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Positivity constraints for lepton polarization in neutrino deep inelastic scattering

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Abstract

We consider the spin polarization of leptons produced in neutrino and antineutrino nucleon deep inelastic scattering, via charged currents, and we study the positivity constraints on the spin components in a model independent way. These results are very important, in particular in the case of $\tau^{\pm}$ leptons, because the polarization information is crucial in all future neutrino oscillation experiments.

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1 Introduction

Recent studies from neutrino oscillation experiments [1, 2, 3] provide evidence for non-zero neutrino masses. Results from the Super-Kamiokande underground experiment [3] measuring the atmospheric neutrino flux, suggest that muon neutrinos oscillate into tau neutrinos with nearly maximal mixing. This $\nu_\mu \to \nu_\tau$ oscillation hypothesis can be tested by means of τ production via ν_τ scattering through charged current interactions, namely

$$\nu_\tau(\bar{\nu}_\tau) + N \to \tau^-(\tau^+) + X \quad , \tag{1}$$

where N is a nucleon target. This process will be studied with underground neutrino telescopes, such as AMANDA, ANTARES, NESTOR and BAIKAL [4], as well as long-baseline neutrino oscillation experiments, such as ICARUS, MINOS, MONOLITH and OPERA [5]. Recently several authors have calculated the τ production cross section for nuclear targets [6, 7], but the τ polarization should be also studied in order to estimate more precisely the background events. This was the motivation for recent calculations of the τ polarization, which have been achieved in the framework of some particular models [8, 9], for deep inelastic scattering (1), but also for quasi-elastic scattering and resonance production.

The relevance of positivity in spin physics, which puts strong restrictions on spin observables in many areas of particle physics, has been already emphasized [10] and the above process is one more example. In this paper we show that the use of model independent positivity constraints reduces considerably the allowed region for the τ polarization. In the next section we recall the kinematics, the general formalism for deep inelastic scattering and the expressions for the components of the τ polarization. In Section 3, we exhibit the positivity conditions and our numerical results, which have a direct relevance to the experiments mentioned above. Concluding remarks are given in Section 4 and some technical considerations about the positivity of the hadronic tensor are given in the Appendix.

2 General formalism and kinematics

In lepton nucleon deep inelastic scattering all the observables involve the hadronic tensor of the nucleon $W_{\mu\nu}(p,q)$, where p, k and $k^{'}$ are the four momenta of the nucleon, incoming $\nu_\tau(\bar{\nu}_\tau)$ and produced τ^- (τ^+), respectively, and $q = k - k^{'}$ is the momentum transfer. Since we consider the scattering of an unpolarized nucleon, using Lorentz invariance and time reversal invariance, we can express $W_{\mu\nu}(p,q)$ in terms of five real structure functions W_i as follows [11, 12, 13],

$$\begin{aligned} W_{\mu\nu}(p,q) &= -g_{\mu\nu}W_1(\nu,q^2) + \frac{p_\mu p_\nu}{M^2}\,W_2(\nu,q^2) - i\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{2M^2}\,W_3(\nu,q^2) \\ &\quad + \frac{q_\mu q_\nu}{M^2}\,W_4(\nu,q^2) + \frac{p_\mu q_\nu + q_\mu p_\nu}{2M^2}\,W_5(\nu,q^2)\ . \end{aligned} \tag{2}$$

Here $\epsilon_{\mu\nu\alpha\beta}$ is the total antisymmetric tensor with $\epsilon_{0123} = +1$ and W_3 appears because of parity violation of weak interactions. All structure functions, which are made dimensionless by including appropriate mass factors, depend on two Lorentz scalars $\nu = p\cdot q/M$ and $q^2 = -Q^2$ $(Q^2 > 0)$, where M is the nucleon mass. In the laboratory frame, let us denote by E_ν, E_τ and p_τ the neutrino energy, τ energy and momentum, respectively and θ the scattering angle. We then have $\nu = E_\nu - E_\tau$ and $Q^2 = 2E_\nu[E_\tau - p_\tau\cos\theta] - m_\tau^2$, where $m_\tau = 1.777$GeV is the τ mass. Finally, the Bjorken variable x is defined as $x = Q^2/2p\cdot q$ and the physical region is $x_{min} \le x \le 1$, where $x_{min} = m_\tau^2/2M(E_\nu - m_\tau)$. The unpolarized cross sections for deep inelastic scattering (1), are expressed as

$$\frac{d\sigma^\pm}{dE_\tau d\cos\theta} = \frac{G_F^2}{2\pi}\frac{M_W^4 p_\tau}{(Q^2+M_W^2)^2}R_\pm\ , \tag{3}$$

where G_F is the Fermi constant and M_W is the W-boson mass. Here

$$\begin{aligned} R_\pm &= \frac{1}{M}\left\{\left(2W_1 + \frac{m_\tau^2}{M^2}\,W_4\right)(E_\tau - p_\tau\cos\theta) + W_2\,(E_\tau + p_\tau\cos\theta)\right. \\ &\quad \left. \pm\frac{W_3}{M}\left(E_\nu E_\tau + p_\tau^2 - (E_\nu + E_\tau)p_\tau\cos\theta\right) - \frac{m_\tau^2}{M}\,W_5\right\}\ , \end{aligned} \tag{4}$$

where the $\pm$ signs correspond to $\tau^\mp$ productions.

Because of time reversal invariance, the polarization vector $\overrightarrow{P}$ of the τ in its rest frame, lies in the scattering plane defined by the momenta of the

incident neutrino and the produced τ. It has a component P_L along the direction of $\overrightarrow{p_\tau}$ and a component P_P perpendicular to $\overrightarrow{p_\tau}$, whose expressions are, in the laboratory frame, [8, 9, 12]

$$P_P = \mp \frac{m_\tau \sin\theta}{MR_\pm}\left(2W_1 - W_2 \pm \frac{E_\nu}{M}W_3 - \frac{m_\tau^2}{M^2}W_4 + \frac{E_\tau}{M}W_5\right), \tag{5}$$

$$P_L = \mp \frac{1}{MR_\pm}\left\{\left(2W_1 - \frac{m_\tau^2}{M^2}W_4\right)(p_\tau - E_\tau\cos\theta) + W_2\,(p_\tau + E_\tau\cos\theta)\right.$$
$$\left. \pm\frac{W_3}{M}\left((E_\nu + E_\tau)p_\tau - (E_\nu E_\tau + p_\tau^2)\cos\theta\right) - \frac{m_\tau^2}{M}W_5\cos\theta\right\}. \tag{6}$$

In addition, it is convenient to introduce also the degree of polarization defined as $P = \sqrt{P_P^2 + P_L^2}$. As previously the $\pm$ signs correspond to $\tau^\mp$ productions and it is clear that if $W_3 = 0$, one has $R_+ = R_-$ and τ^+ and τ^- have opposite polarizations. We also note that if one can neglect the mass of the produced lepton ($m_\tau = 0$), $P_P = 0$, so such a lepton is purely left-handed, if negatively charged, or purely right-handed, if positive.

3 Positivity constraints and numerical results

From Eq. (2) clearly the hadronic tensor $W_{\mu\nu}(p,q)$ is Hermitian

$$W_{\mu\nu}(p,q) = W^*_{\nu\mu}(p,q)\ , \tag{7}$$

and semi-positive. This last property implies that

$$a^*_\mu W_{\mu\nu}(p,q)a_\nu \geq 0\ , \tag{8}$$

for *any* complex 4-vector a_μ. The 4x4 matrix representation of $W_{\mu\nu}(p,q)$ in the laboratory frame where $p = (M,0,0,0)$ and $q = (\nu, \sqrt{\nu^2+Q^2}, 0, 0)$ reads $\begin{pmatrix} M_1 & 0 \\ 0 & M_0 \end{pmatrix}$ where M_1 and M_0 are the following 2x2 Hermitian matrices

$$M_1 = \begin{pmatrix} -W_1 + W_2 + \frac{\nu^2}{M^2}W_4 + \frac{\nu}{M}W_5 & \frac{\sqrt{\nu^2+Q^2}}{M}(\frac{\nu}{M}W_4 + \frac{1}{2}W_5) \\ \frac{\sqrt{\nu^2+Q^2}}{M}(\frac{\nu}{M}W_4 + \frac{1}{2}W_5) & W_1 + \frac{\nu^2+Q^2}{M^2}W_4 \end{pmatrix}\ , \tag{9}$$

and

$$M_0 = \begin{pmatrix} W_1 & \frac{-i\sqrt{\nu^2+Q^2}}{2M}W_3 \\ \frac{+i\sqrt{\nu^2+Q^2}}{2M}W_3 & W_1 \end{pmatrix}\ . \tag{10}$$

The *necessary and sufficient conditions* for $W_{\mu\nu}(p,q)$ to satisfy inequality (8) are that all the principal minors of M_1 and M_0 should be positive definite. So for the diagonal elements we have three inequalities linear in the W_i's namely

$$W_1 \geq 0 \ , \tag{11}$$

$$-W_1 + W_2 + \frac{\nu^2}{M^2} W_4 + \frac{\nu}{M} W_5 \geq 0 \ , \tag{12}$$

$$W_1 + \frac{\nu^2 + Q^2}{M^2} W_4 \geq 0 \ , \tag{13}$$

and from the 2x2 determinants of M_0 and M_1 we get two inequalities quadratic in the W_i's namely

$$W_1^2 \geq \frac{\nu^2 + Q^2}{4M^2} W_3^2 \ , \tag{14}$$

or equivalently

$$W_1 \geq \frac{\sqrt{\nu^2 + Q^2}}{2M} |W_3| \ , \tag{15}$$

and

$$\left(-W_1 + W_2 + \frac{\nu^2}{M^2} W_4 + \frac{\nu}{M} W_5\right)\left(W_1 + \frac{\nu^2 + Q^2}{M^2} W_4\right) \geq \frac{\nu^2 + Q^2}{M^2}\left(\frac{\nu}{M} W_4 + \frac{1}{2} W_5\right)^2 \ . \tag{16}$$

By imposing the last condition, only one of the two inequalities (12) or (13) is needed, the other one follows automatically. Since the hadronic tensor $W_{\mu\nu}(p,q)$ allows the construction of the scattering amplitudes for a vector-boson nucleon Compton scattering process, the five structure functions W_i are related to the five s-channel helicity amplitudes, which survive in the forward direction. As a special case in Eq. (8), if one takes for a_μ the polarization vectors of the vector-boson, the nucleon being unpolarized, these amplitudes are

$$M(h', h) = \epsilon^*_\mu(h') W_{\mu\nu} \epsilon_\nu(h) \ , \tag{17}$$

where h and h' are the helicities of the initial and final vector-boson, respectively [2]. The positivity conditions reflect the fact that the forward

[2] For a complete study of deep inelastic scattering with a polarized nucleon, in terms of fourteen structure functions, see Ref. [13].

amplitudes, which are indeed cross sections, must be positive. The linear conditions correspond to the polarized vector-boson scattering, with longitudinal, transverse or scalar polarizations and the quadratic condition (16), is a Cauchy-Schwarz inequality which corresponds to the scalar-longitudinal interference. The above set of positivity constraints might appear to be different from the ones derived earlier [14, 15], but this is not the case as we will discuss in the Appendix.

In order to test the usefulness of these constraints to restrict the allowed domains for P_P and P_L, we proceed by the following method, without refering to a specific model for the W_i's. We generate randomly the values of the W_i's, in the ranges [0,+1] for W_1 and W_2, which are clearly positive and [-1,+1] for $i = 3, 4, 5$. The most trivial positivity constraints are $R_\pm \geq 0$, but in fact they are too weak and do not imply the obvious requirements $|P_L| \leq 1$ and $|P_P| \leq 1$ or $P \leq 1$ [3]. So we first impose $R_\pm \geq 0$ and $P \leq 1$ for different values of E_ν, Q^2 and x and as shown in Fig. 1, for τ^+ production, the points which satisfy these constraints are represented by grey dots inside the disk, $P_L^2 + P_P^2 \leq 1$. If we now add the non trivial positivity constraints Eqs.(10-15), which also garantee that $P \leq 1$, we get the black dots, giving a much smaller area. In Fig. 1, the top row corresponds to $E_\nu = 10$GeV and $Q^2 = 1\text{GeV}^2$, the row below to $E_\nu = 10$GeV and $Q^2 = 4\text{GeV}^2$ and the next two rows to $E_\nu = 20$GeV and $Q^2 = 1, 4\text{GeV}^2$. Going from left to right x increases from a value close to its minimum to 0.9. It is interesting to note that the black allowed area increases with Q^2 and becomes smaller for increasing incident energy and increasing x. For τ^- production, the corresponding areas are obtained by symmetry with respect to the center of the disk. For increasing x, since P_L is more and more restricted to values close to +1 for τ^+ (-1 for τ^-), it is striking to observe that the non trivial positivity constraints lead to a situation where the τ^+ (τ^-) is almost purely right-handed (left-handed), although it has a non zero mass.

Another way to present our results is seen in Fig. 2, which shows the upper and lower bounds from the non trivial positivity constraints for a given incident energy and different x values, versus Q^2. These bounds are obtained by selecting the larger and smaller allowed values of P_L and P_P, when the W_i's are varied for a fixed bin of E_ν and x. We also indicate the scattering angle which increases with Q^2 and we recall that for $\theta = 0$ we have $P_P = 0$ (see Eq. (4)).

[3] Note that in the trivial case where $W_3 = W_4 = W_5 = 0$, $R \geq 0$ implies $P \leq 1$.

Finally we have tested the effect of some approximate relations among the W_i's, which have been proposed in the literature. First, as an example for a particular kinematic situation we show in Fig. 3 the effect of imposing the Callan-Gross relation [16], namely $Q^2 W_1 = \nu^2 W_2$. It further reduces both the grey dots and the black dots areas, since this has to be compared with the first row of Fig. 1. For the same kinematic situation we also show in Fig. 4, the effect of the Albright-Jarlskog relations [12], namely $MW_1 = \nu W_5$ and $W_4 = 0$, and we observe again that the allowed regions are much smaller. These examples illustrate the fact that a more precise knowledge of the structure functions W_i's, will certainly further restrict the domains shown in Fig.1.

4 Concluding remarks

We have shown in this paper that the positivity conditions on the hadronic tensor of the nucleon $W_{\mu\nu}(p,q)$, is essential to reduce the allowed values for the $\tau^{\pm}$ polarization in neutrino deep inelastic scattering. We have not used a specific model and we have considered only a few kinematic situations, which are relevant for the long baseline neutrino oscillation experiments, but they can be easily applied to other kinematic ranges and in the framework of any given model. They are less usefull for ultra high neutrino energies, because in this case $\theta \simeq 0$, so $P_P \simeq 0$ and $P_L \simeq \pm 1$ for $\tau^{\mp}$. The universality of $W_{\mu\nu}(p,q)$, which occurs in processes we have not studied here (i.e. quasi-elastic scattering etc...), also increases the importance of these positivity constraints.

5 Appendix

The positivity conditions on $W_{\mu\nu}(p,q)$ were first obtained in Refs. [14, 15] and they were reported in Refs. [11, 12] under a slightly different form due to the use of our definition of $W_{\mu\nu}(p,q)$, which differs from that of Ref. [15]. Moreover in Ref. [15] instead of the laboratory system, they were using a frame where q is purely space-like. Although from covariance one expects the equivalence of the different sets of conditions, it seems natural to show it explicitely. Let us consider the frame where $p = (M\sqrt{1+\nu^2/Q^2}, -\nu M/\sqrt{Q^2}, 0, 0)$ and $q = (0, \sqrt{Q^2}, 0, 0)$. The 4x4 matrix representation of $W_{\mu\nu}(p,q)$ is very

similar to the case of the laboratory frame, since it reads $\begin{pmatrix} M_2 & 0 \\ 0 & M_0 \end{pmatrix}$ where M_2 is

$$M_2 = \begin{pmatrix} -W_1 + (1+\frac{\nu^2}{Q^2})W_2 & \frac{\sqrt{\nu^2+Q^2}}{2M}(W_5 - \frac{2M\nu}{Q^2}W_2) \\ \frac{\sqrt{\nu^2+Q^2}}{2M}(W_5 - \frac{2M\nu}{Q^2}W_2) & W_1 + \frac{\nu^2}{Q^2}W_2 + \frac{Q^2}{M^2}W_4 - \frac{\nu}{M}W_5 \end{pmatrix} , \quad (18)$$

and M_0 was given in (10). The momenta p and q defined in the two reference frames are related by a Lorentz transform, so the matrix elements of M_1 and M_2 are simply related. Moreover one can check that, first,

$$\det(M_1) = \det(M_2) \ , \quad (19)$$

second, the difference of the diagonal elements of M_1 and M_2 is the same and these diagonal elements must be both either positive or negative, due to Eq. (19). So in order to establish the equivalence of the positivity conditions in the two reference frames, a simple calculation proves that the two inequalities (12) or (13) imply

$$-W_1 + (1+\frac{\nu^2}{Q^2})W_2 \geq 0 \quad (20)$$

or

$$W_1 + \frac{\nu^2}{Q^2}W_2 + \frac{Q^2}{M^2}W_4 - \frac{\nu}{M}W_5 \geq 0 \ . \quad (21)$$

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References

[1] K. S. Hirata *et al.*, Phys. Lett. B **205**, 416 (1988); ibid **280**, 146 (1992).

[2] D. Casper *et al.*, Phys. Rev. Lett. **66**, 2561 (1991); R. Becker-Szendy *et al.*, Phys. Rev. D **46**, 3720 (1992).

[3] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Lett. B **335**, 237 (1994); ibid **433**, 9 (1998); **436**, 33 (1998); Phys. Rev. Lett. **81**, 1562 (1998); ibid **85**, 3999 (2000).

[4] AMANDA Collaboration, J. Ahrens *et al.*, Nucl. Phys. A **721**, 545 (2003); ANTARES Collaboration, G. D. Hallewell *et al.* Nucl. Instrum. Meth. A **502**, 138 (2003); BAIKAL Collaboration, R. Wischnewski *et al.*, astro-ph/0305302; NESTOR Collaboration, S. E. Tzamarias Nucl. Instrum. Meth. A **502**, 150 (2003); F. Halzen and Dan Hooper, astro-ph/0310152.

[5] ICARUS Collaboration, F. Arneodo *et al.*, Nucl. Instrum. Meth. A **508**, 287 (2003), see also the home page, http://www.aquila.infn.it/icarus/; MINOS Collaboration, V. Paolone *et al.*, Nucl. Phys. (Proc. Suppl.) **100**, 197 (2001), see also MINOS Collaboration home page, http://www-numi.fnal.gov; MINOLITH Collaboration, F. Terranova *et al.*, Int. J. Mod. Phys. A **16S1B**, 736 (2001); A. Rubbia, Nucl. Phys. (Proc. Suppl.) **91**, 223 (2000), see also the OPERA Collaboration home page, http://operaweb.web.cern.ch/operaweb.

[6] E. A. Paschos and J. Y. Yu, Phys. Rev. D **65**, 033002 (2002).

[7] S. Kretzer and M. H. Reno, Phys. Rev. D **66**, 113007 (2002).

[8] K. Hagiwara, K. Mawatari and H. Yokoya, Nucl. Phys. B **668**, 364 (2003).

[9] K. S. Kuzmin, V. V. Lyubushkin and V. A. Naumov, contribution to the 10th Int. Workshop on High Energy Spin Physics (SPIN03), JINR Dubna, Russia, 16-20 Sept. 2003, hep-ph/0312107.

[10] J. Soffer, A.I.P. Conference Proceedings **570**, 461 (2001).

[11] C. H. Llewellyn Smith, Phys. Rep. **3**, 261 (1972).

[12] C. H. Albright and C. Jarlskog, Nucl. Phys. B **84**, 467 (1975).

[13] X. Ji, Nucl. Phys. B **402**, 217 (1993).

[14] T. D. Lee and C. N. Yang, Phys. Rev. **126**, 2239 (1962).

[15] M. G. Doncel and E. De Rafael, Nuovo Cimento A **4**, 363 (1971).

[16] C. G. Callan and D. G. Gross, Phys. Rev. Lett. **22**, 156 (1969).

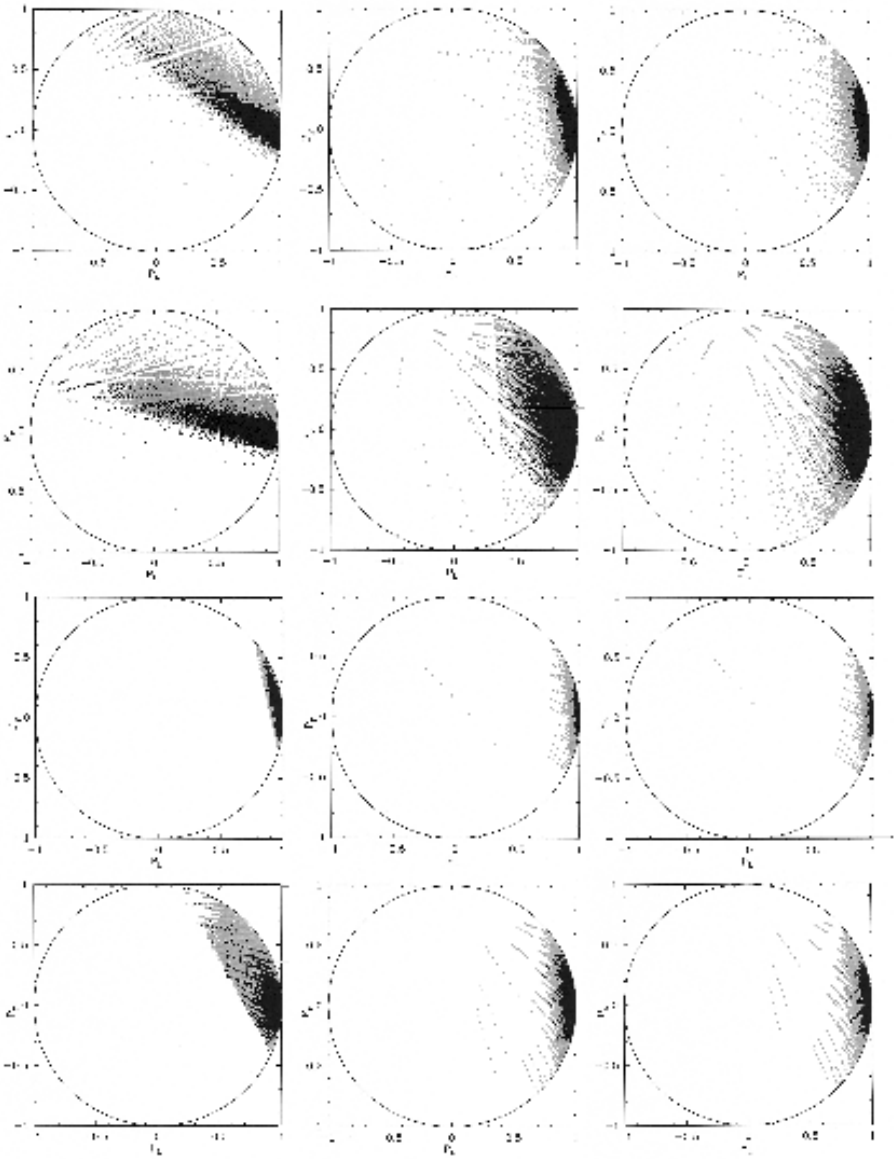

Figure 1: For τ^+ production, P_P versus P_L in a domain limited by $R_+ \geq 0$, $P \leq 1$ (grey area) plus non trivial positivity constraints (black area). From top to bottom and left to right, $E_\nu = 10\text{GeV},\ Q^2 = 1\text{GeV}^2,\ x = 0.25, 0.6, 0.9$, $E_\nu = 10\text{GeV},\ Q^2 = 4\text{GeV}^2,\ x = 0.4, 0.6, 0.9,\ E_\nu = 20\text{GeV},\ Q^2 = 1\text{GeV}^2,\ x = 0.25, 0.6, 0.9,\ E_\nu = 20\text{GeV},\ Q^2 = 4\text{GeV}^2,\ x = 0.25, 0.6, 0.9$.

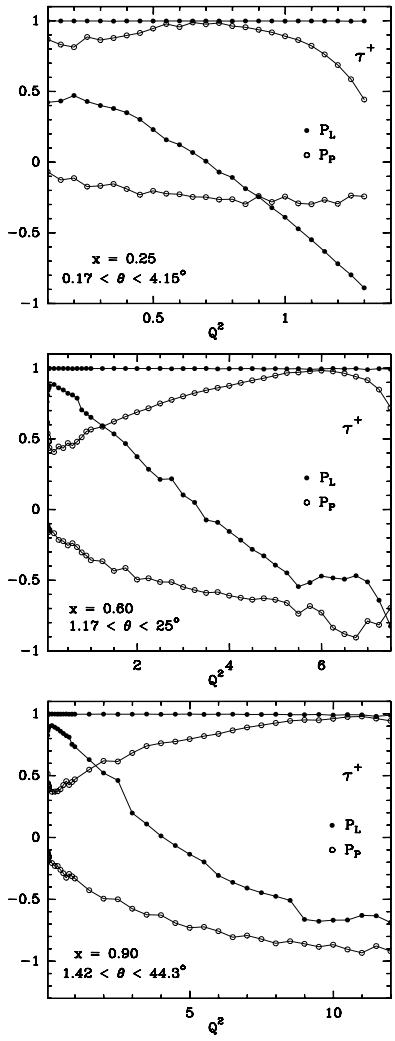

Figure 2: For τ^+ production, upper and lower bounds on P_P (open circles) and P_L (full circles) as a function of Q^2 for $E_\nu = 10$GeV and $x = 0.25, 0.6, 0.9$.

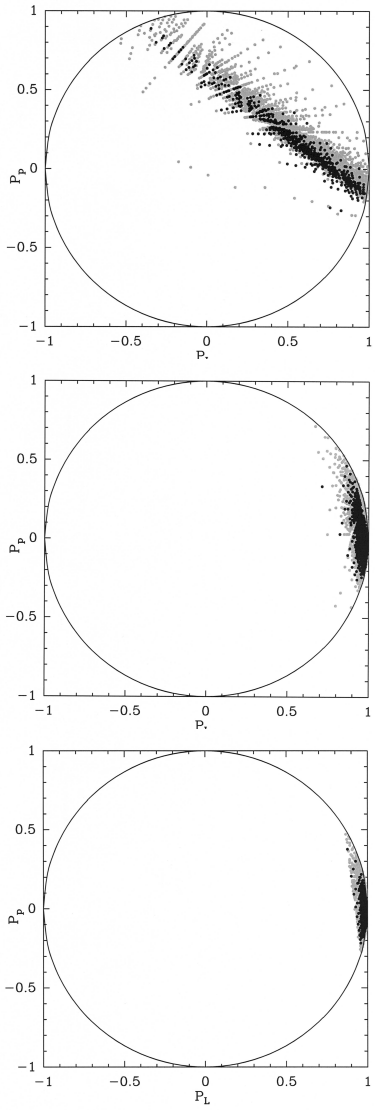

Figure 3: For τ^+ production, P_P versus P_L in a domain limited by $R_+ \geq 0$, $P \leq 1$ assuming the Callan-Gross relation (grey area) plus non trivial positivity constraints (black area). $E_\nu = 10\text{GeV}$, $Q^2 = 1\text{GeV}^2$, from top to bottom, $x = 0.25, 0.6, 0.9$.

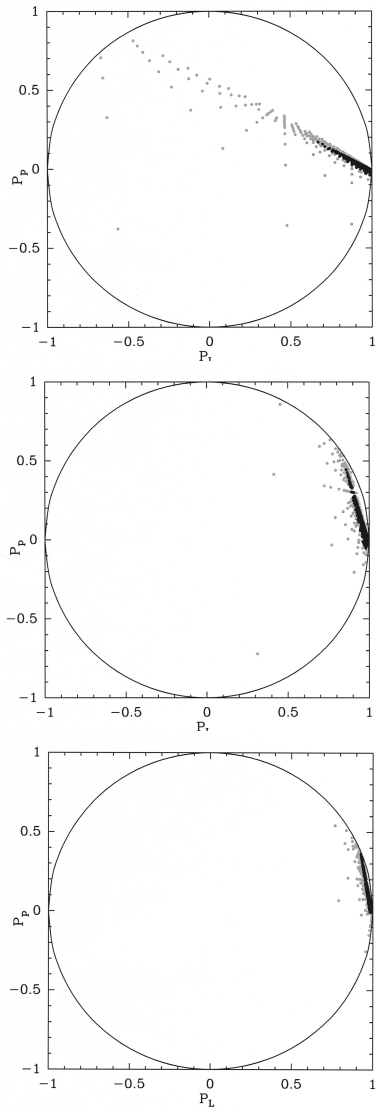

Figure 4: For τ^+ production, P_P versus P_L in a domain limited by $R_+ \geq 0$, $P \leq 1$ assuming the Albright-Jarlskog relations (grey area) plus non trivial positivity constraints (black area). $E_\nu = 10\text{GeV}$, $Q^2 = 1\text{GeV}^2$, from top to bottom, $x = 0.25, 0.6, 0.9$.