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# Aging of dephasing by a classical intermittent noise

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(Dated: 31 Mars 2004)

We propose a phenomenological model for intermittent classical  $1/f^\mu$  noise and study the dephasing of a two level system (*e.g* a qubit) in presence of this noise. Within this model, the evolution of the relative phase of the two level system is described as a continuous time random walk (CTRW). We find exact expressions for the dephasing factor in various regimes. The non-stationarity and pronounced non-Gaussian features of this noise induce new anomalous dephasing scenarii. In particular, for strong coupling between the two level system and the noise, the dephasing time depends explicitly on the age of the noise (aging).

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The question of the phase coherence of quantum two level systems (TLS) in a noisy environment has motivated numerous recent works. It is of crucial interest in the context of quantum computing[1], but might also be relevant to other areas like the dielectric response of glasses at low temperatures[2]. In the case of solid state quantum bits, it is now believed that dephasing of the TLS by a low frequency  $1/f$  noise is a limiting mechanism which certainly deserves further studies. It is the purpose of this letter to propose a model for intermittent classical  $1/f^\mu$  noise and study the corresponding dephasing scenarii of the TLS. Usual models for a  $1/f$  environment consist in a set of harmonic oscillators (spin-Boson model)[3] or a set of independent bistable fluctuators with judiciously chosen parameters[4]. In the last case, these fluctuators can be identified as localized defects interacting electrostatically or elastically with the TLS[5, 6]. The noise is then described by the sum of the contributions of these independent fluctuators. In the limit of a classical longitudinal noise (see below), the phase difference between the two states of the TLS diffuses.

In this letter, we describe the physics corresponding to an anomalous diffusion of this relative phase. It can be considered as an effective model for the situations where the mutual interaction between the fluctuators (of same nature as the one with the TLS) cannot be neglected, as opposed to previous proposals. Indeed, in the analogous physics of glasses at very low temperatures[6], interactions between TLS were found recently to play a crucial role in the dielectric response[7]. As in this last case, we expect these interactions and the associated frustration to bring very slow relaxational cooperative dynamics into the problem. A common picture to describe the evolution of a glassy system is a random walk in phase space, where time intervals  $\tau(E)$  between subsequent rearrangements are related to the energy barriers  $E$  to overcome :  $\tau(E) \simeq \tau_0 e^{E/kT}$  where  $\tau_0$  is a microscopic time. This very non-linear relation implies that the distribution of the trapping times  $P(\tau)$  is generally broad, typically with algebraic tail of the form  $P(\tau) \simeq \tau^{-1-\mu}$ . Here  $\mu \sim kT/E_0$

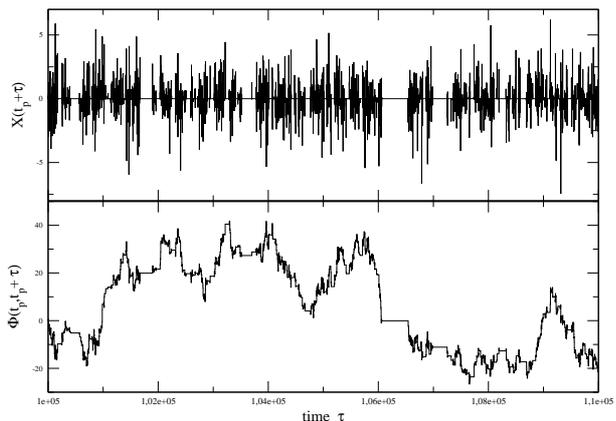


FIG. 1: A configuration of the noise  $X(t_p + \tau)$  in our model (as a function of  $\tau$ ) between  $t_p = 10^5 \tau_0$  and  $t_p + \tau = 1.1 \cdot 10^5 \tau_0$ . In this figure, the waiting times are distributed with an algebraic distribution  $P(\tau_i) \simeq \tau_i^{-2.1}$ . The bottom part of the figure shows the corresponding continuous time random walk of the accumulated phase of the TLS between  $t_p$  and  $t_p + \tau$ .

where  $E_0$  is the typical height of the energy barriers.

In the following, we address the question of the TLS dephasing by such intermittent collective fluctuations of an environment by considering a phenomenological model. The conditions for such a noise to be the dominant source of dephasing is of main importance but deserves a separate study [9].

The Hamiltonian of a dissipative quantum two-level system can be written in the form:

$$\mathcal{H} = \epsilon \sigma_z + \Delta \sigma_x - \frac{1}{2} X \sigma_z + \mathcal{H}_{\text{env}}(X), \quad (1)$$

where  $\epsilon$  and  $\Delta$  are control parameters which we assume to be time-independent, and  $X$  describes the fluctuating extra bias induced by the environment described by  $\mathcal{H}_{\text{env}}$ . For simplicity we will focus on the case of longitudinal coupling  $\Delta = 0$  and a classical noise  $X(t)$ [8]. We model the intermittent noise as a succession of spikes of weight  $v_i$  (see Fig.1). Thus the statistics of the noise is

characterized by the distributions of waiting times  $P(\tau_i)$ , and of weights  $\mathcal{P}(v)$  (related respectively to the energy barrier, and both the energy difference between the successive environment configurations and the coupling with the TLS). The distribution  $\mathcal{P}(v)$  is assumed to have zero mean and a finite width  $g^2 = \langle v^2 \rangle$ : it will be taken as gaussian without loss of generality (see [9] for other cases). Following the above discussion, we consider an algebraic distribution of waiting times characterized by an exponent  $\mu$ :

$$P(\tau) = \frac{\mu}{\tau_0} \left( \frac{\tau_0}{\tau_0 + \tau} \right)^{1+\mu}, \quad (2)$$

where  $\tau_0$  is a microscopic time. Note that the average  $\bar{\tau} = \tau_0/(\mu-1)$  of this distribution is finite only for  $\mu > 1$ . The two point noise correlation decays for  $0 < \mu < 1$  as

$$\overline{X(t+t')X(t)} - \overline{X(t)} \overline{X(t+t')} \simeq \overline{X(t)}^2 (t/t')^{1-\mu}, \quad (3)$$

where the overline corresponds to an average over noise realizations. Therefore its power spectrum exhibits a  $1/f^\mu$  divergence at low frequencies. However let us stress that this noise cannot be described solely by its power spectrum. In particular for  $\mu < 1$ , (3) implies a non stationarity of this noise.

Let us now focus on the dephasing of the TLS by this noise. The two TLS states  $|\pm\rangle$  get between time  $t$  and  $t+t'$  a random relative phase:

$$\Phi(t, t') = \int_t^{t+t'} X(t'') dt''. \quad (4)$$

The corresponding dephasing factor is defined by the average over many realizations of noise of  $D(t, t') = e^{i\Phi(t, t')}$  [10], and the dephasing time  $\tau_\phi$  as the characteristic time scale of the decay of  $\overline{D(t, t')}$ . In the case of gaussian phase fluctuations, this dephasing factor reads  $\overline{D(t, t')} = \exp(-\frac{1}{2}\overline{\Phi^2(t, t')})$ . Hence in this case,  $\tau_\phi$  corresponds to the *typical* time over which the phase of the TLS becomes statistically uncertain:  $\overline{\Phi^2(t, \tau_\phi)} \sim 1$ .

In previous studies [3, 5], stationarity of the noise and consequently of the phase was assumed:  $\overline{D(t, t')} \simeq \overline{D(0, t')}$ , and the noise was characterized by its power spectrum. In the present case, the presence of broadly distributed waiting times is expected to bring non stationarity into the physics. Note that in the context of glassy systems (with quenched disorder as opposed to the present case), the presence of such broad distribution of relaxation times is intimately related to the notion of weak-ergodicity breaking [11]. This formal analogy leads us to pay attention to the *aging* of the dephasing factor  $\overline{D(t_p, t')}$ , *i.e.* its dependence on the *preparation time*  $t_p$  of the system.

We first consider the case of weak coupling (or weak noise amplitude)  $g \ll 1$ . Within the reasonable gaussian approximation (*i.e.* a usual random walk approximation of the CTRW) we focus on the phase fluctuations

$\overline{\Phi^2(t_p, t)}$  to extract the dephasing behavior of the TLS. This phase is expressed as  $\Phi(t_p, t) = \sum_{i=1}^{N(t_p, t)} v_i$  where  $N(t_p, t)$  corresponds to the number of spikes between  $t_p$  and  $t_p + t$ . The dephasing time is simply defined by

$$\overline{\Phi^2(t_p, \tau_\phi(g, t_p))} = \overline{N(t_p, \tau_\phi(g, t_p))} g^2 = 2. \quad (5)$$

Let us first consider the common situation where the past history of the noise is neglected:  $t_p = 0$ . As the  $\tau_i$ s are independent from each other and distributed according to the *same* distribution (2), the generalized central limit theorem (GCLT) determines  $\overline{N(0, t)}$  in the limit of weak  $g$  (large  $\overline{N}$ ) (see *e.g.* [12]). According to the GCLT, three cases must be considered: (i) when  $\bar{\tau}$  and  $\bar{\tau}^2$  are finite ( $\mu > 2$ ), the limit distribution of  $\sum_{i=1}^N \tau_i$  is gaussian and  $\overline{N(0, t)} \simeq t/\bar{\tau}$ , corresponding to

$$\overline{\Phi^2(0, t)} = 2t/\tau_\phi^{(0)}; \quad \tau_\phi^{(0)} \equiv \tau_\phi(g, t_p = 0) = 2\bar{\tau}/g^2. \quad (6a)$$

This case corresponds to the usual model of independent fluctuators considered in [5]. The dephasing factor decays exponentially over a well defined time scale  $\tau_\phi \sim g^{-2}$ . (ii) when  $\bar{\tau}$  is finite but  $\bar{\tau}^2$  diverges ( $1 < \mu < 2$ ), the limit distribution of  $\sum_{i=1}^N \tau_i$  is a fully asymmetric Lévy distribution  $L_{\mu, 1}$  with algebraic tail  $\sim \tau^{-1-\mu}$ . In this case the  $\tau_i$ s strongly fluctuate around their average, which provides the expression for  $\overline{N}$  and consequently for

$$\overline{\Phi^2(0, t)} = \frac{2t}{\tau_\phi^{(0)}} \left( 1 + \frac{1}{1-\mu} \left( \frac{t}{\tau_0} \right)^{1-\mu} \right), \quad (6b)$$

where  $\tau_\phi^{(0)}$  is defined in (6a). The difference with normal dephasing (6a) lies only in the subdominant corrections. Finally in the case (iii) where none of  $\bar{\tau}$ ,  $\bar{\tau}^2$  is finite ( $0 < \mu < 1$ ), the limit distribution is also a Lévy distribution  $L_{1, \mu}$ , but the sum  $\sum_{i=1}^N \tau_i$  is now dominated by its maximum, leading to:

$$\frac{1}{2} \overline{\Phi^2(0, t)} = \left( \frac{t}{\tilde{\tau}_\phi^{(0)}} \right)^\mu; \quad \tilde{\tau}_\phi^{(0)} = \tau_0 \left( \frac{2\pi\mu}{\sin(\pi\mu)} \right)^{1/\mu} g^{-2/\mu}. \quad (6c)$$

In this case, we have both an anomalous decoherence scenario and an anomalous dependence of  $\tau_\phi$  in  $g$ .

Let us now discuss the past history dependence of the dephasing ( $t_p \neq 0$ ). For very broad distributions (*e.g.* for  $\mu < 1$ ),  $\overline{N(t_p, t)}$  is expected to decrease as  $t_p$  increases since larger and larger values of  $\tau_i$  occur as the sampling of  $P(\tau)$  goes on. *A priori*, we can discuss this history dependence of  $\overline{D(t_p, t)}$  within the above random walk picture. The average number of spikes  $\overline{N(t_p, t)}$  is expressed in terms of the density distribution of spikes  $S(t)$  at time  $t$  [12]:  $\overline{N(t_p, t)} = \int_{t_p}^{t_p+t} S(t') dt'$ . The behavior of  $S(t)$  follows from the simple integral relation

$$S(t) = P(t) + \int_0^t P(\tau) S(t-\tau) d\tau. \quad (7)$$

This equation states that a spike occurring at time  $t$  is either the first one, or that a previous spike occurred at time  $t - \tau$ , where  $\tau$  is distributed according to (2). Denoting by  $L[f]$  the Laplace transform of the distribution  $f$ , eq.(7) can be rewritten as  $L[S] = L[P]/(1 - L[P])$ . This exact Laplace transform can be inverted to get the behavior of the dephasing factor. For  $\mu > 2$ , as expected no  $t_p$  dependence is found and we recover (6a). In the case  $1 < \mu < 2$ , only the subdominant corrections to  $\overline{\Phi^2(t_p, t)}$  depend on  $t_p$ :

$$\overline{\Phi^2(t_p, t)} = \frac{2t}{\tau_\phi^{(0)}} + \frac{2}{2 - \mu} \frac{\tau_0}{\tau_\phi^{(0)}} \left[ \left( \frac{t_p + t}{\tau_0} \right)^{2-\mu} - \left( \frac{t_p}{\tau_0} \right)^{2-\mu} \right].$$

In the limit of  $t \gg t_p$ , the memory of the noise history is lost and we recover exactly (6b). The  $t_p$  dependence of the corrections matters only at short times and for  $\mu$  close to 1. This modifies significantly the dephasing scenario only for strong coupling  $g \simeq 1$  (see inset of Fig. 2). Finally, when  $0 < \mu < 1$ , (6c) is replaced by

$$\frac{1}{2} \overline{\Phi^2(t_p, t)} = \left( \frac{t_p + t}{\tilde{\tau}_\phi^{(0)}} \right)^\mu - \left( \frac{t_p}{\tilde{\tau}_\phi^{(0)}} \right)^\mu. \quad (8)$$

Similarly to  $1 < \mu < 2$ , on time intervals  $t \ll t_p$ , the noise *appears* stationary. It is described by an effective switching rate  $S(t_p)$  and therefore  $\overline{N(t_p, t)} \simeq t S(t_p)$ . In this transient aging regime, the dephasing factor is an explicit algebraic function of  $t_p$ . At longer times, where dephasing of the TLS occurs, the  $t_p$  independent behavior (6c) is recovered. Hence in spite of the non stationarity of the noise the dephasing of the TLS seems to always occur outside of the aging regime of  $D(t, t_p)$ . However, the previous analysis fails in two limits: for strong coupling or on times longer than  $\tau_\phi$ . In these regimes, the large fluctuations of  $N(t_p, t)$  cannot be neglected.

Going beyond the above random walk approximation can be achieved by using CTRW techniques (see [13]) and taking special care of initial conditions at  $t_p$ . First, we need to understand the origin of aging in the present case: the  $\tau_i$  being independent from each other (and thus of the past history), the  $t_p$  dependence of *e.g.*  $\overline{D(t_p, t)}$  comes only from the first  $\tau_1$  between  $t_p$  and the first subsequent spike. Indeed the previous spike didn't occur at  $t_p$  but at some previous time  $t_p - \tilde{\tau}_1$ . Hence  $\tau_1$  is distributed according to a new distribution  $H_{t_p}(\tau_1)$  which replaces (2) and depends explicitly on  $t_p$ . As we show, all the aging properties of  $\overline{D(t_p, t)}$  can be derived from those of  $H_{t_p}(\tau_1)$ . Now, all we need is an expression for  $H_{t_p}(\tau_1)$ . Separating noise histories that have their first spike at  $t_p + \tau_1$  from the others straightforwardly leads to:

$$H_{t_p}(\tau_1) = P(t_p + \tau_1) + \int_0^{t_p} d\tilde{\tau}_1 P(\tau_1 + \tilde{\tau}_1) S(t_p - \tilde{\tau}_1), \quad (9)$$

where  $S(t)$  was defined in (7). Note that for  $t_p$  large compared to  $\tau_0$ , the first term can be neglected. For

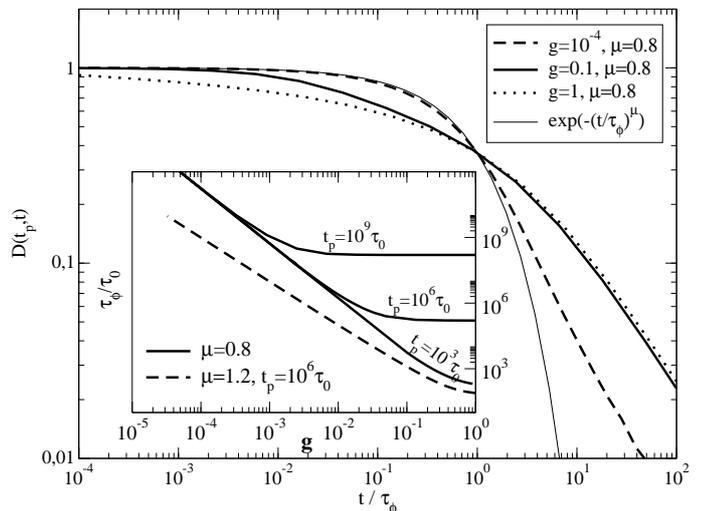


FIG. 2: Dephasing factor  $\overline{D(t_p = 10^6, t)}$  obtained by numerical inversion of (11). For  $\mu = 0.8, g = 10^{-4}$ , the dephasing factor behaves as  $\exp(-(t/\tau_\phi)^\mu)$  up to  $\tau_\phi$  (and apart from a small time aging regime). Beyond  $\tau_\phi$ , it decays algebraically. For strong coupling, it behaves as  $\Pi_0(t_p, t)$  beyond  $\tau_\phi$  ( $g = 10^{-1}$ ) or for any time ( $g = 1$ ). Inset shows the dephasing time  $\tau_\phi(g)$  for  $\mu = 0.8, 1.2$ . Note that for  $\mu = 0.8, g = 10^{-4}$ ,  $\tau_\phi$  scales as  $g^{-2/\mu}$  for weak couplings  $g < g_c(\mu, t_p)$ , and is proportional to the age of the system for  $g > g_c(t_p)$ :  $\tau_\phi \simeq c(\mu)t_p$ .

$\mu > 2$ ,  $S$  is constant and  $H_{t_p}(\tau_1)$  coincides with  $P(\tau_1)$ . This expresses the memoryless property of the noise. For  $\mu < 2$ , the time dependence of  $S(t)$  implies an explicit history dependence of  $H_{t_p}(\tau_1)$ . For instance, for  $1 < \mu < 2$ ,  $S(t) \simeq \bar{\tau}^{-1}(1 + (\tau_0/t)^{\mu-1})$ , and the average value of  $\langle \tau_1 \rangle$  increases as  $t_p^{2-\mu}$ . In the following, we will need an expression of the Laplace transform as a function of  $\tau_1$  of  $H_{t_p}(\tau_1)$  which can be derived from (9). For  $0 < \mu < 1$ , it reads  $L[H_{t_p}](s) = e^{st_p} \Gamma(\mu, st_p) / \Gamma(\mu)$  (see [9]).

Now understanding the origin of aging, we decompose noise configurations into those without any event between  $t_p$  and  $t_p + t$ , and those with at least one event in the same interval, the first of which occurring at time  $t_p + \tau$ . In the second case, at  $t_p + \tau$  the noise starts anew:

$$\overline{D(t_p, t)} = \Pi_0(t_p, t) + f(g) \int_0^t H_{t_p}(\tau) \overline{D(0, t - \tau)} d\tau. \quad (10)$$

In this equation,  $f(g) = \langle e^{iv} \rangle = \exp(-g^2/2)$  is the characteristic function of  $\mathcal{P}(v)$  and  $\Pi_0(t_p, t) = \int_t^{+\infty} H_{t_p}(\tau) d\tau$  denotes the probability that no spike occurs between  $t_p$  and  $t_p + t$ . More conveniently, using the Laplace transform as a function of  $t$ :  $L[\overline{D(t_p, \cdot)}](s) \equiv L[\overline{D(t_p, \cdot)}](s)$  and specializing (10) to  $t_p = 0$ , we find  $L[\overline{D_0}](s) = s^{-1}(1 - L[P](s))/(1 - f(g)L[P](s))$ . Plugging it back into the Laplace transform of (10) we obtain

$$L[\overline{D(t_p)}](s) = \frac{1}{s} \left( 1 - \frac{(1 - f(g))L[H_{t_p}]}{1 - f(g)L[P]} \right). \quad (11)$$

This expression is one of the central result of this approach. Indeed, we can apply a numerical Laplace Transform inversion to obtain the complete behavior of the dephasing factor  $\overline{D}(t_p, t)$ . Some of these results are shown in Figure 2. The behavior of  $\overline{D}(t_p, t)$  is shown for both weak ( $g = 10^{-4}$ ) and strong coupling ( $g = 10^{-1}, 1$ ). We now turn to a discussion of the results we can derive from eq. (11). The derivations being technically involved, we refer the reader to [9] for further details.

For  $\mu > 2$ ,  $H_{t_p}(\tau) = P(\tau)$  and from (11)  $\overline{D}_{t_p} = \overline{D}_0$ . As expected, there is no aging in this case. In the extreme case of  $0 < \mu < 1$ , two regimes appear as a function of the coupling constant  $g$ , separated by  $g_c(\mu, t_p) \simeq \lambda(\mu) t_p^{-\mu/2}$  (see Fig.2). For weak coupling  $g < g_c(t_p)$ , the behavior of  $\tau_\phi$  is given by eq. (6c) : it is insensitive to  $t_p$ . On the other hand,  $\overline{D}(t_p, t)$  is not fully described by the previous random walk approximation which applies only for  $t < \tau_\phi$  : from (8), away from the initial aging regime,  $\overline{D}(t_p, t)$  decreases as  $\exp(-(t/\tau_\phi)^\mu)$ . For  $t > \tau_\phi$  this exponential decay is replaced by a much slower algebraic one. Indeed, this new behavior can be understood by noting that in this case and for  $t > \tau_\phi$   $\overline{D}(t_p, t) \simeq \overline{D}(t_p, \tau_\phi) \Pi_0(\tau_\phi, t - \tau_\phi) \simeq (t/\tau_\phi - 1)^{-\mu}$  [9]. The second term of (10) corresponds to the anomalous random walk spreading of the phase  $\Phi$  : it leads to the exponential decay (8) which is subleading beyond  $\tau_\phi$ . Hence the leading term corresponds to the remaining trace at  $t$  of the phase configuration at  $\tau_\phi$  : it is the contribution of the noise configurations that didn't change between  $\tau_\phi$  and  $t$ . This situation of a main contribution induced by rare configurations is somehow similar to the physics of Griffiths singularities in disordered systems.

In the strong coupling regime  $g > g_c(\mu)$ ,  $\overline{D}(t_p, t)$  is dominated by the first term of eq. (10). In this limit, the distribution of the phase of the TLS starts spreading over  $[0, 2\pi]$ . Hence most noise configurations produce a vanishing contribution to  $\overline{D}(t_p, t)$  and the whole average is dominated by those noise configurations that don't evolve during the experiment. Consequently, the dephasing time becomes proportional to  $t_p$  as seen in Fig.2. This aging physics is indeed related to the physics of mean-field trap models of glassy materials[11] : whereas we are considering an *annealed* disorder in the present case, this difference with the quenched disorder of [11] is irrelevant as for the physics at strong coupling. Indeed the probability  $\Pi_0(t_p, t)$  is exactly the same in both situations. Hence the present aging properties of the dephasing in the strong coupling regime are reminiscent of those of the weak ergodicity breaking scenario in glasses[11]. This strong coupling regime has the very peculiar features that its range increases with the age  $t_p$  of the system :  $g_c \sim t_p^{-2/\mu}$ . In other words, any TLS surrounded by a noise with  $0 < \mu < 1$  will eventually end up in this aging regime. Further consequences of this characteristic

property of this non-stationary noise will be explored in [9]. Finally, let us mention that for  $1 < \mu < 2$ , this strong coupling regime is considerably reduced.

In summary, we have studied decoherence of a TLS due to an intermittent low frequency noise, describing phenomenologically a slow collective environment. We have derived exact expressions for the dephasing factor in various regimes, and discussed the corresponding signatures of such a noise as a function of the coupling with the TLS. The most striking prediction are an anomalous dephasing scenario in the weak coupling regime, an aging dephasing for strong coupling, and an aging critical coupling between these two regimes. More precise consequences for experimental situations, like echo[14] and one shot measurement experiments[15], will be presented in a forthcoming publication. Also of particular interest would be a quantum version of the present intermittent noise, but this certainly goes beyond the scope of the present paper. We hope that such studies will shed some more light on the nature of  $1/f$  noise in solid state qubits and the associated dephasing mechanism and raise new questions about the experimental pertinence of non stationarity in TLS dephasing.

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- [1] Yu. Makhlin, G. Schön, A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001)
  - [2] *Tunneling Systems in Amorphous and Crystalline Solids*, P. Esquinazi (Eds), Springer (Berlin) 1998.
  - [3] A. Shnirman, Yu. Makhlin, G. Schön, Physica Scripta **T102**, 147 (2002)
  - [4] M.B. Weissman, Rev. Mod. Phys. **60**, 537 (1988).
  - [5] E. Paladino *et al.*, Phys. Rev. Lett **88**, 228304 (2002).
  - [6] Y. M. Galperin, B. L. Altshuler, and D. V. Shantsev, cond-mat/0312490
  - [7] S. Ludwig *et al.*, Phys. Rev. Lett. **90** 105501 (2003)
  - [8] Generalizations of our results to  $\Delta, \epsilon \neq 0$  may be treated along the lines of J. Schrieffer and Yu. Makhlin, *in preparation*
  - [9] J. Schrieffer, M. Clusel, D. Carpentier, P. Degiovanni, *in preparation*.
  - [10] Note that this average does not necessarily corresponds to a time average. The statistics of  $D(t, t')$  will be characterized more precisely in [9].
  - [11] J.-P. Bouchaud, J. Physique I **2**, 1705 (1992).
  - [12] Lévy Statistics and Laser Cooling, F. Bardou *et al.*, Cambridge University Press (2002).
  - [13] J.W. Haus and K.W. Kehr, Phys. Rep. **150**, 263 (1987).
  - [14] Y. Nakamura, Yu.A. Pashkin, T. Yamamoto, J.S. Tsai, Phys. Rev. Lett. **88**, 047901 (2002).
  - [15] O. Buisson, F. Balestro, J.P. Pekola, F.W.J. Hekking, Phys. Rev. Lett. **90**, 238304 (2003).