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Inhomogeneous Energy-Density Driven Instability in Presence of a Transverse DC Electric Fields in a Magnetized Plasma Cylinder

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ABSTRACT

Temporal evolution of the inhomogeneous energy density driven instability (IEDDI) is examined in presence of a nonuniform transverse dc electric field in collisional magnetized plasma cylinder. Using experimentally known parameters relevant to IEDDI, we have estimated the growth rate for $\nu=1, 2$ and 3 nonlocal eigenmodes.

1. INTRODUCTION

IEDD waves propagate primarily in the $\vec{E} \times \vec{B}$ direction and have frequencies in a wide range near the ion cyclotron frequency. For $R(r)=k_{\perp} v_E(r)/k_z v_{de} \ll 1$, the case of large v_{de} and small v_E , the electrostatic ion cyclotron (EIC) waves are current-driven (EIC Instability)^{1,2}. For $R \gg 1$, the case of small v_{de} and large v_E , the EIC waves are driven by the transverse, localized, dc electric field (IEDD Instability)³⁻⁴.

2. Instability Analysis

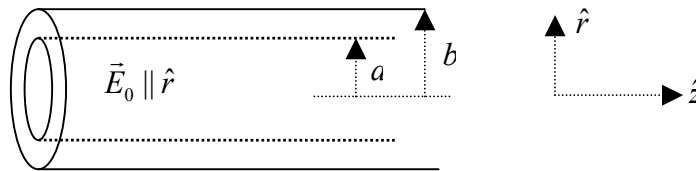


Fig. 1 A schematic diagram of the electric field model

Plasma waveguide radii b_1, a_1 , equilibrium density n_{op} , $T_e \approx T_i$, collisional frequency ($= \nu_e$ for electrons and $=0$ for ions), static magnetic field

B is in the z -direction, dc electric field $E_0(r)$ is in the radial direction, drift

$$\vec{v}(r) = \frac{c\vec{E}_0(r) \times \vec{B}}{B^2}, \text{ Electric fields } E_0(r) \text{ model}$$

$$E_0(r) = E_0 \quad \text{for } 0 < r < a_1 \quad \text{I region} \\ = 0 \quad \text{for } a_1 < r < b_1 \quad \text{II region}$$

In addition electrons have a flow or drift (v_{de}) along the magnetic field direction. A nonlocal wave packet can couple these two regions, and a flow of energy from region I to region II enables the IEDD mode to grow. This gives rise to IEDDI.

Basic Equations

$$m_\alpha \frac{d\vec{v}_\alpha}{dt} = q_\alpha \vec{E} - \frac{e}{c} \vec{v}_\alpha \times \vec{B} - \frac{\nabla(p_\alpha)}{n_\alpha} - m_\alpha v_e \vec{v}_\alpha. \quad (1)$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \vec{v}_\alpha) = 0 \quad (2)$$

$$\nabla \cdot \vec{E} = 4\pi\rho_\alpha \quad (3)$$

where $\alpha=e, i$.

By solving equations (1)-(3), we obtain ν th order Bessel equation for Φ_1

$$\frac{\partial^2 \Phi_1}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \Phi_1}{\partial \eta} + \left(1 - \frac{\nu^2}{\eta^2}\right) \Phi_1 = 0, \quad (4)$$

$$\eta = k_I r \quad \text{for } 0 < r < a_1, \\ = k_{II} r \quad \text{for } a_1 < r < b_1,$$

$$k_I^2 = \frac{\left[\frac{\omega_p^2}{v_{te}^2 k_z^2 \left(-1 + \frac{i v_e \omega_1}{k_z^2 v_{te}^2}\right)} - 1 \right] k_z^2}{\left[1 + \frac{\omega_p^2}{\omega_c^2} - \frac{\omega_{pi}^2}{\omega_2^2 - \omega_{ci}^2} \right]}. \quad (5)$$

$$k_{II}^2 = \frac{\left[\frac{\omega_p^2}{v_{te}^2 k_z^2 (-1 + \frac{i v_e (\omega - k_z v_{de})}{k_z^2 v_{te}^2})} - 1 \right] k_z^2}{\left[1 + \frac{\omega_p^2}{\omega_c^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \right]}. \quad (6)$$

Now dividing Eq. (5) by (6), and cross-multiplying , we get

$$\epsilon_r(\omega, k) + i \epsilon_i(\omega, k) = 0, \quad (7)$$

where

$$\begin{aligned} \epsilon_r(\omega, k) = & 1 + \frac{\omega_p^2}{k_z^2 v_{te}^2} - \frac{\omega_p^2}{k_z^2 v_{te}^2} \frac{\omega_{pi}^2}{[(\omega^2 - \omega_{ci}^2)]} \frac{k_I^2}{(k_I^2 - k_{II}^2)(1 + \frac{\omega_p^2}{\omega_c^2})} \\ & + \frac{\omega_p^2}{k^2 v_{te}^2} \frac{\omega_{pi}^2}{(\omega^2 - \omega_{ci}^2)} \frac{k_I^2}{(k_I^2 - k_{II}^2)(1 + \frac{\omega_p^2}{\omega_c^2})} - \\ & \frac{\omega_{pi}^2}{[(\omega^2 - \omega_{ci}^2)]} \frac{k_I^2}{(k_I^2 - k_{II}^2)(1 + \frac{\omega_p^2}{\omega_c^2})} - \frac{\omega_{pi}^2}{(\omega^2 - \omega_{ci}^2)} \frac{k_{II}^2}{(k^2 - k_{II}^2)(1 + \frac{\omega_p^2}{\omega_c^2})}, \quad (8) \\ \epsilon_i(\omega, k) = & v_e \left[\frac{\omega_1}{k_z^4 v_{te}^4} \frac{\omega_p^2 \omega_{pi}^2}{(\omega^2 - \omega_{ci}^2)} \frac{k_I^2}{(k_I^2 - k_{II}^2)(1 + \frac{\omega_p^2}{\omega_c^2})} - \frac{\omega_1}{k_z^4 v_{te}^4} \frac{\omega_p^2 \omega_{pi}^2}{(\omega^2 - \omega_{ci}^2)} \times \right. \\ & \left. \frac{k_{II}^2}{(k_I^2 - k_{II}^2)} + \frac{(\omega - k_z v_{de})}{k_z^4 v_{te}^4} \frac{k_I^2 \omega_p^2}{(k_I^2 - k_{II}^2)} - \frac{(\omega - k_z v_{de})}{k_z^4 v_{te}^4} \frac{k_I^2 \omega_p^2}{(k_I^2 - k_{II}^2)} \times \right. \\ & \left. \frac{\omega_{pi}^2}{[(\omega^2 - \omega_{ci}^2)](1 + \frac{\omega_p^2}{\omega_c^2})} \right]. \quad (9) \end{aligned}$$

We write $\omega = \omega_r + i\gamma$ ($|\gamma| \ll \omega_r$). The real and imaginary part of the frequencies are given by

$$\omega_r = k_\theta v_E + \left[\omega_{ci}^2 + \frac{k_I^2}{k_p^2 (k_I^2 - k_{II}^2)} \frac{k_z^2 c_s^2}{(1 + \frac{\omega_p^2}{\omega_c^2})} \right]^{1/2}. \quad (10)$$

$$\omega_r = k_\theta v_E + \left[\frac{\omega_{ci}^2 \frac{k_I^2}{(k_I^2 - k_{II}^2)} \frac{k_z^2 c_s^2}{(1 + \frac{\omega_p^2}{\omega_c^2}) k_p^2}}{\omega_{ci}^2 + \frac{k_I^2}{(k_I^2 - k_{II}^2)} \frac{k_z^2 c_s^2}{(1 + \frac{\omega_p^2}{\omega_c^2}) k_p^2}} \right]^{1/2}. \quad (11)$$

$$\gamma = \nu_e \left[\frac{\{1 - \frac{k_z v_{de}(1+R)}{\omega_r}\} \omega_p^2 \omega_{pi}^2 k_I^2 \omega_p^2}{k_z^4 \nu_{ie}^4 \omega_r (k_I^2 - k_{II}^2) \omega_c^2 (1 + \frac{\omega_p^2}{\omega_c^2})} - \frac{(\omega_r - k_z v_{de}) k_I^2 \omega_p^2}{k_z^4 \nu_{ie}^4 (k_I^2 - k_{II}^2)} \left(1 - \frac{\omega_{pi}^2}{[(\omega_r - k_\theta v_E)^2 - \omega_{ci}^2] (1 + \frac{\omega_p^2}{\omega_c^2})} \right) \right] / \left[2 \frac{\omega_{pi}^4}{k_z^2 c_s^2} \frac{1}{(k_I^2 - k_{II}^2) (1 + \frac{\omega_p^2}{\omega_c^2})} \left[\frac{(\omega_r - k_\theta v_E) k_I^2}{[(\omega_r - k_\theta v_E)^2 - \omega_{ci}^2]^2} + \frac{k_{II}^2}{\omega_r^3} \right] \right], \quad (12)$$

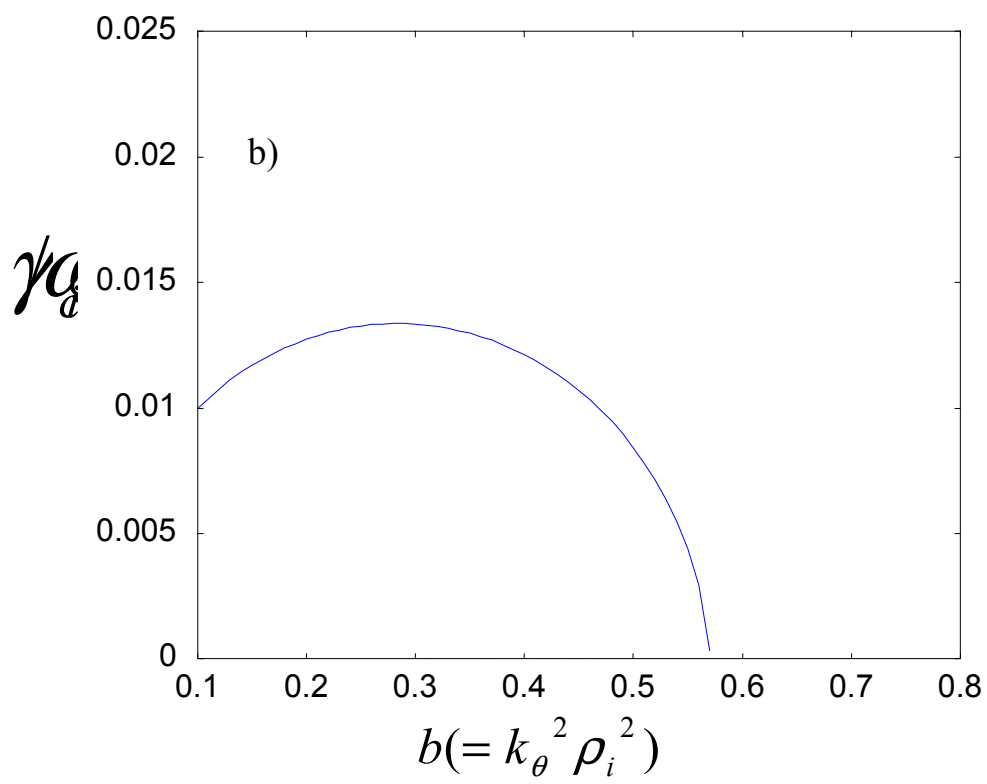
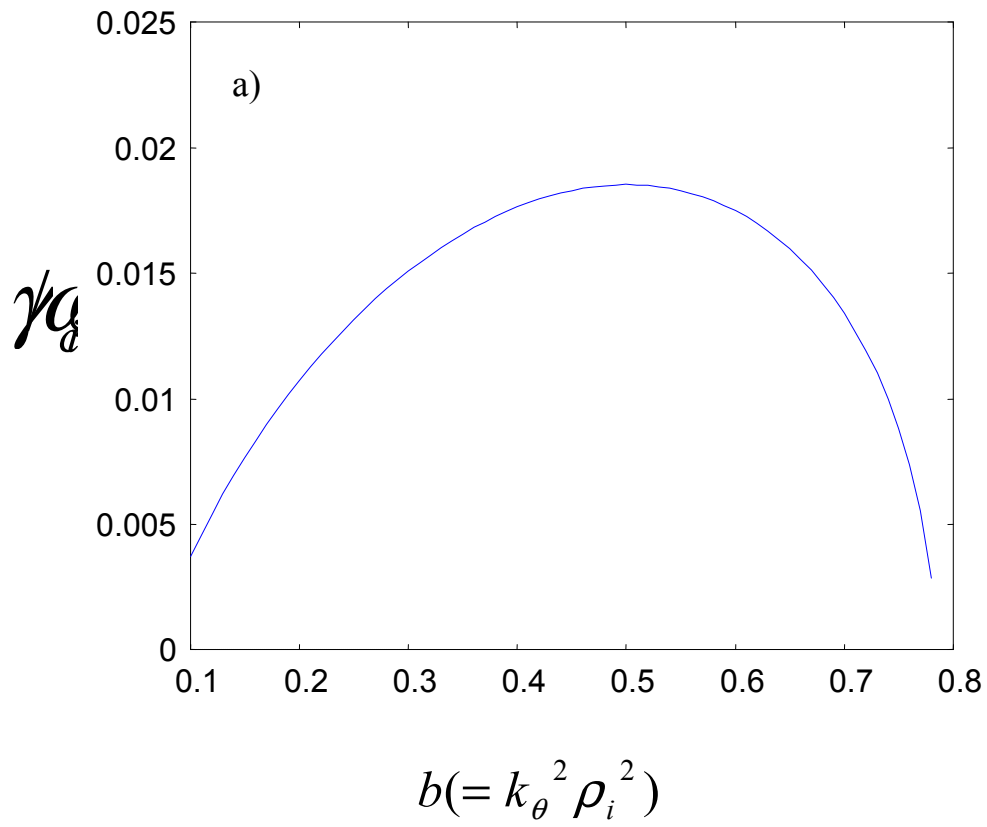
Equation (10) gives the real frequency of IEDD modes in presence of a transverse dc electric fields.

RESULTS

3. Parameters³ :

1. Finite gyroradius parameter $b(=k_\theta^2 \rho_i^2) = 0.1 - 0.8$, $n_{p0} \approx 1 \times 10^9 \text{ cm}^{-3}$, $T_e = T_i \approx 0.2 \text{ eV}$, m_i/m (Sodium) $= 4.6 \times 10^4$, $k_\square = 2.8 \text{ cm}^{-1}$, $k_z = 0.65 \text{ cm}^{-1}$, $\nu_e = 3.22 \times 10^8 \text{ sec}^{-1}$, $\nu_E = 3.3 \times 10^8 / B$ (in Gauss), $\nu_{de} = \sim 1.76 \times 10^4 \text{ cm/sec}$.

We have plotted in Fig. 2 the normalized growth rate (γ/ω_{ci}) [a) for $\nu=1$, b) for $\nu=2$ and c) for $\nu=3$ eigenmodes] of the IEDD instability as a function of the finite gyroradius parameter b for the same parameters as mentioned above. The growth rate of the instability increases with the finite gyroradius parameter b and reaches maximum in accordance with the experimental observations by Koepke *et al*³



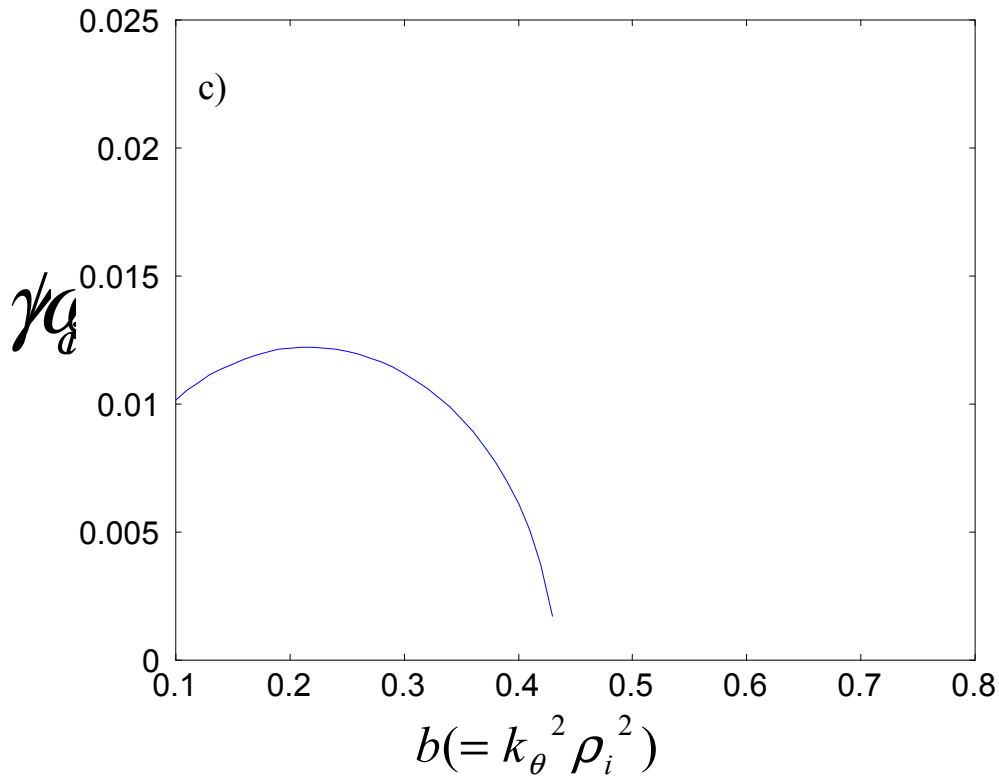


Fig. 2 : Normalized growth rate (γ/ω_{ci}) of the inhomogeneous energy-density driven instability as a function of the finite gyroradius parameter $b(=k_{\theta}^2 \rho_i^2)$ [a) for $v=1$ b) for $v=2$, and c) for $v=3$ eigenmodes]. The parameters are given in the text.

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