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Kinetic fluctuation in the inhomogeneous plasma

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Abstract

Using the Langevin approach and the multiscale technique, a kinetic theory of the time and space nonlocal fluctuations in the collisional plasma is constructed. In local equilibrium a generalized version of the Callen-Welton theorem is derived. It is shown that not only the dissipation but also the time and space derivatives of the dispersion determine the amplitude and the width of the spectrum lines of the electrostatic field fluctuations, as well as the form factor. There appear significant differences with respect to the non-uniform plasma. In the kinetic regime the form factor is more sensible to space gradient than the spectral function of the electrostatic field fluctuations. As a result of the inhomogeneity, these proprieties became asymmetric with respect to the inversion of the frequency sign. The differences in amplitude of peaks could become a new tool to diagnose slow time and space variations in the plasma.

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Fluctuations find an application in diagnostic procedures. Indeed, plasma parameters such as temperature, mean velocity, density and their respective profiles can be determined by incoherent (Thomson) scattering diagnostics [1], i.e. by the proper interpretation of data obtained from the scattering of a given electromagnetic field interacting with the system. The key point of interpreting them is the knowledge of the intensity of the dielectric function fluctuations or equally of the electron form factor $(\delta n_e \delta n_e)_{\omega, \mathbf{k}}$. Here ω and \mathbf{k} are respectively the frequency and wavevector of the autocorrelations. Due to the Poisson equation the electron form factor in the spatially homogeneous system is directly linked to the electrostatic field fluctuations, which have been the object of active study since the early 1960s [1]. In the thermodynamic equilibrium, the electrostatic field fluctuations satisfy the famous **Callen-Welton** fluctuation-dissipation theorem [2]:

$$(\delta \mathbf{E} \delta \mathbf{E})_{\omega \mathbf{k}} = \Theta \frac{8\pi I m \varepsilon(\omega, \mathbf{k})}{\omega |\varepsilon(\omega, \mathbf{k})|^2} \quad (1)$$

linking their intensity to the imaginary part of the dielectric function $\varepsilon(\omega, \mathbf{k})$, and the temperature Θ in energy units. The spectral function (1) has peaks, corresponding to proper plasma frequencies. The matter becomes more tricky in the non-equilibrium case, when the state of the plasma is given by Maxwellian distributions characterized by *different constant* temperatures and velocities per species ($\Theta_a, \mathbf{V}_a; a = e, i$). We have indeed shown [3], that, in the *collisional regime* equations (1) should be revisited. We stressed the fact that a *kinetic approach* should be taken. Introducing fluctuations by the Langevin method, we have elaborated a "revisited" Callen-Welton formula containing, beside the terms appearing in Eq. (1), new terms explicitly displaying dissipative non equilibrium contributions.

$$(\delta \mathbf{E} \delta \mathbf{E})_{\omega \mathbf{k}} = \sum_{a,b=e,i} \frac{8\pi\Theta_a}{(\omega - \mathbf{k} \cdot \mathbf{V}_a) |\varepsilon(\omega, \mathbf{k})|^2} [Im\chi_a + \nu_{ab}(\Theta_a - \Theta_b)\Phi_1 + \nu_{ab}(\mathbf{k} \cdot \mathbf{V}_a - \mathbf{k} \cdot \mathbf{V}_b)\Phi_2], \quad (2)$$

where χ_a ($a = e, i$) is the complex dielectric susceptibility of the a-th component. It is important that these new terms contain the interparticle collision frequency ν_{ab} , the differences in temperatures ($\Theta_a - \Theta_b$) and velocities ($\mathbf{V}_a - \mathbf{V}_b$), and the functions Φ_1 and Φ_2 of **real** parts of the dielectric susceptibilities. It is however not evident that the plasma parameters - temperature, velocities and densities can be kept *constant*. Inhomogeneities in space and time of these quantities will certainly also contribute to the fluctuations. Obviously, to treat the problem, a kinetic approach is *required*, especially when the wavelength of the fluctuations is larger than the Debye wavelength. To derive nonlocal expressions for the spectral function of the electrostatic field fluctuation and for the electron form factor we use the Langevin approach to describe kinetic fluctuations [4, 5]. The starting point of our procedure is the same as in [3]. A kinetic equation for the fluctuation δf_a of the one-particle distribution function (DF) with respect to the reference state f_a is considered. In the general case the reference state is a none-equilibrium DF which varies in space and time both on the kinetic scale (mean free path l_{ei} and interparticle collision time ν_{ei}^{-1}) and on the larger hydrodynamic scales. These scales are much larger than the characteristic fluctuation time ω^{-1} . In the non-equilibrium case we can therefore introduce a small parameter $\mu = \nu_{ei}/\omega$, which allows us to describe fluctuations on the basis of a multiple space and time scale analysis. Obviously, the fluctuations vary on both the "fast" (\mathbf{r}, t) and the "slow" ($\mu\mathbf{r}, \mu t$) time and space scales: $\delta f_a(\mathbf{x}, t) = \delta f_a(\mathbf{x}, t, \mu t, \mu\mathbf{r})$ and $f_a(\mathbf{x}, t) = f_a(\mathbf{p}, \mu t, \mu\mathbf{r})$. Here \mathbf{x} stands for the phase-space coordinates (\mathbf{r}, \mathbf{p}) . The Langevin kinetic equation for δf_a has the form [4, 3]

$$\hat{L}_{axt}(\delta f_a(\mathbf{x}, t) - \delta f_a^S(\mathbf{x}, t)) = -e_a \delta \mathbf{E}(\mathbf{r}, t) \cdot \frac{\partial f_a(\mathbf{x}, t)}{\partial \mathbf{p}}, \quad (3)$$

where

$$\hat{L}_{axt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} + \hat{\Gamma}_a(t, \mathbf{r}); \quad \hat{\Gamma}_a(t, \mathbf{r}, \mathbf{p}) = e_a \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{p}} - \delta \hat{I}_a;$$

$\delta \hat{I}_a$ is the linearized Balescu-Lenard collision operator.

The Langevin source in Eq. (3) is determined [3] by following equation:

$$\widehat{L}_{axt} \overline{\delta f_a(\mathbf{x}, t) \delta f_b(\mathbf{x}', t')}^S = \delta_{ab} \delta(t - t') \delta(\mathbf{x} - \mathbf{x}') f_a(\mathbf{x}', t').$$

The solution of Eq. (3) has the form

$$\delta f_a(\mathbf{x}, t) = \delta f^S(\mathbf{x}, t) - \sum_b \int d\mathbf{x}' \int_{-\infty}^t dt' G_{ab}(\mathbf{x}, t, \mathbf{x}', t') e_b \delta \mathbf{E}(\mathbf{r}', t') \cdot \frac{\partial f_b(\mathbf{x}', t')}{\partial \mathbf{p}'}, \quad (4)$$

where the Green function $G_{ab}(\mathbf{x}, t, \mathbf{x}', t')$ of the operator \widehat{L}_{axt} is determined by

$$\widehat{L}_{axt} G_{ab}(\mathbf{x}, t, \mathbf{x}', t') = \delta_{ab} \delta(\mathbf{x} - \mathbf{x}') \delta(t - t')$$

with the causality condition $G_{ab}(\mathbf{x}, t, \mathbf{x}', t') = 0$, when $t < t'$. Thus, $\overline{\delta f_a(\mathbf{x}, t) \delta f_b(\mathbf{x}', t')}^S$ and $G_{ab}(\mathbf{x}, t, \mathbf{x}', t')$ are connected by the relation:

$$\overline{\delta f_a(\mathbf{x}, t) \delta f_b(\mathbf{x}', t')}^S = G_{ab}(\mathbf{x}, t, \mathbf{x}', t') f_b(\mathbf{x}', t').$$

For the stationary and spatially uniform systems, when DF f_a and the operator $\widehat{\Gamma}_a$ do not depend on time and space, $G_{ab}(\mathbf{x}, t, \mathbf{x}', t')$ can depend only on its time and space variables through the difference $t - t'$ and $\mathbf{r} - \mathbf{r}'$. In the general case, when the one-particle DF $f_a(\mathbf{p}, \mu \mathbf{r}, \mu t)$ and the operator $\widehat{\Gamma}_a$ slowly (in comparison with the correlation scales) vary in time and space, and when non-local effects are considered, the time and space dependence of $G_{ab}(\mathbf{x}, t, \mathbf{x}', t')$ is more subtle.

$$G_{ab}(\mathbf{x}, t, \mathbf{x}', t') = G_{ab}(\mathbf{p}, \mathbf{p}', \mathbf{r} - \mathbf{r}', t - t', \mu \mathbf{r}', \mu t'). \quad (5)$$

For the homogeneous case this non-trivial result was obtained for the first time in [6]. For inhomogeneous systems it has been generalized recently in [7].

The relationship (5) is directly linked with the constitutive relation between the electric displacement and the electric field:

$$D_i(\mathbf{r}, t) = \int d\mathbf{r}' \int_{-\infty}^t dt' \varepsilon_{ij}(\mathbf{r}, \mathbf{r}', t, t') E_j(\mathbf{r}', t').$$

Previously two kinds of constitutive relations were proposed phenomenologically for a weakly-inhomogeneous and slowly time-varying medium:

(i) the so-called *symmetrized* constitutive relation [8]:

$$D_i(\mathbf{r}, t) = \int d\mathbf{r}' \int_{-\infty}^t \int_{-\infty}^t dt \varepsilon_{ij}(\mathbf{r} - \mathbf{r}', t - t'; \mu \frac{\mathbf{r} + \mathbf{r}'}{2}, \mu \frac{t + t'}{2}) E_j(\mathbf{r}', t'). \quad (6)$$

(ii) the *non-symmetrized* constitutive relation [9]:

$$D_i(\mathbf{r}, t) = \int d\mathbf{r}' \int_{-\infty}^t \int_{-\infty}^t dt \varepsilon_{ij}(\mathbf{r} - \mathbf{r}', t - t'; \mu \mathbf{r}, \mu t) E_j(\mathbf{r}', t'). \quad (7)$$

Both phenomenological formulations (i) and (ii) are unsatisfactory. The correct

expression should be

$$D_i(\mathbf{r}, t) = \int d\mathbf{r}' \int_{-\infty}^t dt \varepsilon_{ij}(\mathbf{r} - \mathbf{r}', t - t'; \mu \mathbf{r}', \mu t') E_j(\mathbf{r}', t'). \quad (8)$$

Taking into account the first-order terms with respect to μ from (4) and (5) we have

$$\delta f_a(\mathbf{x}, t) = \delta f_a^S(\mathbf{x}, t) - \sum_b \int d\mathbf{p}' d\rho \int_0^\infty d\tau (1 - \mu\tau \frac{\partial}{\partial \mu t} - \mu\rho \cdot \frac{\partial}{\partial \mu \mathbf{r}}) e_b \delta \mathbf{E}(\mathbf{r} - \rho, t - \tau) G_{ab}(\rho, \tau, \mathbf{p}, \mathbf{p}', \mu t, \mu \mathbf{r}) \cdot \frac{\partial f_b(\mathbf{p}', \mu t, \mu \mathbf{r})}{\partial \mathbf{p}'}, \quad (9)$$

($\rho = \mathbf{r} - \mathbf{r}', \tau = t - t'$).

From the Poisson equation

$$\delta \mathbf{E}(\mathbf{r}, t) = -\frac{\partial}{\partial \mathbf{r}} \sum_b e_b \int \frac{1}{|\mathbf{r} - \mathbf{r}'|} \delta f_b(\mathbf{x}', t) d\mathbf{x}' \quad (10)$$

and performing the Fourier-Laplace transformation

$$\delta \mathbf{E}(\mathbf{k}, \omega) = \int_0^\infty dt \int d\mathbf{r} \delta \mathbf{E}(\mathbf{r}, t) \exp(-\Delta t + i\omega t - i\mathbf{k} \cdot \mathbf{r}).$$

from (9) we have

$$\begin{aligned} \delta \mathbf{E}(\mathbf{k}, \omega, \mu t, \mu \mathbf{r}) &= \delta \mathbf{E}^S(\mathbf{k}, \omega) + \sum_a 4\pi i e_a^2 \int d\mathbf{p} [(1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial \mu t}) \\ &\times \frac{\mathbf{k}}{\mathbf{k}^2} \hat{L}_{a\omega \mathbf{k}}^{-1} \delta \mathbf{E}(\mathbf{k}, \omega, \mu \mathbf{r}, \mu t) \cdot \frac{\partial f_a(\mathbf{p}, \mu \mathbf{r}, \mu t)}{\partial \mathbf{p}} \\ &- i \frac{\partial}{\partial \mu r_i} \delta \mathbf{E}(\mathbf{k}, \omega, \mu \mathbf{r}, \mu t) \frac{\partial}{\partial k_i} \frac{\mathbf{k}}{\mathbf{k}^2} \hat{L}_{a\omega \mathbf{k}}^{-1} \frac{\partial f_a(\mathbf{p}, \mu \mathbf{r}, \mu t)}{\partial \mathbf{p}}]. \end{aligned} \quad (11)$$

Here and in the following for simplicity we omit μ , keeping in mind that derivatives over coordinates and time are taken with respect to the slowly varying variables. The resolvent in (11) is determined by the following relation:

$$\int d\rho \int_0^\infty d\tau \exp(-\Delta\tau + i\omega\tau - i\mathbf{k} \cdot \rho) G_{ab}(\rho, \tau, \mathbf{p}, \mathbf{p}', \mu t, \mu \mathbf{r}) = \hat{L}_{a\omega \mathbf{k}}^{-1} \delta_{ab} \delta(\mathbf{p} - \mathbf{p}').$$

The approximation in which Eq. (11) was derived corresponds to the geometric optics approximation [10]. At first-order and after some manipulations, one obtains from Eq. (11) the transport equation in the geometric optics approximation, which is not considered in the present article, and the equation for the spectral function of the electrostatic field fluctuations:

$$Re\varepsilon(\omega, \mathbf{k}) [(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}} - \frac{1}{|\tilde{\varepsilon}(\omega, \mathbf{k})|^2} (\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}}^S] = 0, \quad (12)$$

where we introduced

$$\begin{aligned}\tilde{\varepsilon}(\omega, \mathbf{k}) &= 1 + \sum_a \tilde{\chi}_a(\omega, \mathbf{k}); \quad \varepsilon(\omega, \mathbf{k}) = 1 + \sum_a \chi_a(\omega, \mathbf{k}) \\ \tilde{\chi}_a(\omega, \mathbf{k}) &= (1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} - i \frac{\partial}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{k}}) \chi_a(\omega, \mathbf{k}, t, \mathbf{r}),\end{aligned}\quad (13)$$

and where

$$\chi_a(\omega, \mathbf{k}, t, \mathbf{r}) = -\frac{4\pi i e_a^2}{k^2} \int d\mathbf{p} \hat{L}_{a\omega\mathbf{k}}^{-1} \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}} f_a(\mathbf{p}, t, \mathbf{r})$$

is the susceptibility for a collisional plasma. In the same approximation the spectral function of the Langevin source $(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}}^S$ takes the form

$$(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}}^S = 32\pi^2 \sum_a e_a^2 Re \int d\mathbf{p} (1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} - i \frac{\partial}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}}) \frac{1}{k^2} \hat{L}_{a\omega\mathbf{k}}^{-1} f_a(\mathbf{p}, \mathbf{r}, t). \quad (14)$$

If $Re\varepsilon(\omega, \mathbf{k}) \neq 0$, it follows from Eqs. (12) and (14) that the spectral function of the nonequilibrium electrostatic field fluctuations is determined by the expression:

$$[(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}}] = \frac{32\pi^2 \sum_a e_a^2 Re \int d\mathbf{p} (1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} - i \frac{\partial}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}}) \frac{1}{k^2} \hat{L}_{a\omega\mathbf{k}}^{-1} f_a(\mathbf{p}, \mathbf{r}, t)}{|\tilde{\varepsilon}(\omega, \mathbf{k})|^2}. \quad (15)$$

The effective dielectric function $\tilde{\varepsilon}(\omega, \mathbf{k})$ in the denominator of Eq. (15) determines the spectral properties of the electrostatic field fluctuations and its imaginary part

$$Im\tilde{\varepsilon}(\omega, \mathbf{k}) = Im\varepsilon(\omega, \mathbf{k}) + \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} Re\varepsilon(\omega, \mathbf{k}, t, \mathbf{r}) - \frac{\partial}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}} Re\varepsilon(\omega, \mathbf{k}, t, \mathbf{r}), \quad (16)$$

determines the width of the spectral lines near the resonance. Note that when expanding the Green function in Eq. (9) in terms of the small parameter μ , there appear additional terms at first order. It is important to note that the *imaginary* part of the dielectric susceptibility is now replaced by the *real* part, which is greater than *imaginary* part by the factor μ^{-1} . Therefore, the second and third terms in Eq. (16) in the kinetic regime have an effect comparable to that of the first term. At second order in the expansion in μ the corrections appear only in the *imaginary* part of the susceptibility, and they can reasonably be neglected. It is therefore sufficient to retain the first order corrections to solve the problem.

For the local equilibrium case where the reference state f_a is Maxwellian, we have the identity: $\int d\mathbf{p} (1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} - i \frac{\partial}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}}) \frac{1}{k^2} \hat{L}_{a\omega\mathbf{k}}^{-1} f_a(\mathbf{p}, t, \mathbf{r}) = \frac{i}{\omega_a} \int d\mathbf{p} f_a(\mathbf{p}, t, \mathbf{r}) - \frac{i\Theta_a}{\omega_a 4\pi e_a^2} \tilde{\chi}_a(\omega, \mathbf{k})$ ($\omega_a = \omega - \mathbf{kV}_a$) and Eq.(15) takes the form

$$(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}} = \sum_a \frac{8\pi \Theta_a}{\omega_a |\tilde{\varepsilon}(\omega, \mathbf{k})|^2} Im\tilde{\chi}_a(\omega, \mathbf{k}). \quad (17)$$

In this case the small parameter μ is determined on the slower hydrodynamic scale. For the case of equal temperatures and $\mathbf{V}_a = 0$ one obtains a generalized expression for the Callen-Welton formula:

$$(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}} = \frac{8\pi}{\omega} \frac{\Theta \text{Im} \tilde{\varepsilon}(\omega, \mathbf{k})}{|\tilde{\varepsilon}(\omega, \mathbf{k})|^2}. \quad (18)$$

To calculate explicitly $(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}}$ we will restrict our analysis to the vicinity of the resonance, *i.e.* $\omega = \pm \omega_0$, where $\text{Re} \varepsilon(\omega_0, \mathbf{k}) = 0$. We can develop $\tilde{\varepsilon}(\omega, \mathbf{k}) = (\omega - \omega_0 \text{sgn} \omega) \frac{\partial \text{Re} \varepsilon}{\partial \omega} \big|_{\omega=\omega_0 \text{sgn} \omega} + i [\text{Im} \varepsilon + (\frac{\partial^2}{\partial \omega \partial t} - \frac{\partial}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}}) \text{Re} \varepsilon] \big|_{\omega=\omega_0 \text{sgn} \omega}$. Thus $(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}} = \frac{\tilde{\gamma}}{(\omega - \omega_0 \text{sgn} \omega)^2 + \tilde{\gamma}^2} \frac{8\pi T}{\omega \partial \text{Re} \varepsilon / \partial \omega} \big|_{\omega=\omega_0}$, where

$$\tilde{\gamma} = [\text{Im} \varepsilon + \frac{\partial^2}{\partial \omega \partial t} \text{Re} \varepsilon - \frac{\partial}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}} \text{Re} \varepsilon] / \frac{\partial \text{Re} \varepsilon}{\partial \omega} \big|_{\omega=\omega_0 \text{sgn} \omega} \quad (19)$$

is the effective damping decrement. For the case where the system parameters are homogeneous in space but vary in time, the correction is still symmetric with respect to the change of sign of ω , but the intensities and broadening are different, and the intensity integrated over the frequencies remains the same as in the stationary case. However, when the plasma parameters are space dependent this symmetry is lost. The spectral asymmetry is related to the appearance of space anisotropy in inhomogeneous systems. The *real* part of the susceptibility $\text{Re} \varepsilon$ is an even function of ω . This property implies that the contribution of the third term to the expression of the damping decrement (19) is an odd function of ω . Moreover this term gives rise to an anisotropy in k space.

Let us estimate this correction for the plasma mode ($\omega_0 = \omega_L$) $\text{Re} \varepsilon = 1 - \frac{\omega_p^2}{\omega^2} (1 + 3 \frac{k^2 \Theta}{m \omega^2})$, $\text{Im} \varepsilon = \frac{\omega_p^2}{\omega^2} \frac{\nu_{ei}}{\omega}$, $\omega_L^2 = \frac{4\pi n e^2}{m} = \frac{\Theta k_D^2}{m}$ and

$$\tilde{\gamma} = [\nu_{ei} + \frac{2}{n} \frac{\partial n}{\partial t} + 6 \frac{\omega_L}{n k_D^2} \mathbf{k} \cdot \frac{\partial n}{\partial \mathbf{r}} \text{sgn} \omega] / 2. \quad (20)$$

For the spatially homogeneous case there is no difference between the spectral properties of the longitudinal electric field and of the electron density. They are connected by the Poisson equation. This statement is no longer valid when considering an inhomogeneous plasma. Indeed the longitudinal electric field is linked to the particle density by the nonlocal Poisson relation (10). In the latter case, an analysis similar to that made above can also be performed for the particle density. From Eq. (4) there follows

$$\begin{aligned} \delta n_a(\mathbf{k}, \omega, \mathbf{r}, t) = & \delta n_a^S(\mathbf{k}, \omega, \mathbf{r}, t) + \sum_b \frac{4\pi i \mathbf{k} e_b e_a}{k^2} \int d\mathbf{p} [(1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial t}) \hat{L}_{a\omega \mathbf{k}}^{-1} \delta n_b(\mathbf{k}, \omega, \mathbf{r}, t) \cdot \frac{\partial f_a(\mathbf{p}, \mathbf{r}, t)}{\partial \mathbf{p}} \\ & - i \frac{\partial}{\partial r_i} \delta n_b(\mathbf{k}, \omega, \mathbf{r}, t) \frac{\partial}{\partial k_i} \hat{L}_{a\omega \mathbf{k}}^{-1} \frac{\partial f_a(\mathbf{p}, \mathbf{r}, t)}{\partial \mathbf{p}}]. \end{aligned} \quad (21)$$

At the first order approximation and after some manipulations, one obtains the following expression for the electron form factor for a two-component ($a = e, i$)

plasma:

$$\begin{aligned}
(\delta n_e \delta n_e)_{\omega, \mathbf{k}} &= \frac{2n_e k^2}{\omega_e k_D^2} \frac{|1 + \tilde{\chi}_i(\omega, \mathbf{k})|^2}{|\tilde{\varepsilon}(\omega, \mathbf{k})|^2} \text{Im} \tilde{\chi}_e(\omega, \mathbf{k}) \\
&+ \left| \frac{\tilde{\chi}_e(\omega, \mathbf{k})}{\tilde{\varepsilon}(\omega, \mathbf{k})} \right|^2 \frac{\Theta_i}{\Theta_e} \frac{2n_e k^2}{\omega_i k_D^2} \text{Im} \tilde{\chi}_i(\omega, \mathbf{k}), \tag{22}
\end{aligned}$$

where we used for local equilibrium the following expression for the "source"

$$\begin{aligned}
(\delta n_a \delta n_b)_{\omega, \mathbf{k}}^S &= \delta_{ab} \frac{\Theta_a}{\omega_a} \frac{k^2}{2\pi e_a^2} \text{Im} \tilde{\chi}_a(\omega, \mathbf{k}), \\
\text{and } \tilde{\varepsilon}(\omega, \mathbf{k}) &= 1 + \sum_a \tilde{\chi}_a(\omega, \mathbf{k}); \tilde{\chi}_a(\omega, \mathbf{k}) = (1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} - i \frac{1}{k^2} \frac{\partial}{\partial r_i} k_j \frac{\partial}{\partial k_i} k_j) \chi_a(\omega, \mathbf{k}, t, \mathbf{r}).
\end{aligned}$$

As above we can expand $\tilde{\varepsilon}(\omega, \mathbf{k})$ near the plasma resonance $\omega = \omega_L$. Thus, for the electron line,

$$(\delta n_e \delta n_e)_{\omega, \mathbf{k}} = \frac{\tilde{\gamma}}{(\omega - \text{sgn} \omega_L)^2 + (\tilde{\gamma})^2} \frac{2n_e k^2}{\omega k_D^2 \partial \text{Re} \varepsilon / \partial \omega} \Big|_{\omega = \omega_L},$$

where

$$\tilde{\gamma} = [\text{Im} \varepsilon + \frac{\partial^2 \text{Re} \varepsilon}{\partial t \partial \omega} - \frac{1}{k^2} \frac{\partial}{\partial r_i} k_j \frac{\partial}{\partial k_i} k_j \text{Re} \varepsilon] / \frac{\partial \text{Re} \varepsilon}{\partial \omega} \Big|_{\omega = \omega_L \text{sgn} \omega} \tag{23}$$

is the effective damping decrement for the electron form factor. At this stage of calculation, let us note that the damping decrements for the electrostatic field fluctuations [Eq. (19)] and for the electron density fluctuations [Eq. (23)] are not the same. The origin of this difference is that the Green function for electrostatic field fluctuation and density particle fluctuations are not the same. This property holds only in the inhomogeneous situation. An estimation for the plasma mode is then:

$$\tilde{\gamma} = [\nu_{ei} + \frac{2}{n} \frac{\partial n}{\partial t} + \frac{\omega_L}{nk^2} \mathbf{k} \cdot \frac{\partial n}{\partial \mathbf{r}} (1 + \frac{6k^2}{k_D^2}) \text{sgn} \omega] / 2. \tag{24}$$

From this equation we see that the inhomogeneous correction in Eq.(24) is greater than the one in Eq. (20) by the factor $1 + k_D^2/6k^2$. For the same inhomogeneity; i.e., the same gradient of the density, we plot the form factor $(\delta n_e \delta n_e)_{\omega, \mathbf{k}}$ together with the $(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}}$ as functions of frequency (Fig. 1). This figure shows that the asymmetry of the spectral lines is present both for $(\delta n_e \delta n_e)_{\omega, \mathbf{k}}$ and $(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}}$. However, this effect is more pronounced in $(\delta n_e \delta n_e)_{\omega, \mathbf{k}}$ than in $(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}}$.

Conclusion 1 *We have shown that the amplitude and the width of the spectral lines of the electrostatic field fluctuations and form factor are affected by new non-local dispersive terms. They are not related to Joule dissipation and appear because of an additional phase shift between the vectors of induction and electric field. This phase shift results from the finite time needed to set the polarization in the plasma with dispersion. Such a phase shift in the plasma with space dispersion appears due to the medium inhomogeneity. These results are important*

for the understanding and the classification of the various phenomena that may be observed in applications; in particular, the asymmetry of lines can be used as a diagnostic tool to measure local gradients in the plasma.

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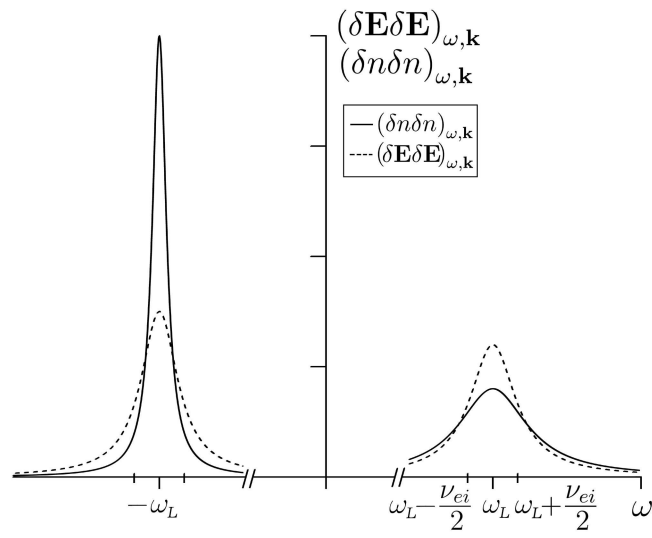


Figure 1: The electron form factor $(\delta n_e \delta n_e)_{\omega, \mathbf{k}}$ (solid line) and the spectral function of electrostatic field fluctuations $(\delta \mathbf{E} \delta \mathbf{E})_{\omega, \mathbf{k}}$ (dashed line) as a function of frequency. $\mathbf{k} \cdot \frac{\partial n}{\partial \mathbf{r}} = \frac{\nu_{ei} n k_D^2}{54 \omega_L}$; $\frac{k_D}{k} = 6$