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Theoretical studies of parallel current in the presence of fluctuations

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Abstract

Effects of electrostatic fluctuations on the current parallel to the magnetic field are theoretically studied for magnetically confined plasmas. Based upon a closed set of equations for the ensemble-averaged distribution function and the response function to an infinitesimal external perturbation, a radial diffusion equation for the bootstrap and Ohmic current densities is derived. The anomalous radial diffusion due to fluctuations as well as the finite-banana width effect is shown to significantly modify the conventional bootstrap current near the magnetic axis. It is also shown that this modification leads to the possibility of the completely bootstrapped tokamak.

We consider a single ion-species plasma in a tokamak with circular close section. Then we assume that a gyrophase-averaged distribution function is slightly perturbed from the Maxwellian f_{a0} and the perturbed distribution function f_{a1} in a presence of electrostatic fluctuations is determined from a drift kinetic equation

$$\frac{\partial f_{a1}}{\partial t} + \zeta v \mathbf{b} \cdot \nabla f_{a1} - C_a(f_{a1}) + \delta \mathbf{v}_\perp \cdot \nabla_\perp f_{a1} = -V_{dar} \frac{\partial f_{a0}}{\partial r} + \frac{e_a \zeta v E_\parallel}{T_a} f_{a0}, \quad (1)$$

where $\mathbf{b} = \mathbf{B}/B$; $\zeta = \mathbf{v} \cdot \mathbf{b}/v$; e_a and T_a are the charge and temperature for species a ; V_{dar} is the radial component of the toroidal drift velocity; E_\parallel is the parallel component of induced electric field; $C_a(f_{a1}) = \sum_b [C_{ab}(f_{a1}, f_{b0}) + C_{ab}(f_{a0}, f_{b1})]$ is the Fokker-Planck collision operator; and $\delta \mathbf{v}_\perp = \mathbf{B} \times \nabla \delta \phi / B^2$ is the velocity fluctuation due to fluctuating electrostatic potential $\delta \phi(\mathbf{r}_\perp, t)$. Starting from the equation (1), we derive a closed set of equation for the ensemble-averaged distribution function $\bar{f}_{a1}(\mathbf{r}_\perp, \zeta, v) \equiv \langle f_{a1}(\mathbf{r}_\perp, \zeta, v, t) \rangle_{\delta \phi}$ and the response function $G_a^\dagger(\mathbf{r}_\perp - \mathbf{r}'_\perp, t - t', \zeta, v; \zeta', v')$ [1]:

$$v_\parallel \mathbf{b} \cdot \nabla \bar{f}_{a1} - C_a(\bar{f}_{a1}) - \nabla_\perp \cdot \int d\mathbf{r}'_\perp \int d\mathbf{v}' \mathbf{F}(\mathbf{r}_\perp - \mathbf{r}'_\perp, t - t') \\ \times G_a^\dagger(\mathbf{r}'_\perp - \mathbf{r}_\perp, t' - t, \zeta', v'; \zeta, v) \cdot \nabla_\perp \bar{f}_{a1}(\mathbf{r}_\perp, \zeta', v') = -V_{dar} \frac{\partial f_{a0}}{\partial r} + \frac{e_a \zeta v E_\parallel}{T_a} f_{a0}, \quad (2)$$

$$-\frac{\partial G_a^\dagger}{\partial t} - \hat{C}_a(G_a^\dagger) - \nabla_\perp \cdot \mathbf{D}_a \cdot \nabla_\perp G_a^\dagger = \frac{1}{2\pi v^2} \delta(t - t') \delta(\mathbf{r}_\perp - \mathbf{r}'_\perp) \delta(\zeta - \zeta') \delta(v - v'), \quad (3)$$

where $\hat{C}_a(G_a^\dagger) = C_a(G_a^\dagger f_{a0})/f_{a0}$;

$$\mathbf{F}(\mathbf{r}_\perp - \mathbf{r}'_\perp, t - t') = \langle \delta \mathbf{v}_\perp(\mathbf{r}_\perp, t) \delta \mathbf{v}_\perp(\mathbf{r}'_\perp, t') \rangle_{\delta\phi}; \quad (4)$$

$$\mathbf{D}_a = \int d\tau \int d\mathbf{r}_\perp \int d\mathbf{v}' G_a^\dagger(\mathbf{r}_\perp, \tau, \zeta, v; \zeta', v') \mathbf{F}(\mathbf{r}_\perp, \tau); \quad (5)$$

the ensemble average over fluctuations is denoted by $\langle \cdot \rangle_{\delta\phi}$; the temporal and spatial Markovian approximations are used in the equation for \bar{f}_{a1} ; and in addition the neglect of the toroidal effect and the Markovian approximation in velocity space are also used in the equation for G_a^\dagger . Note here for the later use that the Legendre polynomial $P_l(\zeta)$ is the eigenfunction of the collision operator, i.e., $C_a(h(v)P_l(\zeta)) = P_l(\zeta)C_a^l(h(v))$.

We first calculate the diffusion tensor defined by (5). This diffusion tensor \mathbf{D}_a is rewritten as

$$\mathbf{D}_a = \int d\tau \int d\mathbf{r}_\perp G_a^{(0)}(\mathbf{r}_\perp, \tau, v) \mathbf{F}(\mathbf{r}_\perp, \tau), \quad (6)$$

where the function $G_a^{(0)}$ satisfies the equation

$$-\frac{\partial G_a^{(0)}}{\partial t} - \hat{C}_a^0(G_a^{(0)}) - \nabla_\perp \cdot \mathbf{D}_a \cdot \nabla_\perp G_a^{(0)} = \delta(t - t') \delta(\mathbf{r}_\perp - \mathbf{r}'_\perp) \quad (7)$$

with $\hat{C}_a^0(G_a^{(0)}) = C_a^0(G_a^{(0)} f_{a0})/f_{a0}$. Let us now assume the correlation function of fluctuating electrostatic potential in the form

$$\langle \delta\phi(\mathbf{r}_\perp, t) \delta\phi(\mathbf{0}, 0) \rangle_{\delta\phi} = \phi_0^2 e^{-r_\perp^2/2\lambda_\perp^2} e^{-|t|/\tau_c}. \quad (8)$$

Then the diffusion tensor is written as $\mathbf{D}_a = D_a \mathbf{I}$, and the diagonal component D_a is determined by the following nonlinear equation

$$D_a = \frac{\bar{\phi}_0}{\sqrt{2}} [1 + x e^x \text{Ei}(-x)]^{1/2}, \quad (9)$$

where $\bar{\phi}_0 = \phi_0/B$, $x = \lambda_\perp^2 \nu_{aa} [1 + 1/(\nu_{aa} \tau_c)] / (2D_a)$, $\text{Ei}(-x) = -\int_x^\infty e^{-t}/t dt$ is the exponential-integral function, and we have approximated the collision operator in (7) by $\hat{C}_a^0 = -\nu_{aa}$ for simplicity.

Taking the velocity moments $m_a v_\parallel B$ and $m_a v_\parallel (v^2/v_a^2 - 5/2)B$ of (2) and averaging over a flux surface, we have the moment equations

$$\langle B A_{a1} \rangle = \langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_a \rangle - \langle B F_{a1} \rangle - e_a n_a \langle B E_\parallel \rangle, \quad (10)$$

$$\langle B A_{a2} \rangle = \langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Theta}_a \rangle - \langle B F_{a2} \rangle, \quad (11)$$

where m_a and $v_a = \sqrt{2T_a/m_a}$ are the mass and the thermal velocity for species a ; $\langle \cdot \rangle$ denotes the flux surface average; and the parallel viscous forces and the frictional forces are given by

$$\begin{bmatrix} \langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_a \rangle \\ \langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Theta}_a \rangle \end{bmatrix} = \langle m_a B \int d\mathbf{v} v_\parallel^2 \begin{bmatrix} 1 \\ v^2/v_a^2 - 5/2 \end{bmatrix} \mathbf{b} \cdot \nabla \bar{f}_{a1} \rangle \quad (12)$$

and

$$\begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix} = \int d\mathbf{v} m_a v_{\parallel} \begin{bmatrix} 1 \\ v^2/v_a^2 - 5/2 \end{bmatrix} C_a(\bar{f}_{a1}). \quad (13)$$

The anomalous diffusion terms are written as

$$\begin{bmatrix} A_{a1} \\ A_{a2} \end{bmatrix} = \nabla_{\perp} \cdot \int d\mathbf{v} m_a \zeta \left(\int d\mathbf{r}'_{\perp} d\tau \mathbf{F}(\mathbf{r}'_{\perp}, \tau) \begin{bmatrix} G_{a1}^{(1)}(\mathbf{r}'_{\perp}, \tau, v) \\ G_{a2}^{(1)}(\mathbf{r}'_{\perp}, \tau, v) \end{bmatrix} \right) \cdot \nabla_{\perp} \bar{f}_{a1}(\mathbf{r}_{\perp}, \zeta, v), \quad (14)$$

where the functions $G_{ak}^{(1)}(\mathbf{r}_{\perp}, t, v)$ ($k = 1, 2$) are determined by

$$-\frac{\partial G_{ak}^{(1)}}{\partial t} - \hat{C}_a^1(G_{ak}^{(1)}) - \nabla_{\perp} \cdot \mathbf{D}_a \cdot \nabla_{\perp} G_{ak}^{(1)} = c_k \delta(t - t') \delta(\mathbf{r}_{\perp} - \mathbf{r}'_{\perp}) \quad (15)$$

with $c_1 = v$ and $c_2 = v(v^2/v_a^2 - 5/2)$. Equation (15) can be solved by expanding $G_{ak}^{(1)}$ with respect of v in a series of the generalized Laguerre polynomials $L_j^{3/2}(v^2/v_a^2)$ of $3/2$ as

$$G_{ak}^{(1)}(\mathbf{r}_{\perp}, t, v) = v \left[\alpha_k(\mathbf{r}_{\perp}, t) + \beta_k(\mathbf{r}_{\perp}, t) L_1^{3/2}(v^2/v_a^2) + \dots \right]. \quad (16)$$

Inserting this expansion into (15) and taking velocity moments $v^3 L_j^{3/2}(v^2/v_a^2) f_{a0}$, we can calculate the expansion parameters. We here consider the lowest-order approximation in this expansion and write as $G_{ak}^{(1)}(\mathbf{r}_{\perp}, t, v) = \alpha_k(\mathbf{r}_{\perp}, t) v$. Then, solving the equation (15) with the use of the diffusion coefficient (9) and substituting its solution into the expression (14), we obtain $A_{a2} = 0$ and

$$\langle B A_{a1} \rangle = \frac{1}{r} \frac{\partial}{\partial r} r \tilde{D}_a \frac{\partial}{\partial r} \langle B n_a u_{a\parallel} \rangle \frac{m_a}{\tau_{aa}} \quad (17)$$

with

$$\tilde{D}_a = \frac{1}{2} \frac{\tau_{aa}^2}{\lambda_{\perp}^2 d_{a1}} \bar{\phi}_0^2 [1 + \eta_a e^{\eta_a} \text{Ei}(-\eta_a)], \quad (18)$$

where $d_a = \tau_{aa} D_a / \lambda_{\perp}^2$, $\eta_e = (Z + \tau_{ee}/\tau_c)/(2d_e)$ and $\eta_i = (\tau_{ii}/\tau_c)/(2d_i)$ with $\tau_{aa} = (3\sqrt{3}/4\nu_{aa})$ and the ion charge number Z .

We now apply the moment method of the conventional neoclassical transport theory to the equations (10) and (11). Then, with the use of (17), the diffusion equation for the parallel current J is obtained in the form

$$\frac{1}{r} \frac{\partial}{\partial r} r D \frac{\partial}{\partial r} \langle B J \rangle - \langle B J \rangle = -\langle B J_b \rangle_0 - \langle B J_E \rangle_0, \quad (19)$$

with

$$D = \frac{\sqrt{2} + 13Z/4}{Z(\sqrt{2} + Z)} \tilde{D}_e, \quad (20)$$

where $\langle B J_b \rangle_0$ and $\langle B J_E \rangle_0$ are the bootstrap and Ohmic current densities in the absence of fluctuations, and we have neglected the anomalous diffusion effect due to fluctuations in the calculation of viscous forces. In the conventional neoclassical transport

theory, the bootstrap current in the absence of fluctuations is expressed in terms of the viscous coefficients μ_{ak} ($k = 1, 2, 3$). However, the finite banana-width effect is known to modify this conventional theory around the magnetic axis. We thus use the modified viscous coefficients μ_{ak}^* presented in ref. 2 in order to take into account the finite banana-width effect around the magnetic axis. These modified viscous coefficients reduce to the conventional ones μ_{ak} for $\sqrt{\epsilon} \gg \delta_a^{1/3}$, and these coefficients on the magnetic axis become $\mu_{ak}^* \propto \delta_a^{1/3}$, where $\epsilon = r/R$ with the major radius R , and $\delta_a = 2q_0\rho_a/kR$ with the safety factor q_0 at the magnetic axis, the Larmor radius ρ_a and the elongation parameter k .

In Fig.1, we plot the diffusion coefficient in the diffusion equation (19) as a function of $\xi = \bar{\phi}_0\tau_{ee}/\lambda_\perp^2$ for $Z = 1$ and $\tau_c \gg \tau_{ee}$. The diffusion coefficients D in the weak turbulent limit ($\xi \ll 1$) and in the strong turbulent limit ($\xi \gg 1$) are analytical calculated as $D/\lambda_\perp^2 = (\sqrt{2}+13Z/4)/[Z^2(\sqrt{2}+Z)]\xi^2$ and $D/\lambda_\perp^2 = (\sqrt{2}+13Z/4)/[\sqrt{2}Z(\sqrt{2}+Z)]\xi$, respectively. The diffusion coefficients for these weak and strong turbulent limits are also plotted in Fig.1 by the dotted and dashed curves.

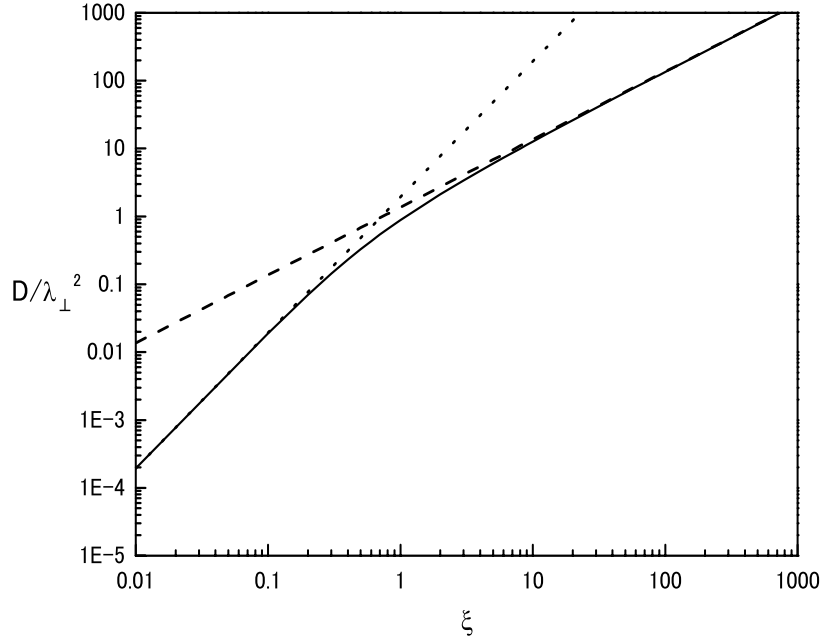


Fig.1: Diffusion coefficient D normalized by λ_\perp^2 versus ξ . The diffusion coefficients for the weak and strong turbulent limits are plotted by the dotted and dashed curves.

The effect of anomalous diffusion is significant for the bootstrap current around the magnetic axis. We finally concentrate on this problem. Assuming the radial profiles for the pressure and the temperatures as $p_e = p_0[1 - (r/a)^2]^2$ and $T_e =$

$T_i = T_0[1 - (r/a)^2]$, we solve the diffusion equation (19) by setting $E_{||} = 0$. The resulting bootstrap current density J_0 on the magnetic axis, normalized by $p_0/a^2 B'_{p0}$ with $B'_{p0} = \lim_{r \rightarrow 0} B_p/r = B/q_0 R$, is shown as a function of $\bar{D} = D/a^2$ in Fig. 2. For the two limiting cases, $\delta_e = 0, D = \text{finite}$ and $D = 0, \delta_e = \text{finite}$, the bootstrap current density J_0 can be obtained analytically

$$J_0(D = 0) = \frac{p_0}{a^2 B'_{p0}} \delta_e^{1/3} \frac{2}{Z(\sqrt{2} + Z)} \left[2 \left(1 + \frac{0.490}{Z} \right) L_1 + L_2 \right] \quad (21)$$

and

$$J_0(\delta_e = 0) = \frac{p_0}{a^2 B'_{p0}} \sqrt{\frac{a}{R}} \bar{D}^{1/4} \frac{3.39}{Z(\sqrt{2} + Z)} \left[2 \left(1 + \frac{0.414}{Z} \right) L_1 + L_2 \right], \quad (22)$$

with

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} (\sqrt{2} + 13Z/4)\hat{\mu}_{e1} + (3Z/2)\hat{\mu}_{e2} \\ (\sqrt{2} + 13Z/4)\hat{\mu}_{e2} + (3Z/2)\hat{\mu}_{e3} \end{bmatrix}, \quad (23)$$

where $\hat{\mu}_{e1} = 0.398 + 0.696Z$, $\hat{\mu}_{e2} = -0.407 - 0.928Z$, $\hat{\mu}_{e3} = 0.961 + 2.05Z$ for the expression (21), and $\hat{\mu}_{e1} = 0.533 + Z$, $\hat{\mu}_{e2} = -0.625 - 1.5Z$, $\hat{\mu}_{e3} = 1.39 + 3.25Z$ for the expression (22). The bootstrap current density on the magnetic axis reduces to $J_0(D = 0)$ when the condition $\delta_e^{1/3} \gg \bar{D}^{1/4}$ is satisfied, and it reduces to $J_0(\delta_e = 0)$ for $\delta_e^{1/3} \ll \bar{D}^{1/4}$. The transition between two regimes is well described by the semi-analytically obtained following formula:

$$J_0 = [1 - x_c K_1(x_c)] J_0(D = 0) + e^{-x_c(x_c+2)/10} J_0(\delta_e = 0), \quad (24)$$

where $x_c = (R/a)\sqrt{\delta_e^{4/3}/\bar{D}}$, and K_1 is the modified Bessel function of the second kind of order 1. The results calculated from this analytic formula (24) are also plotted by the dotted curves in Fig. 2.

The anomalous radial diffusion due to fluctuations as well as the finite-banana width effect lead to the possibility of the completely bootstrapped tokamak. We finally show a simple equilibrium model of completely bootstrapped tokamak by solving the diffusion equation (19) supplemented with the Ampère's law. The resulting radial profiles of the bootstrap current density normalized by $I/\pi a^2$ with the total current I are shown in Figs. 3 and 4 for $\bar{D} = 0.01, 0.001$, $\delta_e = 0.001$ (solid curves), $\bar{D} = 0.01, 0.001$, $\delta_e = 0$ (dotted curves), and $\bar{D} = 0, \delta_e = 0.001$ (dashed-dotted curves). For comparison, the conventional bootstrap current densities ($\bar{D} = 0, \delta_e = 0$) which are calculated by using the values of B'_{p0} for $\bar{D} = 0.01, 0.001$ and $\delta_e = 0.001$ are also plotted by dashed curves. The values of poloidal beta $\beta_p = \int (p_e + p_i) dV / (V B_{pa}^2 / 2\mu_0)$ for the equilibriums of $\bar{D} = 0.01, \delta_e = 0.001$, and $\bar{D} = 0.001, \delta_e = 0.001$ are calculated as $\beta_p = 2.12$ and 1.85 , respectively.

In conclusion, we have shown that the radial diffusion equation for the parallel current density can be derived from the drift kinetic equation in the presence of electrostatic fluctuations. It is also shown that the anomalous radial diffusion due

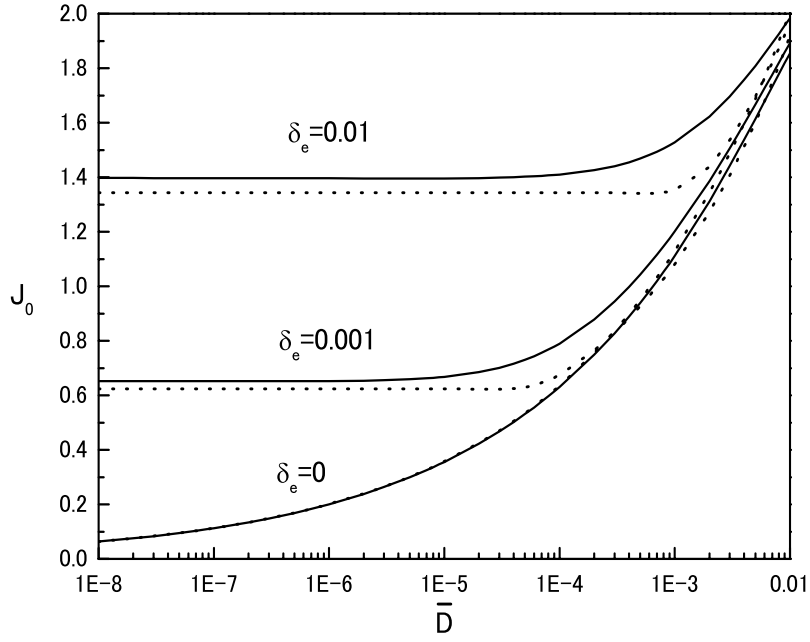


Fig.2: Normalized bootstrap current density at the magnetic axis versus $\bar{D} = D/a^2$. The bootstrap current density calculated from the analytical formula (24) is plotted by the dotted curves.

to fluctuations as well as the finite-banana width effect significantly modifies the conventional bootstrap current around the magnetic axis.

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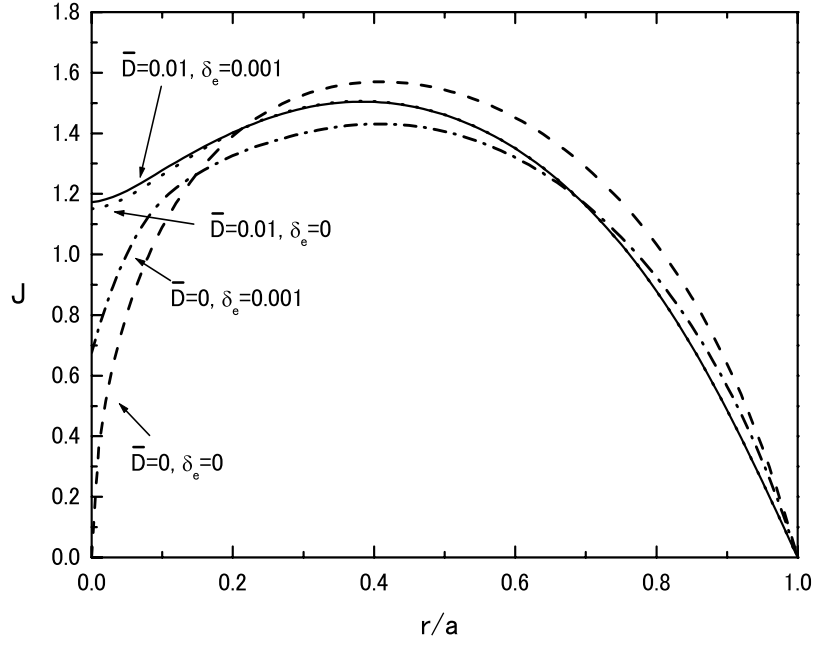


Fig. 3 Radial profile of the bootstrap current density.

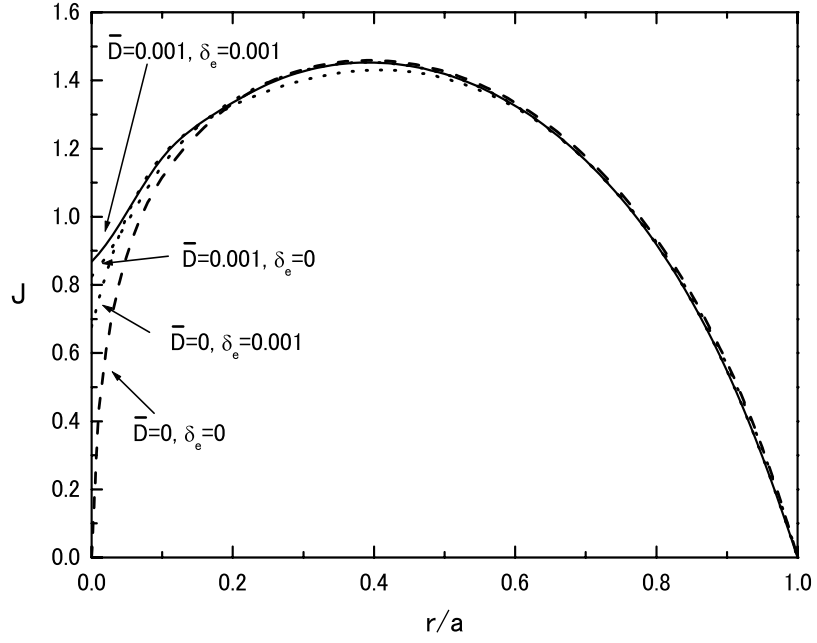


Fig. 4 Radial profile of the bootstrap current density.