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Three-Dimensional PIC Simulation of Electron Plasmas

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Abstract. The three-dimensional evolution of a pure electron plasma is studied by means of a particle-in-cell code which solves the drift-Poisson system where kinetic effects in the motion parallel to the magnetic field are taken into account. Different results relevant to the nonlinear dynamics of trapped plasmas and low-energy electron beams are presented.

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INTRODUCTION

The evolution of an electron plasma in a Malmberg-Penning trap [1] is studied by means of the particle-in-cell (PIC) code MEP [2]. The electron dynamics is analyzed in the frame of a guiding center electrostatic approximation, where the velocity perpendicular to an externally applied uniform axial magnetic field is given by the electric drift, and kinetic effects in the motion parallel to the magnetic field are taken into account. The evolution of the system is followed within a conducting cylindrical surface of radius R and length L on which the (in general time-dependent) boundary conditions for the electrostatic potential are imposed.

The code is applied here to investigate two different situations (see Fig. 1). In the first case, the evolution of a traveling electron beam is considered. It is observed in particular how in a space-charge dominated regime a fraction of the electrons close to the axis is reflected back to the cathode while a high density annulus is formed inside the drift tube, which gives rise to the development of vortex-like structures. In the second case, the injection phase of an electron plasma in a Malmberg-Penning trap is studied. Here, the electrons enter in an initially empty trapping region, are reflected by a fixed potential barrier on the opposite side of the trap and come back to the cathode, thus interacting with new emitted electrons. It is shown how, in dependence of injected current and geometrical parameters, a virtual cathode may form close to the injection surface, and a hollow electron column is formed in the trapping region. Longitudinal kinetic effects are also investigated by varying the velocity distributions of the injected electrons.

MODEL AND PIC CODE

The system is described in the frame of the zeroth order drift approximation, where the velocity perpendicular to the uniform magnetic field $\mathbf{B} = B\mathbf{e}_z$ ($\mathbf{e}_z$ being the unit vector in

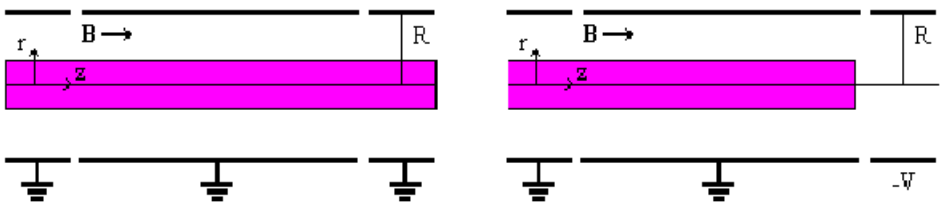

FIGURE 1. Scheme of traveling beam configuration (**left**) and trap filling configuration (**right**). $-V$ is the trap potential.

the z direction) is given by the electric drift, $\mathbf{v}_E = (c/B)\,\mathbf{e}_z \times \nabla\varphi$, with φ the electrostatic potential and c the speed of light. Assuming the guiding centers distribution, f, of the form $f(\mathbf{x},\mathbf{v},t) = F(v_\parallel,\mathbf{x},t)\ \delta(\mathbf{v}_\perp - \mathbf{v}_E)$, where $\mathbf{x},\mathbf{v},t$ are position, velocity and time, respectively, δ denotes the Dirac's distribution, and $v_\parallel$ and $\mathbf{v}_\perp$ denote the components of the velocity parallel and perpendicular to the magnetic field, the Vlasov-Poisson system reads

$$\frac{\partial F}{\partial t} + (\frac{1}{2}\,\mathbf{e}_z \times \nabla\varphi\ + v_\parallel \mathbf{e}_z)\cdot\nabla F + \frac{1}{M_{eff}}\,\mathbf{e}_z\cdot\nabla\varphi\,\frac{\partial F}{\partial v_\parallel} = 0 \quad ; \quad \nabla^2\varphi = n\,, \tag{1}$$

where $n(\mathbf{x},t) = \int F\,dv_\parallel$ is the electron density. Adimensional quantities are used: length, time, density and potential are normalized over R, $\omega_c/2\omega_p^2$, n_0 and $4\pi e n_0 R^2$, respectively, where $\omega_c \equiv eB/mc$ is the non-relativistic electron cyclotron frequency, $\omega_p \equiv (4\pi e^2 n_0/m)^{1/2}$ is the electron plasma frequency, computed for a specified electron density n_0, and $-e$, m are electron charge and mass, respectively. The behavior of the system is therefore characterized by the single parameter M_{eff}, which plays the role of an effective mass,

$$M_{eff} \equiv 4\,\frac{\omega_p^2}{\omega_c^2}\ \simeq 4.115\cdot 10^{-4}\frac{n_0[10^7\mathrm{cm}^{-3}]}{B^2[\mathrm{kGauss}]}\ . \tag{2}$$

It results $M_{eff} = 2n_0/n_B$, n_B being the so-called Brillouin density, $n_B \equiv (B^2/8\pi)/mc^2$, so that $0 \le M_{eff} \le 2$. Introducing a cylindrical system of coordinates (r,θ,z), and using the variable $s \equiv r^2$, the equations of motion corresponding to the trajectories of the kinetic equation in (1) are:

$$\frac{ds}{dt} = -\frac{\partial\varphi}{\partial\theta} \quad ; \frac{d\theta}{dt} = \frac{\partial\varphi}{\partial s} \quad ; \frac{dz}{dt} = v_\parallel \quad ; \frac{dv_\parallel}{dt} = \frac{1}{M_{eff}}\frac{\partial\varphi}{\partial z}\,. \tag{3}$$

In the code, Eqs. (1) are discretized on an equispaced grid (with the only exception of the central cell) in the coordinates s, θ and z. The number of cells is denoted as N_s, N_θ and N_z, respectively. The grid for s is defined as $\underline{s}_0 = 0$, $\underline{s}_1 = 1/(N_s N_\theta + 1)$, $\underline{s}_{j+1} = \underline{s}_j + N_\theta/(N_s N_\theta + 1)$, $s_0 = 0$, $s_j = (\underline{s}_j + \underline{s}_{j+1})/2$, $j = 1\ldots N_s$ (s is the position of the center, while $\underline{s}$ denotes the lower boundary of a "radial" cell). The grid for θ is $\theta_l = 2\pi(l-1)/N_\theta$, $l = 1\ldots N_\theta$, with the periodicity relation $\theta_{N_\theta+1} = \theta_1$, while the

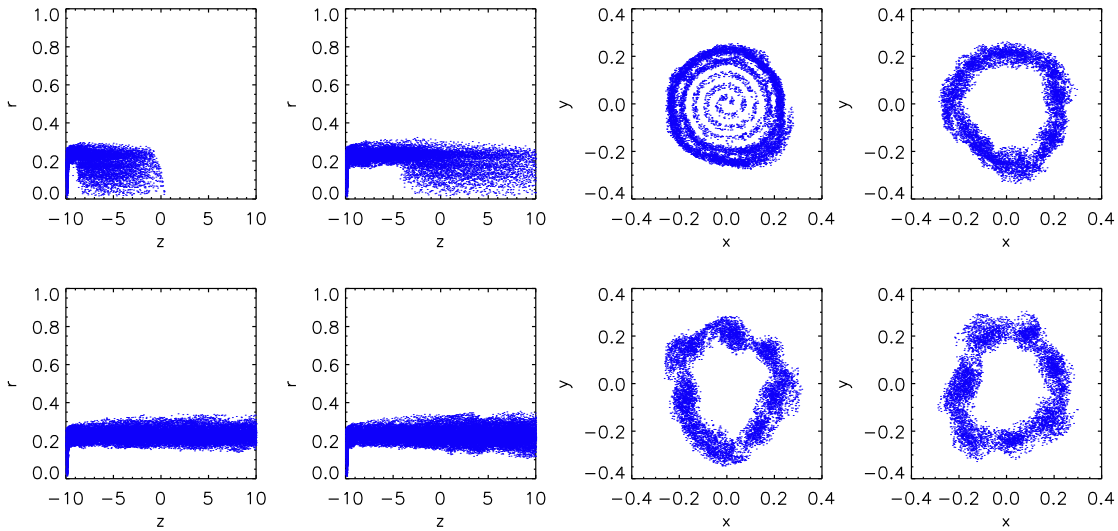

FIGURE 2. **Left:** Time evolution of a beam in the plane (z,r). The parameters of the run are: $M_{eff} = 0.01$, $N_s = 64$, $N_\theta = 64$, $N_z = 128$, $N_p = 10^5$. The beam is injected at $z = -10$, with an initial parallel velocity $v_{\|0} = 9.0$. The current is $I = 0.2215$, I being normalized over the ratio between $e\pi R^2 L n_0$ and the normalization time. Emitter and collector are both at ground. From left to right, top to bottom, the data refer to $t = 0.7$, 1.5, 3.5 and 10.0, respectively. **Right:** Transverse projection $[(x,y)$ plane$]$ of the particles in the interval $-10 \le z \le -5$, $-5 \le z \le 0$, $0 \le z \le 5$ and $5 \le z \le 10$, respectively. The data refer to $t = 10.0$.

grid for z is $z_k = (k-1/2)L/N_z - L/2$, $k = 1 \ldots N_z$. Each cell has the same volume $\Delta V = \Delta s \Delta\theta \Delta z/2$, with $\Delta s = 1/(N_s + 1/N_\theta)$, $\Delta\theta = 2\pi/N_\theta$, and $\Delta z = L/N_z$, respectively. The system governed by Eqs. (1) is simulated numerically as an *ensemble* of macro-particles with fixed sizes Δs, $\Delta\theta$ and Δz, using a PIC method [2].

RESULTS

The code is used to simulate the plasma dynamics in a Malmberg-Penning trap for different possible experimental settings. The physical situation is determined by several geometrical and emission parameters: the magnetic field strength B; the sizes R, L and the geometry of the emitting surface; the potentials which are imposed on cathode, anode and drift tube; the initial electron velocity distribution and the initial current distribution emitted by the source. Malmberg-Penning traps usually use a spiral-wound tungsten filament for the injection [3]. The MEP code is able to simulate this initial spatial distribution of the electrons; the effect of an accelerating grid is considered by suitably "cutting" the spiral along rows and columns of a given width. In addition, the code is able to take into account various initial velocity and current density distributions. At first, a low-energy electron beam traveling in an equipotential (grounded) drift tube is considered (see Fig. 1 *left*). The electrons are continuously emitted from the cathode and collected to the anode. The characteristic time scale of the system is the time of flight of

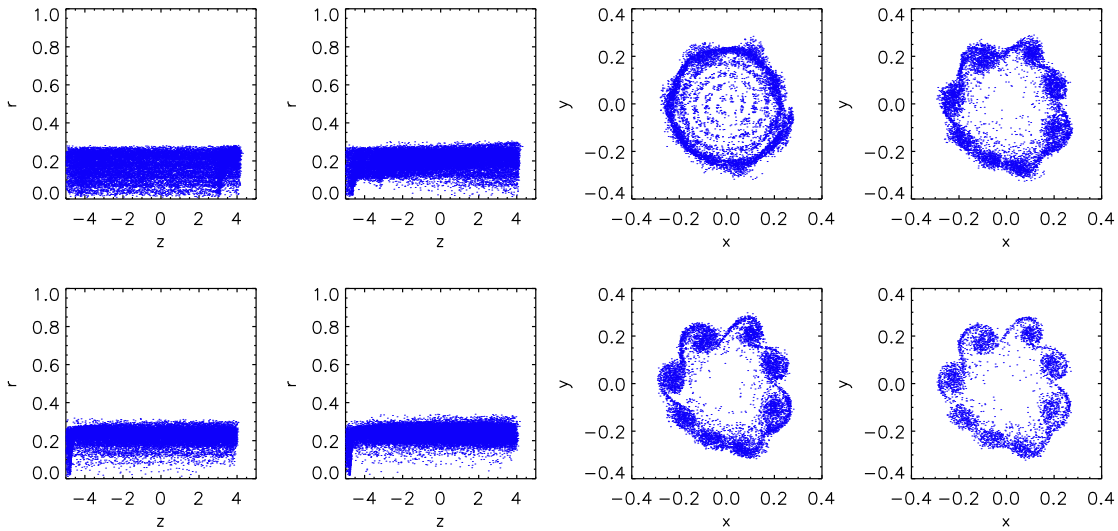

FIGURE 3. **Left:** Time evolution of a plasma filling the trap, in the (z,r) plane. The parameters of the run are: $M_{eff} = 4.64 \cdot 10^{-4}$, $N_s = 64$, $N_\theta = 64$, $N_z = 128$, $N_p = 10^5$. The beam is injected at $z = -5.0$, with $v_{\|0} = 28.90$. The current is $I = 0.1423$, and the potential difference between collector and emitter is -0.97. From left to right, top to bottom, the data refer to $t = 0.7$, 1.5, 4.0 and 8.0, respectively. **Right:** Transverse projection of the particles in the interval $-5 \leq z \leq -2.5$, $-2.5 \leq z \leq 0$, $0 \leq z \leq 2.5$ and $2.5 \leq z \leq 5$, respectively. The data refer to $t = 8.0$.

an electron, $L/v_{\|}$. In a space-charge-limited regime, it is found experimentally that the central part of the beam is reflected, a hollow electron column forms and fast coherent structures arise, possibly due to the development of diocotron instability [4]. The PIC simulations confirm the experimental findings. The time evolution of a mono-energetic beam in the (r,z) plane is shown in Fig. 2 *left*. It is evident, in particular, the formation of a virtual cathode close to the injection surface. The central part of the beam is reflected back to the cathode by the space-charge of the beam itself, and only the outer part of the beam reaches the opposite end of the trap. This annular beam shows a quasi-2D evolution of vortex structures: Fig. 2 *right* represents the electron plasma distribution in (almost) stationary conditions on different transverse slices.

As a second example, the case of an electron plasma filling the trap is studied (see Fig. 1 *right*). This situation simulates the phase of injection in a Malmberg-Penning trap. In this phase, the end plug electrode is maintained at a sufficiently negative potential in order to reflect the electrons entering the trap, while the plug electrode at the entrance is grounded to let the electrons flow into the trap from the cathode (starting from this configuration, the trapping phase is obtained by simply lowering the plug potential at the entrance of the beam to the same value as the end potential, so that the electrons are electrostatically confined along the axis of the device). A mono-energetic beam is considered. It is found that when the input current of the beam is increased, the space charge cloud which forms inside the trap represents a barrier not only for the new incoming particles, but also for the electrons which are already inside and are reflected by the external negative potential barrier. As a result, the electron plasma filling the trap

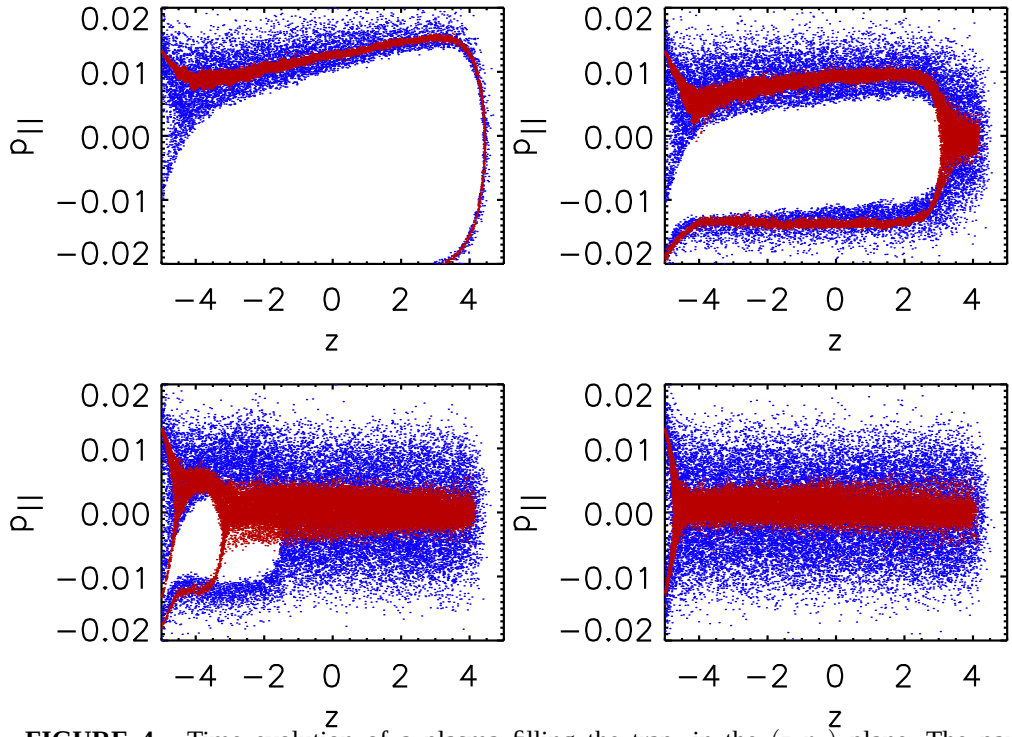

FIGURE 4. Time evolution of a plasma filling the trap, in the $(z, p_{\|})$ plane. The parameters are the same as in Fig. 3 (red dots). From left to right, top to bottom, the data refer to $t = 0.3$, 0.7, 1.5 and 3.0, respectively. The blue dots are relevant to an initial Maxwellian distribution in parallel velocity, $F(v_{\|}, \mathbf{x}, 0) = g(\mathbf{x}) \cdot \exp[-(v_{\|} - v_{\|0})^2/2\sigma^2)$, with the same central value $v_{\|0}$, and a velocity spread $\sigma = 5.78$.

assumes an annular shape, as shown in Fig. 3. The time evolution of the electrons in the phase plane $(z, p_{\|})$ is shown in Fig. 4 (red dots), where $p_{\|} \equiv M_{eff} v_{\|}$. Longitudinal kinetic effects have been investigated in this case. In particular, it has been found that varying the parallel velocity distribution of the injected electrons to a Maxwellian has a dramatic effect (see again Fig. 4, blue dots): space-charge effects are much weaker in this case, and the electron plasma maintains its initial transverse (spiral) shape.

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