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Particle Acceleration by a Short-Intense Elliptically Polarized Electromagnetic Pulse Propagating Along an Externally Applied Homogeneous Magnetic Field

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Abstract. The motion of a charged particle driven by an electromagnetic pulse of elliptical polarization, propagating along the direction of a static and homogeneous external magnetic field is obtained from the exact solution of the relativistic force equations, with the pulse shape of the wave modulated by a Gaussian-like envelope of arbitrary amplitude. The particle dynamics was studied both analytically and numerically and the time evolution of the particle position, momentum and energy was solved in terms of the phase of the wave. It is found that during the interaction the particle acquires energy from the wave and the gain can achieve a much higher level at the cyclotron resonance as compared with the nonresonance case. In addition, it is observed that a broadening of the resonant region around the cyclotron frequency $\omega = \alpha$ occurs between frequencies $\omega = \frac{\alpha N}{N \pm 2}$, where N is the number of optical cycles within the pulse. The existence of this resonant region enhance the energy gain over a wider frequency range $\Delta\omega_r = \frac{4\alpha N}{N^2 - 4}$, implying that the shorter the pulse length the more efficient the acceleration mechanism may result. The acceleration mechanism reported in this work is of concern in the interaction physics of laser-produced plasmas, to study, for instance, plasma heating and radiation phenomena.

I. INTRODUCTION

The interaction between charged particles and electromagnetic waves has been of fundamental importance in the study of a large number of physical phenomena, having applications, for instance, in areas related to plasma physics and particle accelerators. The interaction physics of fields with particles has also been studied with great interest for long time in the description of the particle motion in astrophysical problems concerning the Van Allen particles [1,2], ionospheric plasma processes [3] and plasma in the magnetosphere [4]. In laboratory plasmas, the area of electron-cyclotron resonance heating reveals the importance of studying the motion of charged plasma particles interacting with multifrequency electromagnetic fields [5]. The motion of a charged particle in the field of a transverse electromagnetic wave propagating along a constant magnetic field was investigated by Roberts and Buchsbaum [6], who discussed the

effects at cyclotron resonance of the magnetic field of the wave and the increase with energy of the relativistic mass. They found an exact “synchronous” solution to the equation of motion for the case of circular polarization and showed that when the index of refraction of the medium is not unity, the energy of the particle is periodic in time, and when that index is unity, the effect of the magnetic field compensates for the change in mass with energy and the energy increases indefinitely at resonance. This effect occurs since the particle gains energy parallel and perpendicular to the direction of propagation of the wave. The cyclotron frequency is lowered from the increase in perpendicular energy, while an increase in the parallel changes the velocity of the particle, resulting in a Doppler shift to a lower frequency as seen by the particle. Experimental evidence for these results were reported by Jory and Trivelpiece [7], who also studied numerically the motion of a charged particle in combined inhomogeneous large-amplitude, oscillating electromagnetic waves and steady magnetic fields.

The orbit solutions for the problem of a charged particle under the influence of a radiation pulse have been investigated in the past [8-10]. Krüger and Bovyn [8] integrated the force equation for a particle driven by a plane wave of arbitrary amplitude, in terms of the co-moving and inertial times. Shebalin [9] found an exact solution, as a function of the phase of the wave, for the position and velocity of a particle interacting with an electromagnetic wave. The problem of a particle embedded in an elliptically polarized wave was solved in exact form by Acharya and Saxena [10]. B.-L. Qian [11] studied the motion of a charged particle driven by a circularly polarized wave propagating along a uniform magnetic field and reported an exact solution for certain initial conditions such that the particles can gain energy up to the maximum in the resonance situation for the case of a superposition of plane waves with circular polarization [12].

In Ref. 13 Ondarza solved the relativistic motion of a charged particle driven by an infinite electromagnetic wave of arbitrary amplitude and polarization propagating along the direction of an external magnetic field. The method of solution allowed the Lorentz equations to be integrated analytically yielding expressions for the electron trajectories and drift velocities of the particles. In the presence of an external magnetic field resonance effects are exhibited in the solutions.

In the work reported in Ref. 14 Ondarza and Gómez solved analytically the problem of a charged particle driven by an elliptically polarized electromagnetic field with the inclusion of the pulse shape of the driver, which makes the solutions of interest in aspects of laser-plasma interaction physics, notably for heating mechanisms and particle acceleration. Section II is devoted to outlining the procedure for solving the system of equations of motion of free electrons under the action of both an electromagnetic shaped pulse and a constant magnetic field along the direction of the wave propagation. In Section IV we present a resume of the results reported in this paper.

II. SOLUTION OF THE EQUATION OF MOTION

The vector potential \mathbf{A} associated to the radiation field can be expressed in a normalized form as $\mathbf{a}_0 = e\mathbf{A}/m_0 c^2$, where e, c and m_0 stand for the electron charge, the speed of light in vacuum, and the electron rest mass, respectively. For incident light of wavelength λ_L microns and intensity I_{18} (in units of 10^{18} W/cm², the magnitude of the normalized vector potential can be denoted as $a_0 \sim 0.8544 (I_{18})^{1/2} \lambda_L$. The field amplitude can be expressed in terms of this parameter as $E_0[\text{MeV}/m] \sim 3 \times 10^5 a_0 \lambda_L$. The relativistic Lorentz equation describing

electron motion in an electromagnetic field propagating along the \hat{x} direction of an applied external homogeneous magnetic field \mathbf{B}_0 is

$$\frac{d\mathbf{p}(\mathbf{r};t)}{dt} = e \left(\mathbf{E}(\mathbf{r},t) + \frac{\mathbf{v}(\mathbf{r},t)}{c} \times (\mathbf{B}(\mathbf{r},t) + \mathbf{B}_0) \right), \quad (1)$$

where $\mathbf{p} = m_0 \mathbf{v} \gamma$ and γ is the relativistic factor given by $\gamma = (1 - \beta^2)^{-1/2}$, with $\beta = |\mathbf{v}|/c$. The position of the particle is described by $\mathbf{r} = (x(t), y(t), z(t))$ and \mathbf{E} and \mathbf{B} denote the electric and magnetic fields of the electromagnetic wave, respectively. The vector potential, for a monochromatic plane wave of arbitrary polarization, can be expressed as $\mathbf{A}(\mathbf{r},t) = a(\eta)\mathbf{A}(\eta)$ [15,16], where $\mathbf{A}(\eta) = a_0 \left(0, (1 - \delta^2)^{1/2} \sin \eta, \delta \cos \eta \right)$, $a(\eta)$ gives the shape of the pulse and a_0 the amplitude. Here, $\eta = \omega (t - x/c)$ is the invariant phase. The parameter δ characterizes the degree of polarization ($0 \leq |\delta| \leq 1$) and is given by

$$\delta = \begin{cases} 0, \pm 1 & \text{linear polarization} \\ \pm \frac{1}{\sqrt{2}} & \text{circular polarization} \end{cases}.$$

Assuming that the particle is initially at rest at the origin, *i.e.* $\eta = 0$, $\mathbf{r} = 0$ and $\mathbf{v} = 0$ at $t = 0$, it can be shown that $1 - \frac{\dot{x}}{c} = \frac{1}{\gamma}$, where the dot denotes the time derivative. Since the rate of change of the phase η is $\frac{d\eta}{dt} = \omega \left(1 - \frac{\dot{x}}{c} \right) = \frac{\omega}{\gamma} = \dot{\eta}$, we can integrate the differential equations for the y and z components of the particle velocity

$$\begin{aligned} y' &= -\frac{a_0 c}{\omega} (1 - \delta^2)^{1/2} a \sin \eta + \frac{\alpha}{\omega} z, \\ z' &= -\frac{a_0 c \delta}{\omega} a \cos \eta - \frac{\alpha}{\omega} y + \frac{\epsilon}{\omega}, \end{aligned} \quad (2)$$

where the prime stands for the derivative with respect to the phase η , and $\epsilon = a_0 c \delta a(0)$. Choosing a pulse shape in the form $a = a(\eta) = \sin^2 \left(\frac{\eta}{N} \right)$, with N denoting the number of optical cycles in the pulse and solving the equation for the longitudinal displacement from the equation for the total energy $E^2 = p^2 c^2 + m_0^2 c^4$ and the expression $m_0 \gamma v_x = -m_0 c (1 - \gamma)$, we have that

$$x' = \frac{k}{2} (y'^2 + z'^2). \quad (3)$$

The solution is finally given by

$$\begin{aligned}
y &= -\frac{a_0 c}{2} \sum_{i=1}^3 f_i \frac{\omega \beta_i (1 - \delta^2)^{1/2} + \alpha \delta}{\omega^2 \beta_i^2 - \alpha^2} \left(\cos \beta_i \eta - \cos \frac{\alpha}{\omega} \eta \right) \\
z &= \frac{a_0 c}{2} \sum_{i=1}^3 f_i \frac{\omega \beta_i \delta + \alpha (1 - \delta^2)^{1/2}}{\omega^2 \beta_i^2 - \alpha^2} \left(\sin \beta_i \eta - \frac{\omega}{\alpha} \beta_i \sin \frac{\alpha}{\omega} \eta \right) \\
x &= \frac{a_0^2 c \omega}{4} \left\{ \left[\sum_{i=1}^3 \left(\frac{\omega^2 \beta_i^2 + \alpha^2 + 4 \alpha \delta \omega \beta_i (1 - \delta^2)^{1/2}}{4 (\omega^2 \beta_i^2 - \alpha^2)^2} \right) \beta_i^2 c_i \right. \right. \\
&\quad + \frac{\alpha}{2 \omega} \sum_{i=1}^3 \left(\frac{\omega \alpha \beta_i^2 + \delta \beta_i (1 - \delta^2)^{1/2} (\omega^2 \beta_i^2 + \alpha^2)}{(\omega^2 \beta_i^2 - \alpha^2)^2} \right) c_i \\
&\quad + \frac{\alpha}{4 \omega} \sum_{i=1}^3 (-1)^i \left(\frac{\omega \alpha [\delta^2 (\beta_i^2 + \beta_i^{*2}) + 2 (1 - \delta^2) \beta_i \beta_i^*]}{d_i (\omega^2 \beta_i^2 - \alpha^2) (\omega^2 \beta_i^{*2} - \alpha^2)} \right. \\
&\quad \left. \left. + \frac{\delta (1 - \delta^2)^{1/2} (\beta_i + \beta_i^*) [\omega^2 \beta_i \beta_i^* + \alpha^2]}{d_i (\omega^2 \beta_i^2 - \alpha^2) (\omega^2 \beta_i^{*2} - \alpha^2)} \right) \right] \eta \\
&\quad - \frac{(1 - 2 \delta^2)}{8} \sum_{i=1}^3 \frac{c_i \beta_i}{\omega^2 \beta_i^2 - \alpha^2} \sin 2 \beta_i \eta \\
&\quad + \frac{(1 - 2 \delta^2)}{4} \sum_{i=1}^3 (-1)^{i+1} \frac{\beta_i \beta_i^*}{(\beta_i + \beta_i^*)} \left(\frac{\omega^2 \beta_i \beta_i^* - \alpha^2}{d_i (\omega^2 \beta_i^2 - \alpha^2) (\omega^2 \beta_i^{*2} - \alpha^2)} \right) \sin(\beta_i + \beta_i^*) \eta \\
&\quad + \frac{1}{4} \sum_{i=1}^3 (-1)^i \frac{\beta_i \beta_i^*}{(\beta_i - \beta_i^*)} \left(\frac{\alpha^2 + \omega^2 \beta_i \beta_i^* + 2 \omega \alpha \delta (1 - \delta^2)^{1/2} (\beta_i + \beta_i^*)}{d_i (\omega^2 \beta_i^2 - \alpha^2) (\omega^2 \beta_i^{*2} - \alpha^2)} \right) \sin(\beta_i - \beta_i^*) \eta \\
&\quad \left. - \frac{1}{2} \left(\sum_{i=1}^3 f_i \frac{\omega \beta_i (1 - \delta^2)^{1/2} + \alpha \delta}{\omega^2 \beta_i^2 - \alpha^2} \right) \sum_{i=1}^3 f_i \alpha \beta_i \frac{\delta \mp (1 - \delta^2)^{1/2}}{(\omega \beta_i \pm \alpha)^2} \sin(\beta_i \pm \frac{\alpha}{\omega}) \eta \right\}.
\end{aligned} \tag{4}$$

The constants $c_i = (1, \frac{1}{4}, \frac{1}{4})$, $d_i = (1, 2, 1)$ and $f_i = (-1, \frac{1}{2}, \frac{1}{2})$ where introduced to allow reducing the expressions. Furthermore, an additional simplification was done by setting $\beta_i = (1, a, b)$ and $\beta_i^* = (a, b, 1)$, with $a = 1 + \frac{2}{N}$ and $b = 1 - \frac{2}{N}$.

This exact solution was tested by performing a single-particle simulation, for different arbitrary polarizations and field amplitudes of the electromagnetic wave, modulated by a sine-squared envelope. An external homogeneous magnetic field was also introduced in the dynamics for different magnitudes. The pulse shape was introduced by setting $a(\eta) = 0$ for $\eta < 0$ and $\eta > N\pi$, and $a(\eta) = \sin^2(\eta/N)$ for $0 < \eta < N\pi$. Fig. (1) shows both the analytical and numerical solutions for a particular choice of parameters and displays the evolution of an electron trajectory in the three dimensional space xyz . Atomic units (a.u.) were used for coordinates and physical parameters. For this example, a Kr-F laser pulse ($\omega = 0.248 \mu\text{m} =$

0.18 a.u.) of $N = 25$ optical cycles ($\tau_p = 20$ fs) and field amplitude of $a_0 = 3.97$, is incident on an electron initially at rest at the origin. The polarization of the wave is elliptical with $\delta = \frac{1}{2\sqrt{2}}$ and the magnitude of the magnetic field was chosen as $B = 1$ T. In Fig. (1) the electron trajectory for both solutions, the numerical and the analytical, follow closely each other the solid line.

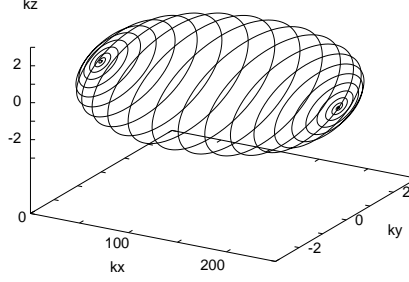


Figure 1: *Electron trajectory for a laser pulse of $\omega = 0.248 \mu m = 0.18$ a.u., $\tau_p = 20$ fs, $a_0 = 3.97$, $\delta = \frac{1}{2\sqrt{2}}$ and a magnetic field $B = 1$ T.*

III. ENERGY GAIN OF A CHARGED PARTICLE

In another application we have studied the motion of a charged particle accelerated by a microwave pulse of wavelength $\lambda = 1$ cm and duration $\tau_p = 1$ nsec, for different polarizations and intensity of 2×10^5 W/cm² ($a_0 \sim 3.81 \times 10^{-3}$). Fig. 2 shows the energy gain of a particle $\gamma - 1 = kx'$, accelerated during the pulse for different values of the magnetic field. It is shown that as the magnitude of the magnetic field approaches that of the corresponding cyclotron frequency $B_r = 1.072$ T, the electron is resonantly accelerated.

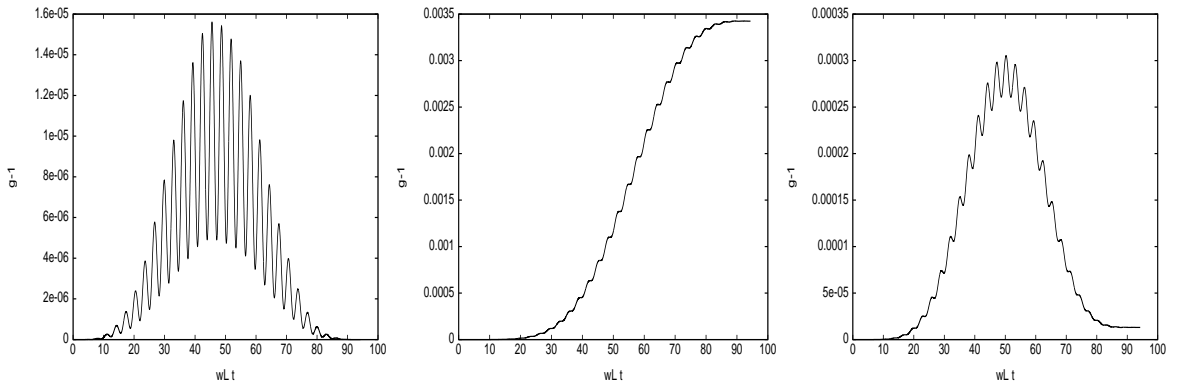


Figure 2: *Energy gain $\gamma - 1 = kx'$, for $\delta = 0$, $\tau_p = 1$ nsec ($N=30$), $a_0 = 3.21 \times 10^{-3}$ and $\lambda = 1$ cm, (a) $B=0.6$ T, (b) $B=1.05$ T and (c) $B=1.2$ T.*

For circular polarization, Fig. 3 shows energy gain during the pulse, at a higher level than for a linearly polarized wave.

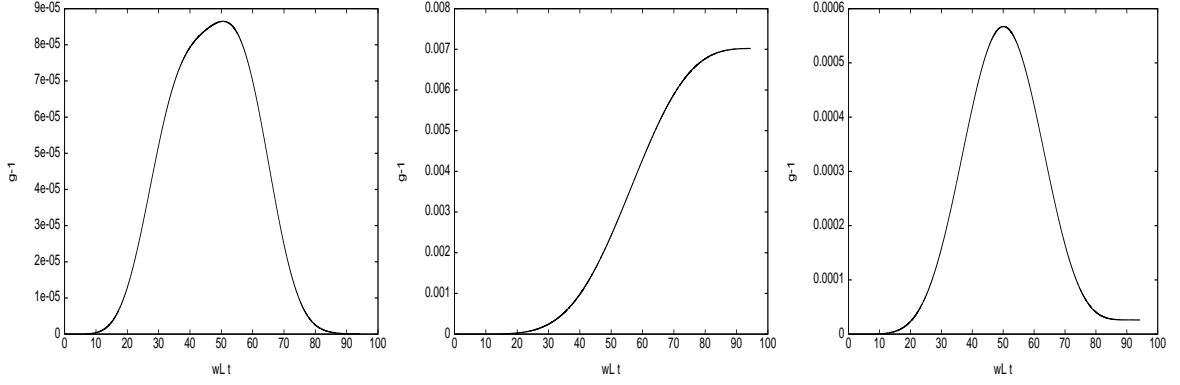


Figure 3: *Energy gain $\gamma - 1 = kx'$, for $\delta = 1/\sqrt{2}$, $\tau_p = 1$ nsec ($N=30$), $a_0 = 3.21 \times 10^{-3}$ and $\lambda = 1$ cm, (a) $B=0.85$ T, (b) $B=1.05$ T and (c) $B=1.2$ T.*

Fig. 4 shows, for linear and circular polarization, the maximum energy gain of a charged particle as a function of the magnetic field for the same choice of parameters. These plots show the resonant region around B_r extending over the range $\Delta\omega_r = \frac{4\alpha N}{N^2 - 4}$.

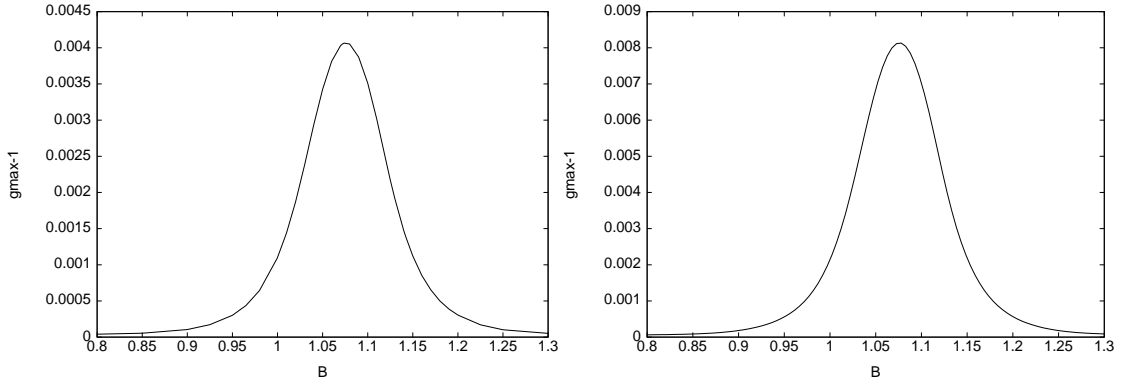


Figure 4: *Maximum energy gain vs. the magnetic field, for $\tau_p = 1$ nsec ($N=30$), $a_0 = 3.21 \times 10^{-3}$ and $\lambda = 1$ cm, (a) $\delta = 0$ and (b) $\delta = 1/\sqrt{2}$.*

The effect of the pulse length in the energy gain of the particle was also obtained for different polarizations, and magnitudes of the magnetic field. In Figs. 5 and 6 the energy gain is shown as a function of the number of wavelengths within the pulse. It is shown that as the magnetic field approaches the resonant magnitude the energy of the particle increases for longer pulses up to a maximum that corresponds to the resonance. If the magnetic field increases beyond that value the maximum energy corresponds again to pulses of shorter duration. This behaviour is shared for both linear and circular polarizations, although the energy gain is greater for the latter, as expected.

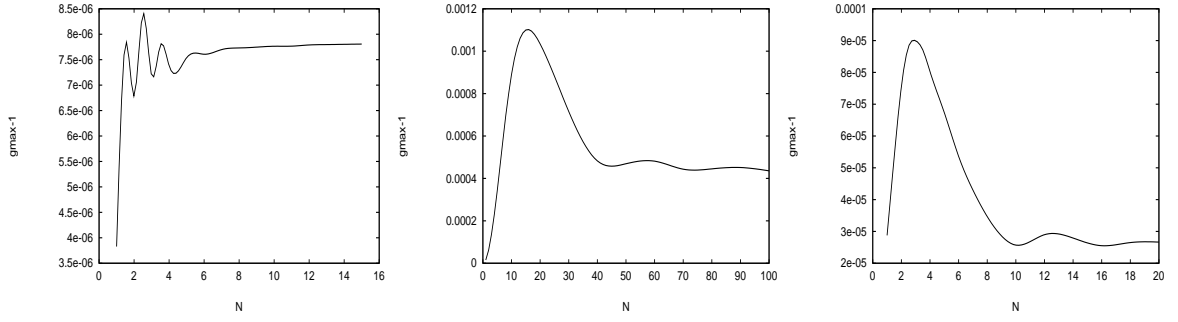


Figure 5: *Maximum energy gain vs. pulse length N , for $\delta = 0, \tau_p = 1$ nsec ($N=30$), $a_0 = 3.21 \times 10^{-3}$ and $\lambda = 1$ cm, (a) $B=0.2$ T, (b) $B=1.0$ T and (c) $B=1.4$ T.*

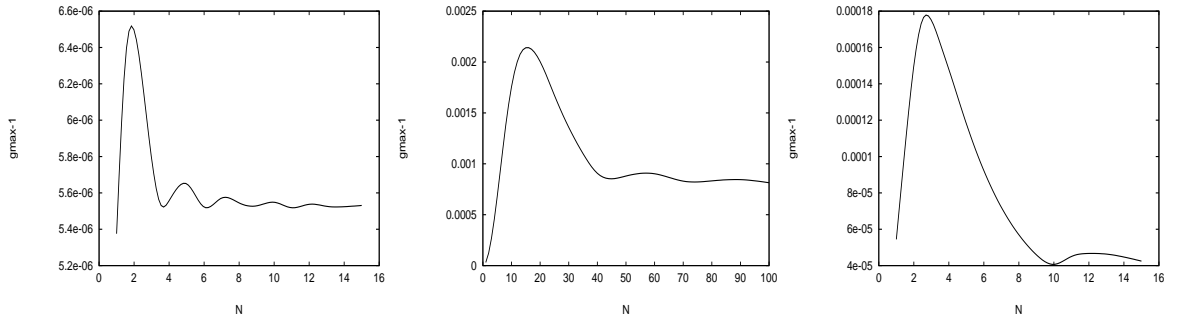


Figure 6: *Maximum energy gain vs. pulse length N , for $\delta = 1/\sqrt{2}, \tau_p = 1$ nsec ($N=30$), $a_0 = 3.21 \times 10^{-3}$ and $\lambda = 1$ cm, (a) $B=0.2$ T, (b) $B=1.0$ T and (c) $B=1.4$ T.*

From this results the optimum pulse length for maximum energy gain was obtained for both linear and circular polarizations. It was obtained that for a wavelength of 1 cm and intensity of $I \sim 3.8 \times 10^5$ W/cm² the maximum energy achieved by an electron at resonance corresponds to a pulse of 15 wavelengths, as shown in Fig. 7.

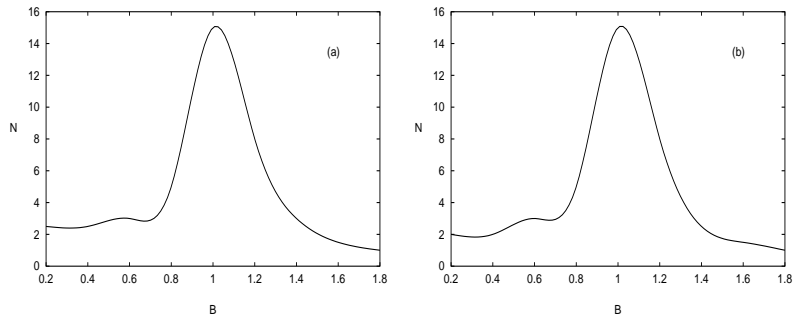


Figure 7: *Optimum pulse length for a maximum energy gain vs. the magnetic field, for $a_0 = 3.21 \times 10^{-3}$ and $\lambda = 1$ cm, (a) $\delta = 0$ and (b) $\delta = 1/\sqrt{2}$.*

IV. CONCLUSIONS

It was observed from the solution to the equations of motion that there exists a resonant region around the cyclotron frequency $\omega = \alpha$ that extends over the frequencies $\omega = \frac{\alpha N}{N \pm 2}$. In contrast with the case of an infinite pulse, where the resonance corresponds to the situation when the wave frequency is at or near the cyclotron frequency of the particle, the interaction of a finite shaped pulse may exhibit a resonant enhancement within the interval $\Delta\omega = \frac{2\alpha}{N \pm 2}$. The existence of this resonant region would produce a more efficient energy gain over a wider range of frequencies $\Delta\omega_r = \frac{4\alpha N}{N^2 - 4}$. A further implication is that, for an electromagnetic wave of arbitrary amplitude, the shorter the pulse length the wider $\Delta\omega_r$ and the more efficient the acceleration mechanism results. Because of the finite duration of the pulse, the particle is no longer accelerated indefinitely, as occurs in the case of an infinite wave, and thus the energy an electron can gain depends on the interaction time of the radiation pulse. It was found that the maximum energy gain of the particle strongly depends on the pulse length, for an optimal number of optical cycles. This in turn offers the possibility of using pulse shaping to generate highly energetic particles.

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