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Direct measurement of the phase coherence length in a GaAs/GaAlAs square network

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The low temperature magnetoconductance of a large array of quantum coherent loops exhibits Altshuler-Aronov-Spivak oscillations which periodicity corresponds to $1/2$ flux quantum per loop. We show that the measurement of the harmonics content in a square network provides an accurate way to determine the electron phase coherence length L_φ in units of the lattice length without any adjustable parameters. We use this method to determine L_φ in a network realised from a 2D electron gas (2DEG) in a GaAs/GaAlAs heterojunction. The temperature dependence follows a power law $T^{-1/3}$ from 1.3 K to 25 mK with no saturation, as expected for 1D diffusive electronic motion and electron-electron scattering as the main decoherence mechanism.

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The characteristic scale on which quantum interference can occur in a conductor, the phase coherence length L_φ , is the key parameter of quantum transport. In particular, the dependence of L_φ on temperature can discriminate between the various scattering mechanisms which limit phase coherence: electron-electron (e-e), electron-phonon or electron-magnetic impurity interactions. Interference on the scale of L_φ gives rise to two different types of contributions to the conductance in a transport experiment. Some are sample specific and depend on the particular disorder configuration. These are conductance fluctuations (magnetofingerprints) and the ϕ_0 periodic Aharonov-Bohm (AB) oscillations ($\phi_0 = h/e$ is the flux quantum). Their amplitudes are governed both by L_φ and the thermal length L_T which in most cases is smaller than L_φ . This makes an accurate determination of L_φ difficult [1, 2, 3]. The second type of contribution, called the weak localisation (WL) correction, is obtained after ensemble averaging of quantum interferences on many configurations of disorder. It originates from interferences between time reversed electronic trajectories, which are the only ones surviving the disorder average. It is also observed in samples of size $L \gg (L_\varphi, L_T)$ and only depends on L_φ since it involves trajectories at the same energy. Manifestations of WL are the magnetoconductance (MC) of large connex samples [1, 2, 3] and the Altshuler-Aronov-Spivak (AAS) $\phi_0/2$ periodic oscillations resulting from the ensemble average of AB oscillations in a long cylinder or large arrays of connected phase coherent rings [4, 5, 6]. The WL provides thus in general a much more direct measurement of L_φ than sample specific corrections.

The analysis of the MC in 1D diffusive metallic wires (with transverse dimensions smaller than L_φ) has led to accurate determinations of L_φ . It was found that the dominant phase breaking mechanism at very low temperature, in the absence of magnetic impurities, is due to e-e scattering and is well described by the Altshuler-Aronov-

Khmelnitskii (AAK) theory [1, 7] yielding $L_\varphi \propto T^{-1/3}$ with no saturation down to 40 mK [2, 3]. This analysis requires a good knowledge of transport parameters such as the diffusion coefficient (D), and the width of the wire (W) or the electronic density (n_e). On the other hand these parameters are not straightforwardly determined in wires fabricated from GaAs/GaAlAs 2DEG, and therefore much less evaluations of L_φ exist [8, 9].

Here we show that L_φ can be determined *without adjustable parameters* from the analysis of the AAS oscillations in a periodic network (Fig. 1), when the circumference L of the elementary loop is of the order of L_φ . It is indeed known that the amplitude of AAS oscillations decreases exponentially with L/L_φ . In this letter, following [10, 11], we explain how to calculate the harmonics content of the oscillations. Since it depends only on L/L_φ it is possible to determine unambiguously L_φ as a function of temperature, in a 2DEG square lattice containing 10^6 loops in the diffusive transport regime. Moreover once L_φ is determined it is also possible from the analysis of the high field positive MC to deduce the elastic mean free path (l_e), the width of the wires, and to make a detailed comparison with theoretical predictions of the AAK theory on dephasing by e-e interactions. We find a very good agreement in this regime, never explored before, of very few conducting channels.

In the weakly disordered diffusive regime corresponding to $k_F l_e \gg 1$, the WL correction is directly related to the Cooperon, which can be computed from the time integrated return probability $P_c(\vec{r}, \vec{r})$ for a diffusive electron. In a cylinder or an array of connected loops the contribution to the Cooperon of trajectories enlacing at least one loop oscillates with a flux periodicity of $\phi_0/2$ giving rise to the AAS oscillations. A systematic way of calculating WL in a mesoscopic network of diffusive wires was derived by Douçot & Rammal [10]. More recently Pascaud & Montambaux [11] found a relation between the WL correction and the spectral determinant

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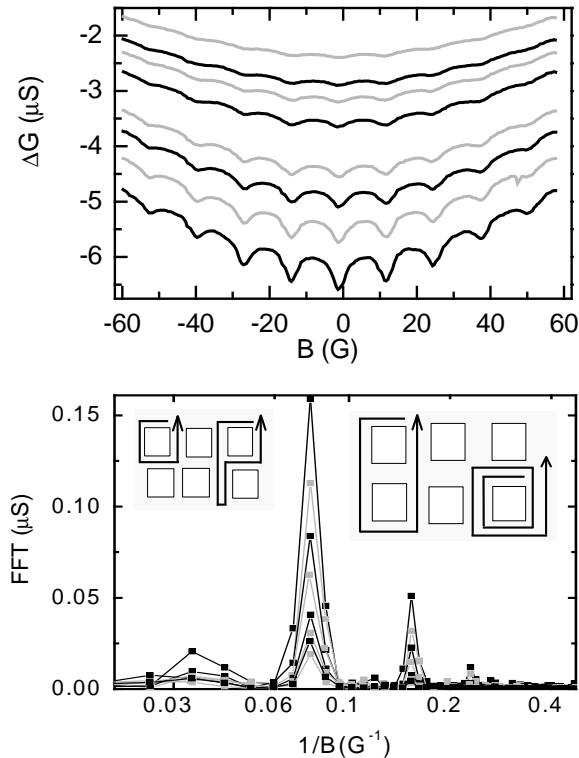


FIG. 1: a) Conductance versus magnetic field for temperatures varying from 25 mK to 220 mK. b) Fourier transform of the MC for different temperatures. Left inset: some orbits contributing to the first harmonic. Right inset: some orbits contributing to the second harmonic.

$S(\gamma) = \det(\gamma - \Delta)$ of the Laplace operator Δ defined on the network :

$$\langle \Delta \sigma \rangle = -\frac{2e^2}{\pi \hbar} \int \frac{d\vec{r}}{\text{Vol}} P_c(\vec{r}, \vec{r}) = -\frac{2e^2}{\pi \hbar} \frac{1}{\text{Vol}} \frac{\partial}{\partial \gamma} \ln S(\gamma) \quad (1)$$

where $\gamma = 1/L_\varphi^2$. This approach, which is meaningful only for regular networks, is particularly efficient because $S(\gamma)$ can be computed systematically for any given network in terms of the determinant of a finite size matrix encoding the information about the network (topology, lengths of the wires, magnetic flux). It can also be shown that the WL can be expressed, in the small L_φ limit, as a trace expansion over periodic orbits, denoted \mathcal{C} ,

$$\frac{\partial \ln S(\gamma)}{\partial \gamma} = \frac{1}{2\sqrt{\gamma}} \left[\mathcal{L} + \frac{V-B}{\sqrt{\gamma}} + \sum_{\mathcal{C}} l(\tilde{\mathcal{C}}) \alpha(\mathcal{C}) e^{-\sqrt{\gamma} l(\mathcal{C}) + i\theta(\mathcal{C})} \right]$$

where V , B , is the total number of vertices, bonds respectively. $\tilde{\mathcal{C}}$ is the primitive orbit related to \mathcal{C} . $\text{Vol} = \mathcal{L}$ is the total length. We explain briefly this formula, demonstrated in [12] and discussed in detail in [13]. Each orbit contributes to the MC with a phase factor which depends on the enclosed flux: $\theta(\mathcal{C}) = 4\pi\Phi(\mathcal{C})/\phi_0$. It is also characterised by its length $l(\mathcal{C})$ and by a geometrical weight

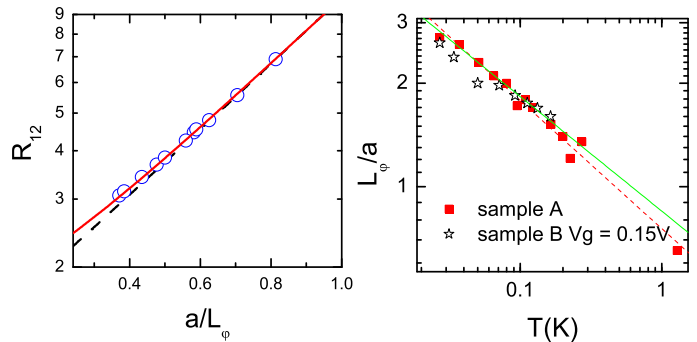


FIG. 2: Left: relation between L_φ/a and R_{12} on a semi-log scale. The continuous line comes from the numerical calculation of $S(\gamma)$. The dashed line is deduced from the expansion (2). The thick points are the experimental values of R_{12} from which L_φ is determined.

Right: L_φ versus temperature on a log-log scale. The fit (dotted line) yields the power law $L_\varphi \propto T^{-0.36}$. The point at 1.3K is obtained from the fit of the envelope. The continuous line is the AAK theory for sample A.

$\alpha(\mathcal{C})$. In the case of a square lattice of periodicity a , the periodically oscillating conductance can be decomposed in the Fourier space as a sum of harmonics of the fundamental periodicity corresponding to $\phi_0/2$ per elementary cell. The first terms of this expansion in $x = e^{-2a/L_\varphi}$ read:

$$\langle \Delta \sigma \rangle = -\frac{e^2 L_\varphi}{h} \left[2 - \frac{L_\varphi}{a} + x + \dots + \frac{x^2}{2} \cos \theta \left(1 - \frac{3}{2} x + \dots \right) + \frac{3x^3}{8} \cos 2\theta \left(1 - \frac{19}{12} x + \dots \right) + \frac{3x^4}{8} \cos 3\theta \left(1 - \frac{15}{8} x + \dots \right) \right] (2)$$

where $\theta = 4\pi\phi/\phi_0$, ϕ being the flux per elementary cell. The amplitude of the n th harmonics is evaluated by counting the paths enclosing n fluxes ϕ . The counting becomes rapidly cumbersome, for instance 156 orbits are involved in the last term, but the crucial point is that the coefficient of each term depends only on the lattice geometry.

More generally, the WL correction can be obtained for all values of L_φ/a from the numerical computation of the determinant in Eq. (1). The numerical FFT of the computed MC yields the ratio R_{12} of the 2 first harmonics as a function of L_φ/a (Fig. 2a). It appears that the small orbit expansion (2) is a good approximation up to $L_\varphi/a = 2$. In any case the ratio of two harmonics is completely determined by L_φ/a and provides a method for a direct evaluation of L_φ without any adjustable parameter. The square lattice is particularly appropriate for such a determination of L_φ due to its large harmonics content: the second harmonic is dominated by orbits of length $6a$ instead of $8a$ for a statistical ensemble of single rings or a necklace of identical rings, for example.

We now use this method to determine the phase coherence length of square networks etched in a 2DEG of

a GaAs/AlGaAs heterostructure. The networks consist of 10^6 square loops of side $a = 1 \mu\text{m}$ and nominal width $W_0 = 0.5 \mu\text{m}$ and cover a total area of 1mm^2 . A metallic (Au) gate deposited 100nm above the 2DEG offers the possibility to increase or decrease the number of electrons in the network. Measurements were done on two identical networks (A and B), giving the same results. Except when specified, figures show the data for sample A. We have measured the MC up to 4.5T between 25mK and 1.3K , using a standard lock-in technique (ac current of 1nA at 30Hz).

The samples were in general strongly depleted at low temperature because of the etching. The intrinsic electron density of the 2DEG, $n_e = 4.4 \times 10^{15} \text{m}^{-2}$, was recovered after illuminating the samples during several minutes at 4.2K . This density was determined from the period of Shubnikov-de Haas oscillations visible above 1T . It was found to vary only by less than 30% when the gate voltage was varied between -0.3V and $+0.3 \text{V}$, whereas the network resistance varied by a factor 5. This shows that the main effect of the gate voltage is to change the width of the wires and disconnect some bonds. Because of depletion after etching of the 2DEG, it is difficult to estimate the real width of the wires and the elastic mean free path. The mobility of the carriers was estimated to be $\mu = 3 \text{m}^2 \text{V}^{-1} \text{s}^{-1}$ *i.e.* 10 times smaller than the mobility of the original 2DEG.

In the following we focus on the MC data at low magnetic field (Fig. 1a). The data exhibit large AAS oscillations with a period 12.6G corresponding to a flux $\phi_0/2$ in a square cell of area a^2 . The oscillations are clearly not purely sinusoidal. At the lowest temperature, 25mK , three harmonics are visible in the Fourier spectrum of the MC (Fig. 1b). Moreover, as shown in Fig. 3 which represents the conductance for a wider range of magnetic field, the oscillations disappear above 60G but the WL magnetoconductance due to the penetration of the magnetic field through the finite width of the wires constituting the network is still clearly visible. At high temperature, above 400mK , the AAS oscillations disappear even at low field. Only the positive MC remains with a smaller amplitude. The same experiments for different gate voltages were also performed.

We first concentrate on the AAS oscillations (Fig. 1a). The Fourier spectrum of the MC exhibits a series of peaks corresponding to successive harmonics of the $\phi_0/2$ periodicity. The finite width of the peaks (Fig. 1b) is due to the penetration of the magnetic field in the wires which damps the AAS oscillations at high field. We have checked that the mechanisms responsible for the broadening of the Fourier components do not affect their integrated values which we have computed in order to compare with the theory depicted above. From the ratio R_{12} of integrated peaks of the two first harmonics it is thus possible to directly determine L_φ/a via the theoretical relation between L_φ and R_{12} . The temperature depen-

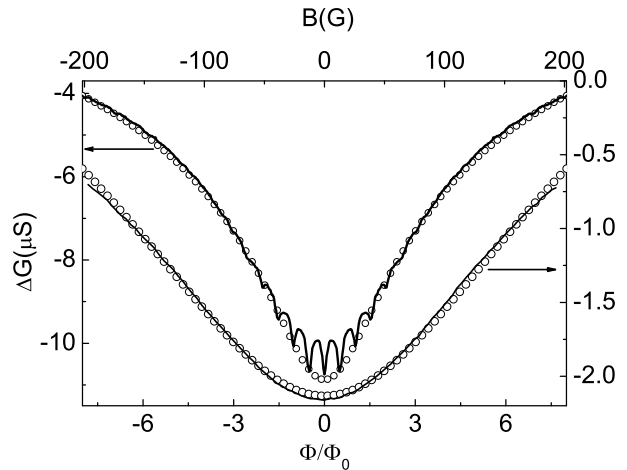


FIG. 3: High field MC, the continuous lines are the experimental data, the dots are the fits with formula (4). At 30mK , parameters of the fit are l_e and W . At 1K , parameters are L_φ and l_e .

dence is deduced this way between 25mK and 250mK as shown in Fig. 2. We find that L_φ follows a power law $T^{-\eta}$, where $\eta = 0.36 \pm 0.05$. The coherence length reaches almost $3 \mu\text{m}$ at 25mK and there is no sign of saturation.

Once L_φ is determined, the sample parameters (W, l_e) can be deduced from the WL envelope. The magnetic field appears as an additional effective phase breaking rate for the time reversed trajectories responsible of the WL leading to an effective L_φ given by [14]:

$$\frac{1}{L_\varphi(\phi)^2} = \frac{1}{L_\varphi(0)^2} \left[1 + \frac{1}{3} \left(2\pi \frac{\phi}{\phi_0} \frac{W_{\text{eff}} L_\varphi(0)}{a^2} \right)^2 \right] \quad (3)$$

where $W_{\text{eff}} = W \sqrt{(3W)/(C_1 l_e)}$ can be understood as a renormalized width which appears in the WL correction for a semi ballistic wire in the regime $l_e \gg W$ due to the phenomenon of flux cancellation. The coefficient C_1 depends on the specific boundary conditions. The samples under consideration are close to the case of specular boundary scattering [8] for which $C_1 = 9.5$. The envelope of the MC, given by $\langle \Delta\sigma(\phi = 0, L_\varphi(\phi)) \rangle$, can be analytically computed for the square lattice geometry and is given by:

$$\langle \Delta\sigma \rangle = -\frac{e^2 L_\varphi}{h} \left[\coth \frac{a}{L_\varphi} - \frac{L_\varphi}{a} + \frac{2}{\pi} \text{th} \frac{a}{L_\varphi} K \left(\frac{1}{\text{ch} \frac{a}{L_\varphi}} \right) \right] \quad (4)$$

where $K(x)$ is a complete elliptic integral. We can then fit the measured $\Delta G/G_D = \Delta\sigma/\sigma_D$ where σ_D and G_D are the Drude conductivity and conductance. G_D is deduced from the high field measurements when the WL vanishes, and the parameters of the fit are W_{eff} and l_e . By fitting the WL envelope at different temperatures we

extract the parameters l_e and W which are found independent of temperature: this confirms the consistency of our analysis. We obtain $W_{\text{eff}} = 85$ nm and $l_e = 220$ nm, yielding $W = 170$ nm. This shows that the networks are indeed in the diffusive regime. Knowing $k_F^{-1} = 6$ nm from the Shubnikov-de Haas measurement, these numbers correspond to $M = k_F W / \pi = 9$ transverse channels per wire and the number of effective conducting channels $M_{\text{eff}} = M l_e / a \sim 2$ on the scale of a is of the order of unity. At higher temperature where no oscillations are visible, we can nevertheless deduce L_φ and l_e by fitting the envelope of the MC, knowing the values of W and G_D (Fig. 3). The knowledge of these parameters makes possible a quantitative comparison of L_φ with the theoretical prediction of AAK [7] $L_\varphi = \sqrt{2} \left(\frac{D^2 m^* W}{\pi k_B T} \right)^{1/3}$ written for a 2D wire. This theory applies to diffusive metallic wires with a large number of conducting channels in a limit where e-e interaction is treated perturbatively. We find a very good agreement (see Fig. 2b) which is surprising for two reasons: (i) we are confronted here with a small number of conducting channels where strong interaction effects could be expected, (ii) the result of AAK was not extended to network geometry. Recently it was found in [15] that L_φ extracted from the AB or AAS oscillations in a single ring of perimeter L behaves rather like $L_\varphi \propto (LT)^{-1/2}$. This behaviour is not observed in our experiment. For the second sample we have applied gate voltages between -0.3 V and 0.3 V changing the resistance from 30 k Ω to 400 k Ω . A good filtering of the gate voltage line is needed to avoid saturation of the coherence length. We find again a $T^{-1/3}$ law for L_φ . It is also possible to estimate for each gate voltage the values of W and l_e . The width W is not found to change within our experimental accuracy but l_e decreases with the number of electrons by a factor 1.5 when the gate voltage varies between -0.15 V and 0.15 V; at the same time G_D decreases by a factor 5. This shows that the effect of the gate is also to disconnect bonds of the network.

As a consistency check we have computed numerically the oscillating part of the MC with formula (1) et (3) using the value of W_{eff} determined above from the WL envelope of the MC curves. We find that this value also precisely describes the damping of the AAS oscillations, if the oscillations amplitude is multiplied by a factor ranging from 1.6 to 2 depending on the gate voltage. This can be explained by the existence of broken bonds in the network which influences envelope and oscillations differently. We obtain a very good agreement between theory and experiments (Fig. 4).

In conclusion we have shown that magnetoconductance experiments in GaAs/GaAlAs networks can be described very accurately by the diagrammatic theory of quantum transport in diffusive networks. It is remarkable that this agreement is achieved in a limit where the conductance on the scale of the period of the network, $M l_e / a$,

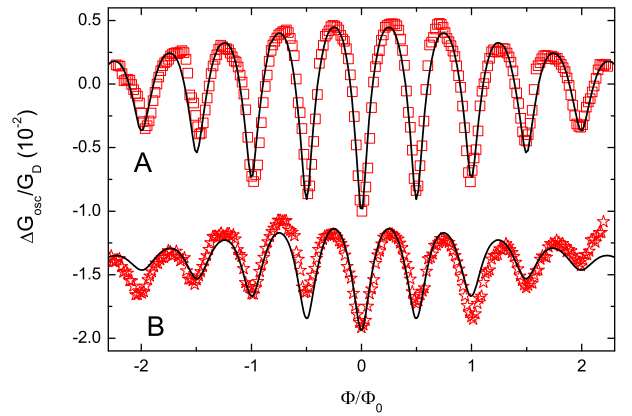


FIG. 4: Comparison between experiment (symbols) and theory (continuous line) for the oscillating part of the conductance for sample A (\square) with $V_g = 0$ and sample B (\star) (shifted down for visibility) with $V_g = -0.15$ V. The only adjustable parameter is the amplitude of the oscillations.

is of the order of the quantum conductance, in contrast with metallic wires where the number of channels is of order 1000. Moreover we extracted from the AAS oscillations the temperature dependance of the phase coherence length $L_\varphi \propto T^{-1/3}$ that agrees with AAK theory down to 25 mK.

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