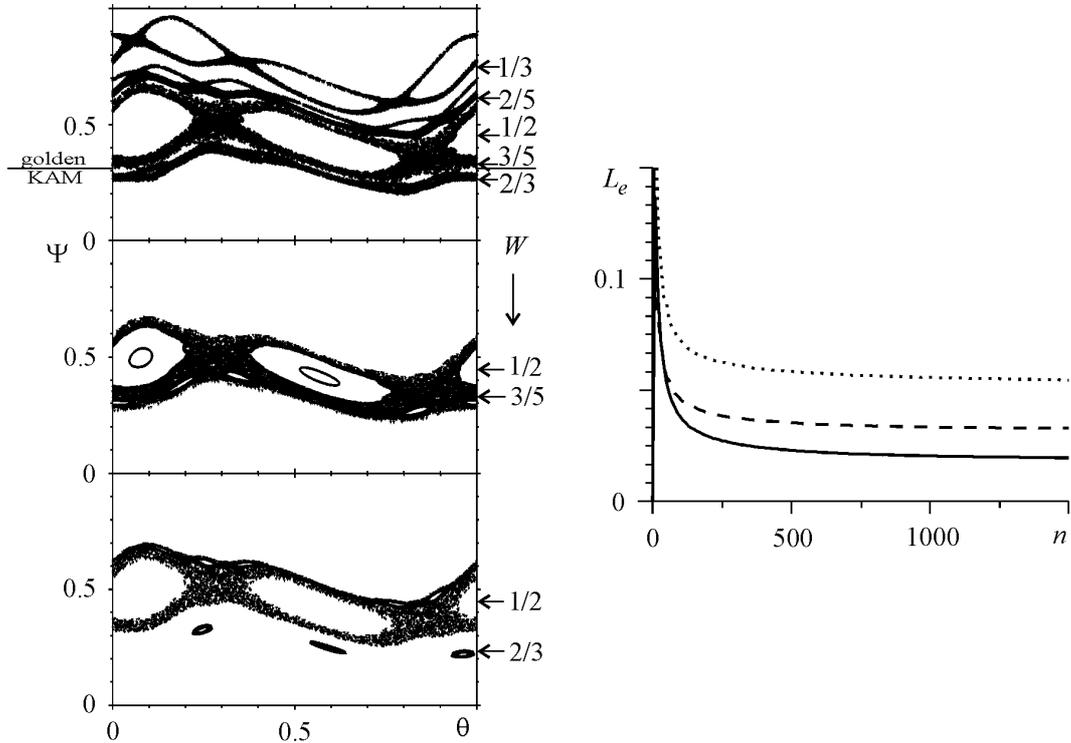


Fig. 3. The L_e vs n ($t = n \cdot dt$) is plotted for tokamap with $K = 3.7$. The test trajectory ensembles are started from $\psi_0 = 2/3, 1/2$ and $1/3$ - solid, dashed and dotted line, respectively. The corresponding phase diagrams are given on the left in present figure: $\psi_0 = 2/3, 1/2$ and $\psi_0 = 1/3$, from top to down.

the most of test particles wander over the whole allowed bounded stochastic region.

The unexpected monotonous growth of the L_e with time for extremely high values of K (e.g. $K = 6.28$) can be associated with the growth of the particle escaping rate (more and more particles escape from the physical domain of the revtokamap as noted also in [12]).

The similar scenario of the stochasticity development is described for tokamap [15] and the radial diffusion of test particles in tokamak [10]. The characteristics of the two last cases is the bounded stochastic sea without regular structures (uniform particle distribution). This regime does not exist in revtokamap (nontwist map) case due to the particle escape in the non-physical domain. Thus, the semi-global chaoticity is generically associated with the revtokamap.

It is worth to mention that in the semi-global chaoticity regime of revtokamap or global stochasticity region, which is characterized by the mixture of the regular and irregular domains in tokamap [15], the value of L_e depends on the initial positions of particles (an example is illustrated in Figure 3 for tokamap [15]). Note that the number of particles with significant value of the Liapunov exponent increases with the development of stochasticity [10],[15].

4.2 Coexistence of the magnetic and collisional stochasticities

There are two kinds of the stochasticity whose nature is completely different: magnetic (deterministic) stochasticity and collisional (statistical) stochasticity. While the first one is generated in the configuration space as the growth of the irregular magnetic domains, the second one is generated in phase (moment) space as a pitch angle scattering [10]. Both of them govern the particle radial diffusion which appears as the standard diffusion, strange diffusion or uniform mixing (non-diffusive) process [10]. The standard diffusion regime is an exception and corresponds only to the case with highly frequent collisions and regular magnetic field structure, or magnetic field with partially destroyed structure.

Let us briefly consider the parameters used in [10]. The Fourier harmonics of the magnetic field perturbation have been chosen as

$$\frac{n}{m} = \frac{7}{10}, \frac{2}{3}, \frac{7}{11}, \quad (8)$$

where $\Delta\psi/\psi_a = 0.1$, $s_b/s_{bc} = 0.33, 1.3, 3.3$, and 33 , and the collision parameters: $\nu/\nu_t = 0.45, 4.5$, and 45 , where ν_t is the characteristic frequency of passing particle motion [10].

In the absence of collisions under the overlapping threshold, $s_b/s_{bc} < 1$, for all time $L_e(t) < 0$ (fig. 4). In all other cases, i.e. $s_b/s_{bc} \geq 1$ and arbitrary ν/ν_t , and $s_b/s_{bc} < 1, \nu/\nu_t > 0$ the effective radial Liapunov exponent $L_e(t)$ almost monotonically increases with time from negative to positive values, asymptotically leading to saturation as shown in figure 4 and modeled by

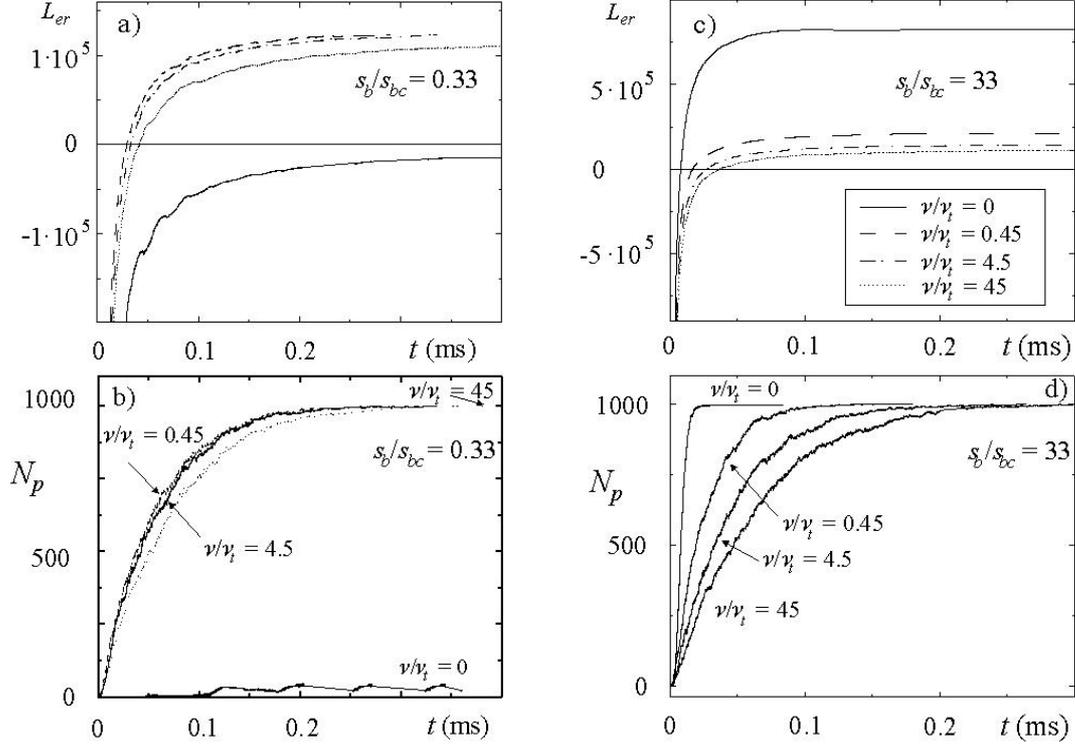


Fig. 4. The numerically calculated L_e and N_p vs time: (a), (b) for $s_b/s_{bc} = 0.33$, and (c), (d) for $s_b/s_{bc} = 33$. The solid, dashed, dot-dashed and dotted curves correspond to the $\nu/\nu_t = 0, 0.45, 4.5$ and 45 , respectively. Note, that the end time in calculation is $t_s \approx 10 \times t_d$ (except for $s_b/s_{bc} = 0.33$ where $t_{end} \approx 0.4$ ms) and $N = 1000$.

equation:

$$L_e(t) = \langle l_e \rangle \left(1 - \frac{t_d}{t} \right), \quad (9)$$

where $\langle l_e \rangle$ is the saturated value obtained by numerical fitting [10]. Additionally, the Kolmogorov entropy is always positive in the global stochasticity regime [10], [15].

Time $t = t_d$ at which $L_e(t) = 0$ (table 1 in [10]) for $s_b/s_{bc} < 0, \nu/\nu_t = 0$ tends to infinity. Correspondingly, the number of particles with the positive Liapunov exponent changes from a few particles below threshold without collisions, to a value which tends to N as the collisions get more frequent and irregular structures grow (Figures 4b and 4d). Note, that the above mentioned value t_d is characterized by $N_p = N/2$. In the regular and partially destroyed magnetic field configurations, when the characteristic times are $\tau_c (= 1/\nu) \ll t_d$, the nature of radial diffusion is changed from strange to standard diffusion [10]. On the other hand in the highly developed magnetic stochasticity region the collisions additionally rearrange the uniform particle distribution which appears as a 'correlation effect' and the radial diffusion is never of the standard

type.

From the point of view of the effective Liapunov exponent, both stochasticities manifest themselves equally which can be seen in Figure 4. In conclusion, the L_e is a universal measure of the degree of stochasticity in the observed physical system.

4.3 Statistics of the Liapunov exponent

The statistical properties of the Liapunov exponent in tokamak models are briefly considered here. In preceding publications, which considered the tokamak case [15] and the radial diffusion [10], it was shown that the distributions of the Liapunov exponents are not the elementary ones. The exception is the case of the radial diffusion for $s_b/s_{bc} = 0.33$, $\nu/\nu_t = 45$, where the L_e distribution is Gaussian. The conclusions are made by calculating the statistical measures: the first four cumulant coefficients and autocorrelation coefficient for all cases.

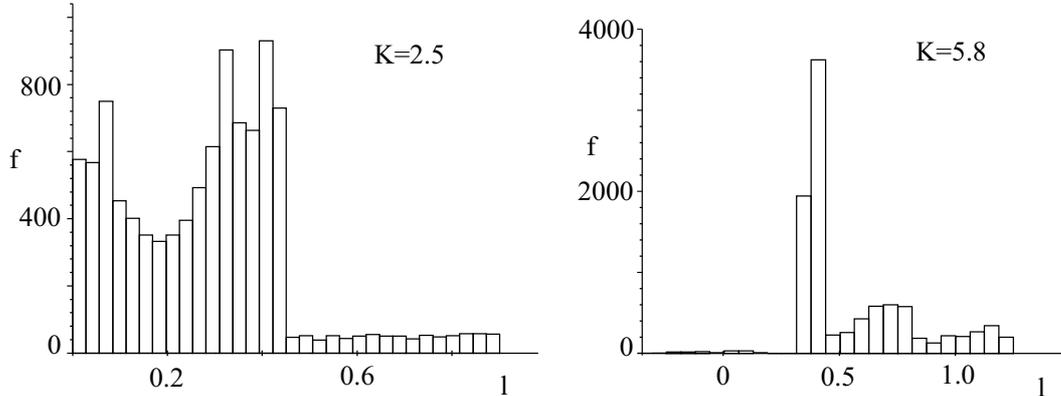


Fig. 5. The distributions of particles with the positive Liapunov exponent for revtokamak with $K = 2.5 > K_T$ and $K = 5.8 \gg K_T$ are not the elementary ones.

Here, as an example, the time development of the above mentioned statistical measures is briefly commented for the revtokamak and parameters in Figure 2. Observation of the first and third cumulant coefficients over whole parametric space indicates highly asymmetric distribution of the Liapunov exponents. Additionally the distribution is more peaked with increasing K , as can be seen in Figure 5.

The values of the second cumulant increase with K as illustrated in Figure 6. It is according to the conclusion that the test particles experience larger stochastic domain with developing stochasticity, as well as the discussion considering Figure 2.

Not considering the details, it is worth mentioning that the autocorrelation

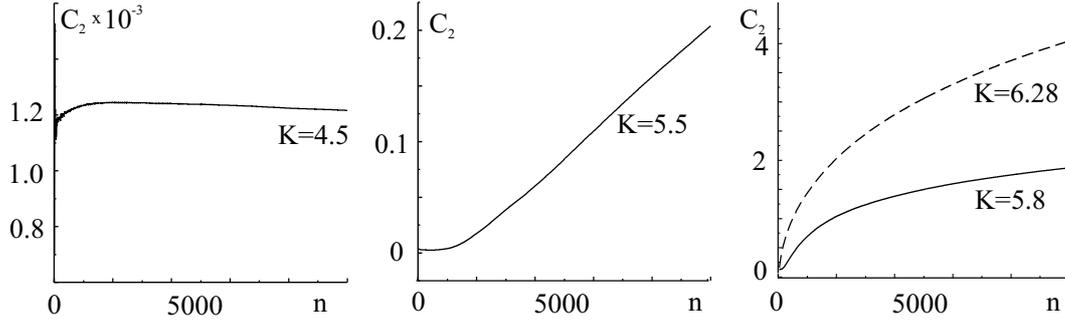


Fig. 6. The numerically calculated C_2 vs time for $K = 4.5, 5.5, 5.8$ and 6.28 . As mentioned in main text the qualitative changes in the second cumulant time behaviour can be associated with growth of stochasticity (medium K) and the effect of escaping particles (high K).

coefficients show the increase of the time correlation with the decrease of K (similarly to earlier cases [10], [15]). In other words the distribution of the Liapunov exponents is not the elementary one.

It seems that a detailed statistical analysis of the Liapunov exponent distribution could be a new breakthrough in the theory of transport in tokamak with partially developed stochasticity.

5 Conclusion

- The concept of the effective Liapunov exponent is applicable for both the magnetic field stochasticity and the collisional stochasticity;
- The sticking effect (sticking of particles to the regular structures) is clearly observable in the evolution of the effective Liapunov exponent for regimes with partially developed stochasticity;
- The presence of the internal transport barrier affects the evolution of the $L_e(t)$;
- The growth of the particle escaping rate for revtokamak is followed by the growth of $L_e(t)$;
- The diverse types of diffusivity can be considered through the different dynamical behaviours of the $L_e(t)$;
- The distribution of the $L_e(t)$ is not the elementary one.

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