Propagation of finite amplitude electrostatic disturbances in a magnetized Vlasov plasma

Maurizio Lontano, Laura Galeotti, and Francesco Califano

1Istituto di Fisica del Plasma, C.N.R., EURATOM-ENEA-CNR Association, Milan, Italy
2Dipartimento di Fisica, Università di Pisa, Pisa, Italy

A 1D2V open boundary Vlasov-Ampere code has been implemented with the aim of making a detailed investigation of the propagation of finite amplitude electromagnetic disturbances in an inhomogeneous magnetized plasma. The code is being applied to study the propagation of an externally driven electromagnetic signal, localized at one boundary of the integration interval, through a given equilibrium plasma configuration with inhomogeneous plasma density and magnetic field.

I. INTRODUCTION

Self-consistent electromagnetic electromagnetic fields in spatially non-uniform plasmas represent one of the fundamental aspects of plasma physics with several implications both in microwave and laser-based experiments. Since the '70es extensive theoretical and experimental investigations have been devoted to ponderomotive effects, particle acceleration, wavebreaking, resonant absorption in plasmas with density gradients. The problem is relevant for magnetized plasmas as well, when Bernstein waves are excited as a consequence of mode conversion close to the hybrid plasma resonances. Recently, a renewed interest for electron Bernstein wave physics has appeared due to the possibility of implementing an attractive radiation emission diagnostic in fusion plasmas [8, 9, 10]. Then it is interesting to investigate the kinetic aspects of the propagation of electromagnetic as well as electrostatic fields in a non uniform plasma with arbitrary density and magnetic field scales, and electric field amplitudes.

A 1D2V open boundary Vlasov-Ampere code has been implemented with the aim of achieving a more realistic investigation of the propagation of a finite amplitude signal in an inhomogeneous plasma, and it has been applied to study the propagation of an externally driven, localized charge density fluctuation in an unmagnetized plasma with an equilibrium spatially nonuniform density. Previous analyses of magnetized plasmas, carried out in a slab geometry with periodic boundary conditions, have shown that strongly anisotropic distribution functions are produced both in the electron and in the ion populations during the interaction of plasma with an externally applied propagating electrostatic wave. In those cases the background plasma inhomogeneities could be modelled by varying the ratio of the pump frequency to the electron plasma frequency.

Here, the propagation of the spatially localized finite amplitude electromagnetic perturbation in a homogeneous magnetized collisionless plasma is preliminarily investigated, the final aim being the study of electromagnetic fields propagating in a given equilibrium plasma configuration where both plasma density and magnetic field are inhomogeneous. The solution of the Vlasov equation, coupled with the Ampere equation, provides the electron distribution function and the electrostatic field in the whole spatial range $0 < x < L$, at any time. Results are presented relevant to moderate amplitude electromagnetic field perturbations.

II. THE PHYSICAL MODEL

Let us consider a one dimensional magnetized plasma, localized in the region $0 < x < L$, homogeneous in the plane $(y,z)$, with unperturbed density $n_0 = n_{e0} = Zn_{i0}$, and magnetic field $B_0 = B_0 \hat{e}_z$. The relevant dimensionless non relativistic Vlasov equations take the form

$$\frac{\partial f_a}{\partial t} + v_x \frac{\partial f_a}{\partial x} - \mu_a \left[ E_{x}^{\text{tot}} + v_y B_z \right] \frac{\partial f_a}{\partial v_x} - \mu_a \left[ E_{y}^{\text{tot}} - v_x B_z \right] \frac{\partial f_a}{\partial v_y} = 0,$$

where $a = e,i$, $\mu_e = 1$, and $\mu_i = -Zm_e/m_i$. Normalized variables are defined as follows: $t \rightarrow t \omega_{pe}, x \rightarrow x/d_e, v \rightarrow v/c, E(B) \rightarrow cE(B)/m_e \omega_{pe}$. Moreover, $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}, d_e = c/\omega_{pe}$. As a result, in our units, the normalized length scale is the electron skin depth $d_e = 1$, while the (dimensionless) electrostatic length scale, the Debye length, is equal to the normalized thermal velocity, $\lambda_D = v_{th,c}/c$. In Eq. (1) the components of the electric field are the sum of two parts $E_{x,y}^{\text{tot}} = E_{x,y}(x,t) + E_{x,y}^{\text{dr}}(x,t)$, the consistent field, satisfying the Maxwell equations, and the externally applied driving field, respectively. Moreover, the magnetic field is also the sum of the constant background field and of the self-consistent field, $B_z^{\text{tot}} = B_0 + B_z(x,t) + B_z^{\text{dr}}(x,t)$. The self-consistent fields satisfy the Maxwell
FIG. 1: The $k$-spectra of the electric field for $\omega = 2.1$, frame (A), and $\omega = 2.1$, frame (B).

equations:

$$\frac{\partial E_x}{\partial t} = -j_x, \quad \frac{\partial E_y}{\partial t} = -\frac{\partial B_z}{\partial x} - j_y,$$

where the two relevant components of the current density are $j_x = Zn_iV_{ix} - n_eV_{ex}$ and $j_y = Zn_iV_{iy} - n_eV_{ey}$. Finally, Poisson equation (used as a check in the code) takes the form

$$\frac{\partial E_x}{\partial x} = \rho,$$

where the charge density is $\rho = Zn_i - n_e$. The driving electric field is modelled as

$$E_x^{dr}(x, t) = \epsilon_1A; \quad E_y^{dr}(x, t) = \epsilon_2A; \quad B_z^{dr}(x, t) = \epsilon_2A; \quad A = \delta(x - x_0)e^{-t^2/\tau^2} \sin \omega t$$

where $\epsilon_{1,2}$ are constant amplitudes, $x_0 \in [0, L]$ is the position where the driving field acts (usually, $x_0 = 0$), $\omega$ is the pump frequency. The disturbance is switched on and off continuously, with a typical time scale $\tau = 44.7$. This model allows one to perturb the system at one boundary of the range $[0, L]$ either by an electromagnetic ($\epsilon_1 = 0$, $\epsilon_2 \neq 0$) or by an electrostatic ($\epsilon_1 \neq 0$, $\epsilon_2 = 0$) disturbance. The consistent polarization and wave vector are then defined by the kinetic plasma response, which comprises the nonlinear coupling between particle motion and fields contained in the Vlasov Eq.(1).

III. THE RESULTS OF NUMERICAL EXPERIMENTS

Tests aimed at reproducing the wave-plasma interaction in the low-amplitude (linear) regime have been performed by injecting a "pure" electromagnetic wave (i.e. $E_y^{dr} = 0$) at the left boundary, $x = 0$, with $\epsilon_2 = 0.005$, or by exciting an electrostatic perturbation (i.e. $E_x^{dr} = B_z^{dr} = 0$) nearby the left boundary, $x \approx 0$, using a normalized amplitude $\epsilon_1 = 0.01$. In all cases $v_{th,e}/c = 0.14$, which corresponds in dimensional units to $|E_{x,y}|^2/(4\pi n_0 T_e) = 5 \times 10^{-3}$. In dimensionless units the magnetic field is equal to the electron cyclotron frequency and the value $B = \Omega_{ce} = 2$ has been chosen in all simulations. Two values of the pump frequency have been considered: $\omega = 0.95$ (a) and $\omega = 2.1$ (b). Note that the upper hybrid frequency $\omega_{uh} = \sqrt{1 + B^2} = 2.24$ and the upper cutoff $\omega_{co} = B/2 + \sqrt{1 + B^2}/4 = 2.41$.

According to the linear theory of cyclotron waves the cold branches of the dispersion relation of an extraordinary mode propagating perpendicularly to the magnetic field have a dimensionless wave vector $k \approx 3.2$ and $k \approx 0.94$ for two chosen frequencies, 2.1 and 0.95, respectively. As it is seen in Fig. 1 the $k$-spectra of the electric field manifest a sharp maximum around the corresponding wave vectors, together with other features at higher $k$'s, independently
of the method of excitation (electrostatic or electromagnetic). Moreover, since we are dealing with a hot plasma, we should expect also a component at higher $k$, due to the conversion of the incoming electromagnetic energy into electrostatic electron Bernstein waves. In this respect the most interesting spectrum is that of case (b), since the $\Omega_{ce} < \omega < \omega_{uh}$, where modes with $k \approx 10.8$ should be excited. However, in this preliminary work where we deal with "small" amplitude external pumps and relatively short time of propagation, this effect is not seen. In Fig. 2 we plot the longitudinal and transversal components of the electric field, first and second frame, respectively, in the case of an electromagnetic external driver of amplitude $\epsilon_2 = 0.005$ and frequency $\omega = 2.1$. We see that near the left boundary the energy is transferred to the electrostatic counterpart and then, after propagating to the right a few $d_e$ at $x \simeq 20$, the amplitudes become nearly constant with $E_x \simeq E_y$, despite we are in the limit $k^2c^2 \gg \omega_{pe}$ where the electromagnetic (applied) field should dominate. We think that this is due to the warm plasma response. In Fig. 3 we draw the isolines of the electron distribution function at $t = 100$ in the $(x,v_x)$ phase space at fixed $v_y$ velocity, namely $v_y = -0.038$, first frame, and $v_y = -0.19$, second frame (B). This figure shows the rotation of the particles around the magnetic field corresponding vortex like structures of typical size of the order of the selected $v_y$ value.

In the present analysis the value $v_{th,c}/c = 0.14$ has been chosen, which corresponds to an electron temperature of $T_e \approx 10keV$. It is well known that in order to describe correctly the collisionless cyclotron waves propagating perpendicularly to the magnetic field, the velocity dependence of the electron mass should be considered. The propagation of cyclotron waves is however well described already by the non relativistic theory. Moreover, for the frequency values which have been considered, that is $\omega = 0.95$ and 2.1 even in the relativistic case, no appreciable collisionless damping is expected.

IV. CONCLUSIONS

In this work the Vlasov equation for electrons, coupled with the Maxwell equations, has been numerically solved in an open plasma slab, where the source of the electromagnetic fields is localized at one of the two spatial boundaries. The magnetized plasma responses to both purely electromagnetic or electrostatic excitations, at small amplitudes, have been studied. The field disturbance is applied in the form of a purely transverse propagating EM wave or, alternatively, as a purely ES field. Then, the injected EM energy goes into the longitudinal and transverse components of the field, almost independently on the excitation details, and the wavevectors are generated consistently during the wave propagation. It is seen that, even in the case of a purely EM excitation, a large fraction of the injected energy can go into the ES field.

The code is particularly suitable for exploring the non linear stage of the wave-plasma interaction. The application of the code to large amplitude driving fields is under way.
FIG. 3: The electron distribution function at $t = 100$ in the $(x, v_x)$ phase space with $v_y = -0.038$, frame (A), and $v_y = -0.19$, frame (B).

Acknowledgments

One of us, FC, is glad to acknowledge the Plasma Physics Institute (IFP) of Milan for supporting in part the research activity on the kinetic study of electrostatic and electromagnetic waves propagation in a plasma.