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Analysis of the effect of the register hole on the tuning of a clarinet

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Abstract

Even if the tuning between the first and second register of a clarinet has been noticeably improved by instrument makers, the inharmonicity of the lowest twelfths (which are too large) is still an unsolved problem. In this paper, we attempt to understand the physical reasons for this problem and to explore whether or not it is possible to improve the accuracy of these twelfths. The starting point is a study of the inharmonicity associated with different bore perturbations inserted in cylindrical instruments, including bore flare, open and closed holes, etc. This study shows that the wide twelfths in question result mostly from the effects of the register hole. Using an elementary model of the clarinet as well as optimization techniques, an optimum location for the register hole is computed (we do not take into account the use of the register hole as a B flat tone hole); the result turns out to be close to the location chosen by the makers. Then small perturbations on the higher part of the cylindrical resonator are introduced and optimized in order to find out whether a simple solution exists for the improvement of the harmonic relationship between the first and quasi-third resonance frequencies. The result is negative, which probably explains why this fundamental problem of the clarinet has not yet been solved. As a consequence, one has to resort to more complicated local solutions, for instance with individual corrections for each tone hole.

Key words: Clarinet, register hole, twelfth inharmonicity, length corrections.

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1 Introduction

In the last forty years many contributions, either experimental or theoretical, have been made to improve our understanding of single reed woodwind instruments (see e.g.[1,2,3,4]). Our knowledge on the linear behaviour of the resonator is now very satisfactory, so that most of the recent literature actually deals with the understanding of the sound production and oscillation regimes. Nevertheless, interesting questions concerning the resonator can still be asked in perspective of possible improvements or modifications of the instrument design. In this line, Benade [5] proposed in the seventies some basic ideas and methods allowing to characterize the qualities of a wind instrument (impedance peak alignment); Meynial and Kergomard [6] designed simple acoustical systems that shift the scale of a woodwind of a given micro-interval; ref. [7] discusses in general how it is possible to predict the emission frequencies, and even some aspects of the clean intonation and tone colour, which naturally leads to the design of modified instruments as soon as appropriate optimization criteria are defined. Recently, optimization techniques have been used in order to define longitudinal profiles of brass instruments [8].

Concerning the clarinet, many instrumentalists and instrument makers agree that, even if generally speaking it is now a very well tuned instrument, the twelfths corresponding to the three lowest notes are slightly too large, sometimes causing problems for the musician. Despite many efforts, this problem has apparently not yet been solved, so that it sometimes requires artificial corrections from the clarinetists during performance. One could think a priori of many reason for this problem: deviation of the bore from purely cylindrical shape, existence of open or closed side holes (cavities), register hole, etc. All these elements make the problem of perfect twelfths for a clarinet a difficult task.

The first purpose of the present article is to analyse the origin of this tuning problem; we will see that the main culprit is actually the register hole. Then the question is to decide whether it is possible, for a simplified shape of a clarinet, to design a register hole allowing for the two complete first registers to be perfectly well tuned (the definition of the registers will be discussed in the next section). Because it is easy to adapt the exact position of the tone holes in order to get a correct tuning for one register, what really matters is the values of the intervals between the two registers. For the sake of clarity, we will reason with a simplified shape, cylindrical and without tone holes, the different values of the tones being adjusted simply by choosing different lengths for the tube.

Two questions will be discussed successively :

- i) what is the location of the register hole optimizing the tuning of the twelfth intervals between the two first registers?
- ii) which simple system, located upstream from the smallest length (corresponding to the highest tone hole), can correct for the best the residual tuning defaults?

A great simplification is obtained if we assume that the playing frequencies are the resonance frequencies of the input impedance of the air column (i.e. the first one for the first register and the second one for the second register). This is a good approximation for low intensity (piano) playing levels, but not necessary for high intensities (forte), where other impedance peaks play a role. The outline of this paper is as follows: the second section recalls some general features of the clarinet; the third section analyzes the length corrections and their variation with frequency for the main kinds of discontinuities encountered in wind instruments. The important rôle of the register hole can be deduced, as well as the possible corrections which can be explored. Finally section 4 and 5 answer to the above questions i) and ii), respectively. Some useful formulae are given in appendix A.

2 Generalities on the clarinet and on the work method

2.1 Description of a clarinet

The clarinet became of major importance in the orchestra at the end of the eighteenth century. Its history is younger than other modern woodwinds, such as flute, oboe or bassoon. This is probably due to its capital characteristic: when the instrumentalist overblows, the clarinet gives an interval of one twelfth (often called fifth) instead of one octave given by the other instruments. A consequence is the compass of 19 semi-tones for the first register, and the difficulty to provide a system of keys adapted to 10 fingers.

For a given length of the tube, several resonance frequencies of the resonator exist, thus several oscillation regimes can be obtained for a given fingering. We call register the set of tones obtained for the same regime: the *first register* involves the tones corresponding to the first mode (i.e. the first resonance frequency) of the resonator, the frequencies being noted f_1 , and the *second register* involves the tones corresponding to second mode, the frequencies being noted f_2 , and close to $3f_1$. The opening of the register hole allows to jump from a given tone of the first register to the corresponding tone

of the second register, one octave and a fifth above, the two first registers covering ideally more than 3 octaves. We notice that while this division into two registers seems evident to the scientist, the analysis of the characteristics of the tone colour is not so clear, and musicians divide this musical compass in four registers: chalumeau, throat, clarinet and extrem [9]. A fact is that for the fingerings corresponding to the higher tones of the first register, i.e. for fingerings from $f'\#$ to $a'\#$ ¹, the opening of the register hole is not sufficient in order to get the proper twelfth, and it is necessary to modify slightly the fingering in order to ensure the well tuning. Thus, in order to play the corresponding twelfths (i.e tones from $c'''\#$), the third resonance, corresponding to an interval of a 17th, is used, corresponding to that the physicist can call the third register (see figure ??). We will understand that an important reason of this fact lies in the imperfect action of the register hole.

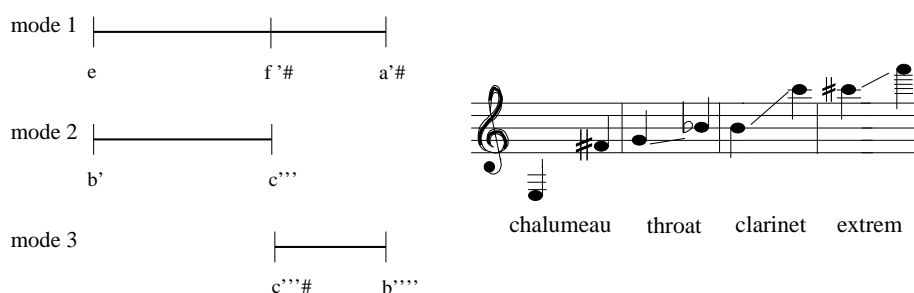


Fig. 1. *Compass and operative resonance frequency for a clarinet.*

The most commonly used clarinet is the clarinet in B flat, which sounds one tone below the written tone. The scope of the paper is limited to this instrument, using especially the geometrical parameters given by Nederveen [1]. Five parts can be distinguished on a clarinet : the mouthpiece, the barrel, the upper and lower joints and the bell. The deviation from a regular, cylindrical shape of the instrument bore (see figure 2), the existence of tone and register holes, the dispersion due to visco-thermal effects alter harmonicity of the resonance frequencies, with consequences discussed hereafter.

2.2 Sound production

The sound of the clarinet is produced by self-sustained oscillations. By studying these oscillations, it is possible to show that at weak level the playing frequencies are imposed by the zero values of the imaginary part of the input

¹ The notation system adopted in this paper is the same as that used by Baines [10].

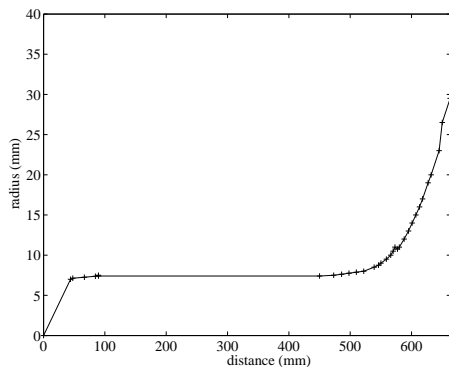


Fig. 2. *Radius of a clarinet as a function of the distance from the reed tip. Measurements by Nederveen [1].*

admittance of the resonator. Therefore they are very close to the frequencies of the maxima of the input impedance modulus. It can be said that the instrument is excited by a "weak reed", because the frequency is imposed by the resonator. Actually the reed has a small influence on the tuning, by two different ways: on one hand the volume velocity created by the reed movement is added to the one produced by the pressure difference across the reed opening; on the other hand the damping of the reed acts also on the playing frequency. It is classically shown that the two effects can be taken into account as corrections to the length of the instrument, almost independently of the frequency, i.e. of the played tone (see [1,7,11]). When the player blows stronger the playing frequency can slightly change, partly because of the inharmonicity of the resonances of the resonator and the damping and inertia of the reed. This question is intricate because of the influence of many parameters, but in what follows, the considered tuning corresponds to low levels, the goal being to achieve a satisfactory tuning at least for pianissimo levels. We notice that to achieve a proper harmonicity of the two first resonances is important for a note alone: the frequency can remain independent of the playing level and no intonation difficulty occurs when the two first resonances have similar magnitude.

For the present purpose, the problem of inharmonicity of the two first resonances is slightly different: the first resonance is when the register hole is closed and the second one is when the hole is open, in order to overblow with a correct tuning. The targeted interval is almost exactly a pure twelfth, i.e. a ratio of 3. Actually for a tempered scale, the ratio f_2/f_1 is not exactly 3, because the tempered intervals are different from the natural ones, except for the octave: the exact value is $2^{19/12} = 2.9966$. The relative difference is 0.11%, i.e. 2 cents (it is the difference between the tempered fifth and the harmonic one, called the "skhisma"). Nevertheless this difference is very small and for simplicity in what follows the ratio 3 is considered. We notice that 2 cents are inaudible, and anyway the instrumentalist, by adjusting his embouchure, has a certain "liberty extent", certainly larger than that, especially for higher

notes.

As a conclusion, an important work of the maker is to make identical the resonance frequencies of the air column and the desired playing frequencies, ensuring that the relationship $f_2 \simeq 3f_1$ is satisfied. If this goal is imperfectly reached, the maker can chose to favour one or another register, by adapting the location of the tone hole: in practice it seems that it is the first register which is "sacrificed", as shown by the tuning diagram presented in the following subsection. Finally because of the residual rôle of the open register hole it is necessary to find a compromise between a correct tuning of the twelfths when using the register hole and harmonicity of the two first resonances for the notes of the first register, wanted for a correct tuning at fortissimo levels. The first requirement is probably the most important, and the present paper focused on it.

2.3 Tuning diagram

Figure 2 shows an example of tuning and inharmonicity diagrams obtained with an instrument Buffet Crampon Prestige, played by Pierre Mallet. Measurements were done playing each twelfth in an ascending chromatic scale, softly. We present results for the part of the scale where musicians use the same fingering to play the fundamental and the associated twelfth, the only difference being in the register key: closed for producing the first register tone and open for producing the associated twelfth.

The first thing to note about the tuning is the intermediate compromise made

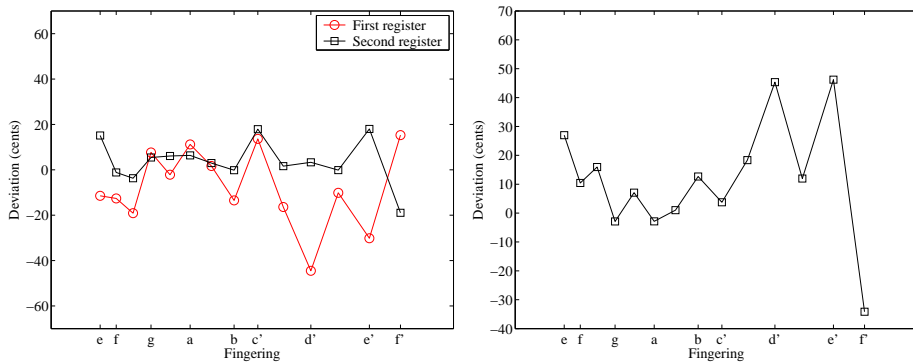


Fig. 3. Deviation of the tempered scale expressed in cent for the playing frequency and measured deviations of the second resonance frequency for a clarinet Buffet Crampon Prestige.

by the maker in order to get an accurate twelfth between tones in the low and second register. The tuning of the first register tones appears to be sacrificed in order to improve the register jump. However, the diagram of inharmonicity between the undisturbed first and the shifted second resonance frequencies shows that the register jump is not so perfect across the entire scale. It ap-

pears clearly that the register hole opening pulls mode 2 upward in frequency at both ends of the scale. The associated twelfths are widened, but thanks to the "liberty extent" of the musician, only the first lower twelfths, which are too high an amount to about 30 cents, are musically annoying. Even if this tuning default appears to be more or less important according to the player embouchure and the dynamic level (piano, mezzoforte or fortissimo), it has also been reported by many authors [12,13,7] who used an artificial mouth allowing to maintain a constant embouchure. As a consequence, the origin of this problem appears to lie in the physic of the clarinet only. Then, makers are presented with a dilemma: either they give clarinets a good low register or a perfect *clarinet* register. Finally, it should be mentioned that even if each instrument has its own harmonic structure, this problem of the register hole effect is a general feature for a lot of clarinets.

2.4 Work hypotheses

A perfectly cylindrical tube is considered as a starting point, with effective length ℓ_{eff} , the ideal playing frequencies being given by the following approximations:

$$f_1 = \frac{c}{4\ell_{eff}} \quad ; \quad f_2 = \frac{3c}{4\ell_{eff}} \quad ,$$

where c is the sound speed in air. The effective length ℓ_{eff} is the length between the effective input, defined by taking into account the length corrections due to reed and mouthpiece, and the effective output, located at the distance $0.6R$ of the end of the tube, taking into account the radiation of the tube. When a tone hole is open, a first approximation is to consider that the tube is cut at the centre of the hole, the corresponding length correction being now well known [14] and almost independent of the frequency. The goal is to achieve harmonicity of the two frequencies f_1 and f_2 , for the effective lengths corresponding to each hole, i.e. between two extreme values of ℓ_{eff} , written ℓ_{min} and ℓ_{max} . The precise intermediate values of the length is without importance for the present objective. As a consequence, the absolute effect of small discontinuities, e.g. cavities or tapers, is not important as well, only their relative effect between these two registers being important. It is therefore convenient to consider a continuous variation of the length between the two extreme values. After the optimization of the intervals between the two registers between these two values, it will be possible to find the precise location of the tone holes achieving the desired scale. On this point of view, the method is similar to that used for the design of micro-interval systems provided at the input of an instrument [6].

How do we achieve harmonicity of the frequency f_1 of the tone of the first register and the frequency f_2 of the tone obtained when the register hole is open, for all lengths between ℓ_{\min} and ℓ_{\max} ? First we will find an optimal location for the register hole, located upstream of ℓ_{\min} , secondly we will study if a correction system can compensate the residual defaults of this register hole. The dimensions of the hole are considered to be optimized by the practice of makers, and therefore are regarded as imposed. It is actually a difficult question, related to nonlinear effects as well as water effects, the important fact being that the linear behaviour, at low level, is well known. We do not take into account the use of the register hole as a B flat tone hole. The correction systems are sought in order to be without manipulation by the instrumentalist, and therefore to act on both the first and the second registers, contrary to the register hole itself.

All calculations are done ignoring the different kinds of dissipation (due to visco-thermal effects in the boundary layers, to radiation, etc...): dissipation is known to have a negligible effect on the resonance frequencies. In Appendix B the effect of the resistance of a small hole is discussed, and even if nonlinear effects are taken into account, it is shown to be negligible. A consequence is the systematic use of purely imaginary impedances. Perturbation to the planar mode theory is classically taken into account using lumped elements representing the effects of higher order, evanescent modes of the tubes.

For the study of any perturbation, the used method, very simple and intuitive, is to convert the perturbation effect into length correction.

3 Length correction and inharmonicity produced by the insertion of a discontinuity

3.1 Length corrections: definition and general theoretical formulation

Modifying an air column alters the resonance frequencies and their relationship from the original values provided by the non-perturbed system. This effect may be expressed conveniently in terms of a length correction, noted $\Delta\ell$, that must be added to the length L of the perturbed system, in order to compute easily its eigenfrequencies as if it was not perturbed (see figure 4). The concept is ideally independent of the frequency, but can be extended when it is not the case. Using an exact formulation of $\Delta\ell$ is possible, the prediction of the resonance frequencies being possible by an exact, iterative procedure. In the present study, we prefer looking for an approximation of the length

corrections to the first order. This gives a sufficiently accurate determination of the resonance frequencies and the advantage is that the length corrections associated with different perturbations can be simply added. The method used by Meynial and Kergomard [6] to translate a complete register from a given value was based on calculation of length corrections looking backward from the open end to the top-end (the mouthpiece) of the instrument where the impedance must be infinite for self-sustained oscillations. The interest lies in the fact that the expression of the length corrections depends on the fingering only via the playing frequency.

Considering the problem of a tone hole, or any discontinuity in parallel, branched on a straight cylindrical tube (see figure 4), the following equation can be written at the location of the discontinuity ($x = \ell$)

$$Y_{up} = Y + Y_{down} , \quad (1)$$

where

- Y_{up} and Y_{down} are the main tube admittances upstream and downstream the discontinuity, respectively (throughout the paper the admittance is defined as a ratio of an acoustic volume velocity and an acoustic pressure);
- Y is the admittance of the inserted discontinuity.

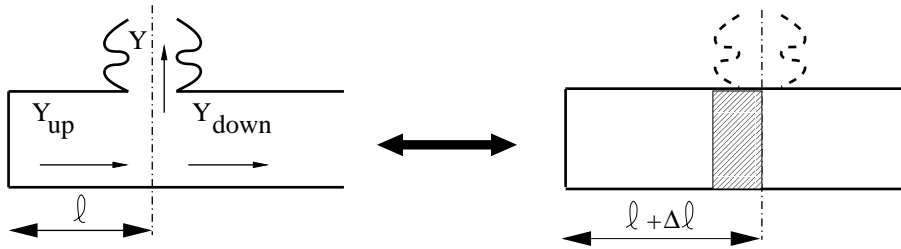


Fig. 4. *Small perturbation on a straight cylindrical tube and equivalent $(\ell + \Delta\ell)$ non-perturbed system: the two systems have the same eigenfrequencies.*

Looking backward from $x = \ell$ to $x = 0$ as mentioned earlier and writing $Y_{down} = -jY_c \tan k(\ell + \Delta\ell)$, equation (1) becomes:

$$-jY_c \tan k\ell = Y - jY_c \tan k(\ell + \Delta\ell) , \quad (2)$$

where

- $\Delta\ell$ is the length correction;
- $j = \sqrt{-1}$;
- $Y_c = S/\rho c$ is the characteristic admittance of the main tube (ρ is the density of air, c is the speed of sound and S the cross section area of the main tube);
- $k = 2\pi f/c$ is the wavenumber.

After some algebra, the following result is deduced:

$$k\Delta\ell = \text{Arctan} \left(\frac{-j\frac{Y}{Y_c} \cos^2 k\ell}{1 - \frac{1}{2}j\frac{Y}{Y_c} \sin 2k\ell} \right) , \quad (3)$$

which can be approximated in the limit of small Y/Y_c by

$$k\Delta\ell \simeq -j\frac{Y}{Y_c} \cos^2 k\ell . \quad (4)$$

Therefore, the discontinuity is described by a quantity called "length correction" which depends on the pitch of the played tone, the geometry and the location of the discontinuity. It can be reminded that the formula of the first order perturbation (see equation (4)) can be directly deduced from the well known Rayleigh variational principle. Otherwise the previous formula is useful for different kinds of discontinuities, but for some particular cases, other formulae need to be derived, as it will be seen below.

Remark 1 *When the discontinuity is in series, the admittances Y and Y_c need to be replaced by the impedances Z and Z_c , respectively.*

3.2 Inharmonicity of the resonance frequencies

Inharmonicity can be defined as the relative difference between the resonance frequency f_n and n times the first resonance f_1 , as follows:

$$IH = \frac{f_n - nf_1}{nf_1} = \frac{\ell_{eff} + \Delta\ell_1}{\ell_{eff} + \Delta\ell_n} - 1 = -\frac{\Delta\ell_n - \Delta\ell_1}{\ell_{eff}} + o\left(\frac{\Delta\ell}{\ell_{eff}}\right) , \quad (5)$$

where ℓ_{eff} is the acoustic length of the non-perturbed system and $\Delta\ell_n$ is the length correction associated to the n th resonance.

In the case $IH > 0$ the basic intervals are enlarged. On the contrary, when $IH < 0$ the intervals are reduced. In the present paper, results are given in *cent*. The cent is the micro-interval equal to one hundredth of a tempered semi-tone:

$$1 \text{ cent} = 5,78.10^{-4} \quad \text{and} \quad (IH)_{cents} = \frac{IH}{5,78.10^{-4}} .$$

A fact to be mentioned is that the smallest frequency deviation perceptible by human hear is estimated to be 4 cents corresponding to $IH \simeq 0.25\%$ [15].

open side hole	
closed-hole	
abrupt change in cross section in the upper part of the instrument	
localized enlargement/contraction	
truncated cone	

Table 1

Acoustic basic systems and associated length corrections.

3.3 Length corrections and inharmonicity formulae for acoustic basic systems

This section is devoted to the analysis of inharmonicity associated with simple acoustical systems depicted in table 1. The starting point is the calculation of first order length corrections. Then analytical expressions of inharmonicity between the first and second resonance frequencies are derived for each case. For exact formulations of length corrections, the reader is invited to see appendix A.

3.3.1 Open or closed hole

Since the size of a tone hole is much smaller than the wavelength in the frequency range of interest, the acoustic effect of the hole may be represented

by means of the common T-shaped circuit shown in figure 5. The series and shunt impedances Z_a and Z_s correspond to the antisymmetrical and symmetrical field in the chimney, respectively [16]. As mentioned earlier, energy losses are ignored in this study so that the real parts of the tone hole impedances are equal to zero. Under these assumptions, the series impedance Z_a appears to be a negative inertance. Concerning the shunt impedance Z_s , it is written as

$$Z_s = Z_{h0} + Z_{s0} ,$$

where Z_{h0} is the planar mode impedance at the input of the side branch and Z_{s0} is an inertance due to the higher order modes in the tube and the chimney. The impedance Z_{h0} is inductive for the case of an open tube, or capacitive for the case of a closed tube. Moreover, due to the smallness of the correction as-

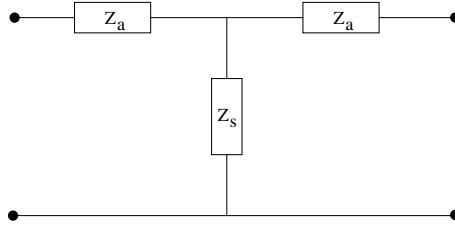


Fig. 5. *Equivalent electrical circuit for a branched tube: the series and shunt impedances Z_a and Z_s correspond to the antisymmetrical and symmetrical field in the chimney.*

sociated with the series impedances [17], only the impedance Z_s is considered.

Open hole Even if the acoustical rôle of a register hole and a tone hole are strongly different, their modelling uses the same formalism. However, in the limit of zero frequency, it is important to note that the effect of an open hole increases to infinity (see equation (3)): the main tube behaves as if it was cut at the location of the hole. For the resonance frequencies, the hole has a finite effect which depends on two quantities, the ratio Y/Y_c and the location ℓ of the hole. Assuming $Y/Y_c \ll 1$ et $k\ell \ll 1$, equation (4) is valid: this is the case of a register hole.

For tone holes, the ratio Y/Y_c becomes large and equation (4) cannot be used. As a consequence, in order for the length correction to be small, its calculation differs from the general case: the considered non-perturbed tube is the tube cut at the location of the hole instead of the tube with total length. For an open hole of height h , radius r at distance ℓ_d from the open end, the length correction is no longer obtained from equation (3), but from the following equation:

$$-jY_c \cotan k\Delta\ell = Y_h - jY_c \cotan k\ell_d , \quad (6)$$

where Y_h is the open hole admittance.

By rewriting equation (6) and using Taylor's formula to the third order in $k\ell_d$

and to the first order in $k\Delta\ell$, the following result is obtained

$$\Delta\ell \simeq \frac{1}{q + \frac{1}{\ell_d(1+(k\ell_d)^2/3)}} , \quad (7)$$

where $q = r^2/R^2h'$ is relative to the geometry of the tone hole.

In the lower frequency limit, the length correction becomes a constant value given by $\Delta\ell = \ell_d\ell_{hole}/(\ell_d + \ell_{hole})$, where $\ell_{hole} = (S/s)^2 h'$. When frequency increases, the length correction increases too so that the resonance frequencies are lowered.

Concerning the f_2/f_1 -mode frequency ratios, equation (7) leads to an expression for inharmonicity in terms of the tone hole geometry q and its location ℓ_d given by:

$$IH \simeq -\frac{16}{3\pi} \frac{k_1^3 \ell_d^3}{(1 + q\ell_d)^2} , \quad (8)$$

where k_1 is the wavenumber of the fundamental frequency of the played tone. Finally, inharmonicity associated with an open hole is negative and decreases with the frequency as shown on figure 6.

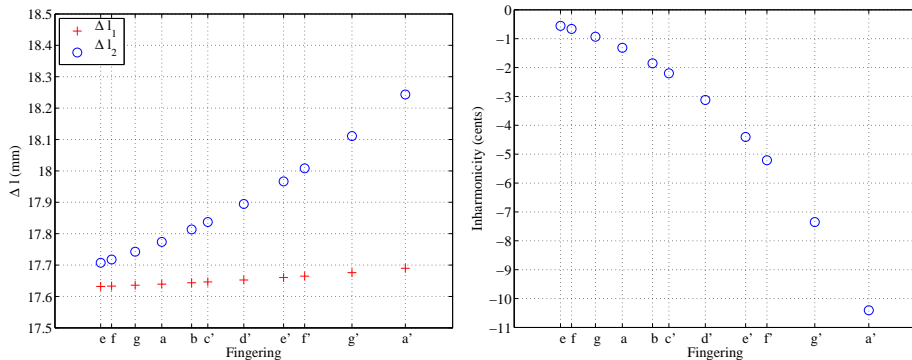


Fig. 6. Length corrections (left) and corresponding inharmonicity (right) for a tone hole of radius $r = 3$ mm, height $h = 3$ mm, located at a distance $\ell_d = 30$ mm from the open end.

Register hole Since the tone hole dimensions are small, the shunt impedance for an open side chimney appears to be an acoustic inductance (or acoustic mass) given by $L = \rho h'/s$ where $h' = h + h_r + h_m$ is the height of the chimney including the length corrections associated with radiation, h_r , and with the matching volume h_m . Then, the admittance Y to substitute in equation (4) is given by $Y = 1/jL\omega$. Since $Y/Y_c \ll 1$ is valid, the length correction becomes:

$$k\Delta\ell \simeq -\frac{sl}{Sh'} \frac{\cos^2 kl}{kl} , \quad (9)$$

where

- s is the cross section area of the register hole;
- ℓ is the distance of the hole from the clarinet reed tip.

Equation (9) states that a small open side hole entails systematically a negative length correction, i.e. an increase in the resonance frequencies (see figure 7). It also shows that $\Delta\ell$ decreases as the inverse of the square of frequency.

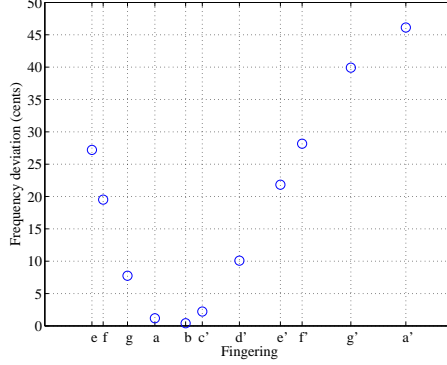


Fig. 7. Deviation of the second resonance frequency due to a register hole of radius $r = 1.75$ mm, height $h = 12.5$ mm, located a distance $\ell = 135$ mm down the reed tip. For the acoustic length corresponding to fingering b , the register hole does not alter the second vibration mode: the register hole is located at one third the effective length for this fingering.

Using equation (9) and noting that the register hole has no effect on tones of the first register, for which it is closed, the frequency shift of the second resonance frequency due to the register hole opening can be derived as follows:

$$\begin{aligned}
 IH &\simeq -\frac{\Delta\ell_2}{\ell_{eff}}, \\
 &\simeq \frac{2}{3\pi} \frac{s}{Sh'} \frac{\cos^2 k_2 \ell}{k_2}, \tag{10}
 \end{aligned}$$

where $k_2 = 3k_1$ is the wavenumber of the played tone. This expression points out that the frequency deviation depends on 2 parameters, s/Sh' and ℓ , as for a tone hole. Moreover, the register hole generates positive inharmonicity by pulling the second vibration mode upward in frequency at both ends of the register scale and has no effect at a pressure node (see figure 7).

Closed hole The input impedance of the planar mode in a closed hole is given by $Z_s = 1/jC\omega$, where $C = v/\rho c^2$ is the acoustic compliance due to

volume v , equal to the closed-hole volume. Using equation (3) and noting $X = v/S\ell$, the length correction is given by:

$$\Delta\ell \simeq X\ell \cos^2 k\ell . \quad (11)$$

This expression shows that the effect of a closed hole inserted on a cylindrical tube is described by a positive length correction entailing a decrease in the resonance frequencies. In addition, since $k\ell \ll 1$, the effect of a cavity is proportional to the ratio of the inserted volume to the volume of air included between the reed tip and the closed hole. At low frequencies, equation (11) also states that the virtual volume of air corresponding to the flow induced by the reed movement introduced by Nederveen, does not produce inharmonicity.

With the use of equation (11), the inharmonicity between first and second resonance frequencies is derived as follows:

$$\begin{aligned} \Delta\ell_1 - \Delta\ell_3 &= -\frac{v}{S}(\cos^2 3k_1\ell - \cos^2 k_1\ell) \\ &= \frac{2v}{S} \sin^2 2k_1\ell \cos 2k_1\ell , \end{aligned}$$

so that:

$$IH \simeq \frac{4}{\pi} \frac{v}{S} k_1 \sin^2 2k_1\ell \cos 2k_1\ell . \quad (12)$$

The first thing to notice about equation (12) is that the f_2/f_1 ratio depends on 2 parameters, the location ℓ and the ratio v/S . Moreover, as a consequence of the term $\cos 2k_1\ell$, either negative or positive inharmonicity is associated with a closed hole (see figure 8) the critical point being $k_1\ell = \pi/4$. An additional fact to be mentioned is that since the magnitude is proportional to the wavenumber k_1 , inharmonicity increases with the fundamental frequency of the played tone.

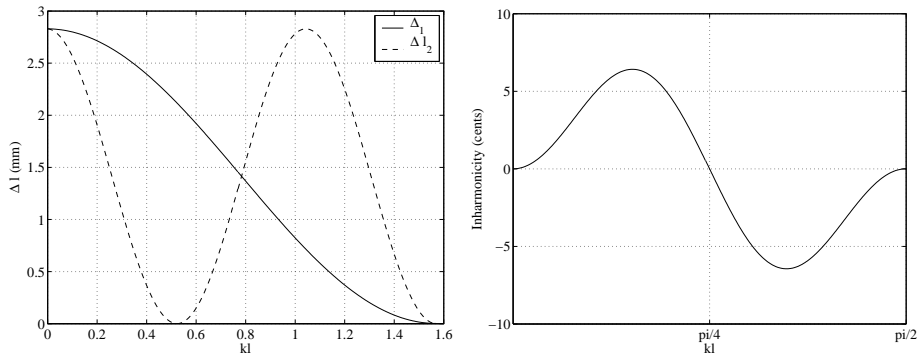


Fig. 8. Length corrections (left) and corresponding inharmonicity (right) for a closed hole whose volume $v = 0.6 \text{ cm}^3$ as a function of $k_1\ell$.

3.3.2 Abrupt change in cross section in the upper part of the instrument

Calculating the length correction to the main tube, it can be written at the discontinuity ($x = \ell$):

$$-jY'_c \tan k\ell = -jY_c \tan k(\ell + \Delta\ell) ,$$

where $Y'_c = S'/\rho c$ and $Y_c = S/\rho c$ are the characteristic admittances of the tube located upstream and downstream the discontinuity, respectively. In the limit of $\alpha = S'/S$ approaching unity, it follows:

$$k\Delta\ell \simeq \frac{1}{2}(\alpha - 1) \sin 2k\ell . \quad (13)$$

Under this assumption, $\Delta\ell$ can be added either to the tube above or below the discontinuity.

Inharmonicity associated with a change in cross section is derived from equation (13) and results in

$$IH \simeq \frac{4(\alpha - 1)}{3\pi} \sin^3 2k_1\ell . \quad (14)$$

In this equation, the term $\sin 2k_1\ell$ still remains positive since $k\ell \in [0, \pi/2]$. As a consequence, the kind of inharmonicity associated with the discontinuity depends only on the value of α as it is shown on figure 9. As expected, the behaviour of an enlargement is similar to that of a closed cavity.

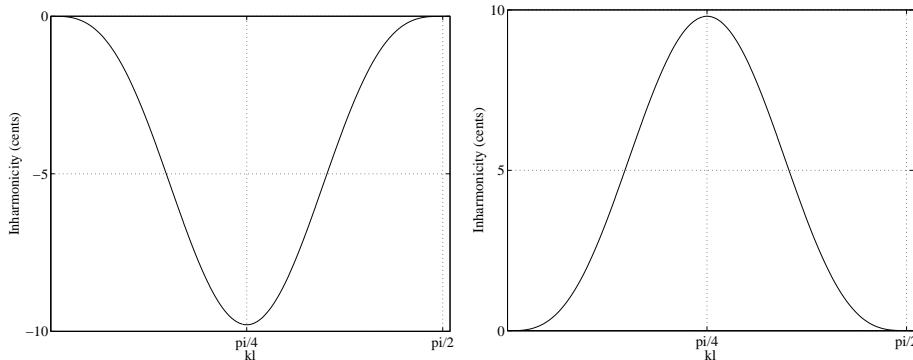


Fig. 9. *Inharmonicity between the first and second resonance frequencies due to a abrupt change in cross section area as a function of $k_1\ell$: $S' = 0.98 S$ (left) and $S' = 1.02 S$ (right).*

3.3.3 Localized enlargement/contraction

Consider a localized enlargement (or contraction) of length ℓ' located at distance ℓ from reed tip in a cylindrical air column and let $\alpha = S'/S$ be the ratio

of the cross section area of the enlargement (or contraction) over the one of the main tube respectively. Assuming the discontinuity in cross section α close to the unity, the change in resonance frequencies can be expressed by means of length correction with the expression

$$k\Delta\ell = (1 - \alpha) \sin k\ell' \cos k(2\ell + \ell') . \quad (15)$$

When $\ell = 0$, one can note that equation (15) gives the expression obtained for an abrupt change in cross section area in the case $S' < S$ (see equation (13)).

Concerning the relationship between first and second resonance frequencies, equation (15) leads to the following expression for the inharmonicity:

$$IH \simeq \frac{4}{3\pi}(\alpha - 1)(\sin^3 2k_1\ell - \sin^3 2k_1(\ell + \ell')) , \quad (16)$$

which states that the two diameter discontinuities located at points ℓ and $\ell + \ell'$ entail either positive or negative inharmonicity as shown on figure 10.

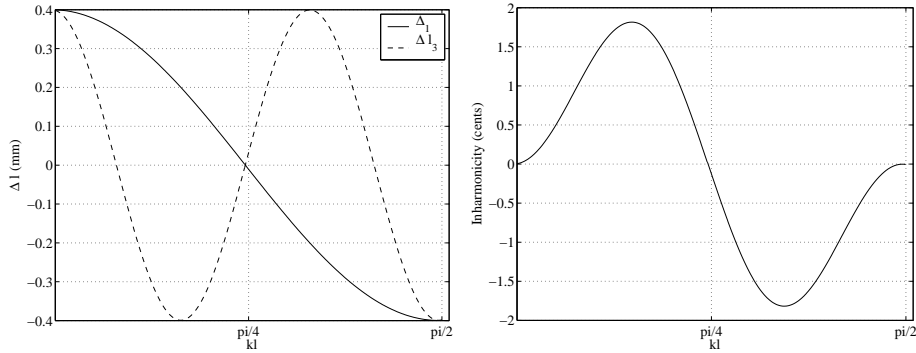


Fig. 10. Length corrections (left) and corresponding inharmonicity (right) for a localized enlargement as a function of $k_1\ell$ ($R = 7.5$ mm, $\ell' = 10$ mm, $S' = 1.04 S$).

3.3.4 Change in taper close to the tube input

The acoustical behaviour of a change in taper over a length ℓ_c can be represented by means of an equivalent electrical circuit (see figure 11) including two inductances of opposite sign and the elements of a cylindrical tube of length ℓ_c [18]. Writing $X_1 = \ell/x_1$ and $X_2 = (\ell + \ell_c)/x_2$, the length correction calculated to first order is given by

$$k\Delta\ell \simeq X_2 \frac{\cos^2 k(\ell + \ell_c)}{k(\ell + \ell_c)} - X_1 \frac{\cos^2 k\ell}{k\ell} . \quad (17)$$

Equation (17) is valid either for a positive or negative taper change, the difference being in the sign of the x_i . The x_i quantities are positive for a positive

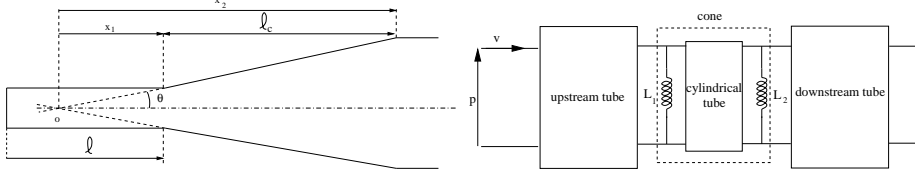


Fig. 11. *Equivalent electrical circuit for truncated cone involving a pair of inertances of opposite sign and a non tapered tube.*

cone and negative for a negative cone. With equations (9) and (17), we notice that a single taper change is equivalent to an open side hole.

In order to evaluate inharmonicity generated by a small truncated cone, it is convenient to reformulate the expression of the length correction in terms of two control parameters by rewriting equation (17). Since the approximations $kl_c \ll 1$ and $l_c/x_1 \ll 1$ still remain, it is possible to write:

$$\cos^2 k(\ell + \ell_c) = \cos^2 k\ell - k(\ell + \ell_c) \sin 2k\ell + o(k\ell_c),$$

and

$$\frac{1}{kx_2} = \frac{1}{kx_1(1 + \ell_c/x_1)} \simeq \frac{1}{kx_1}.$$

Therefore a simplified expression for the length correction is derived

$$k\Delta\ell \simeq -\frac{\ell_c}{x_1} \sin 2k\ell, \quad (18)$$

where ℓ_c/x_1 and ℓ are the two parameters. Under these conditions and with the use of equation (18), inharmonicity is given by:

$$IH \simeq -\frac{8}{3\pi} \frac{\ell_c}{x_1} \sin^3 2k_1\ell. \quad (19)$$

Looking at figure 12 and equation (18), one can notice the equivalence between a positive truncated cone and an abrupt change in cross section for the case $S' < S$.

3.4 Inharmonicity associated with radiation and dispersion

Because of dispersion due to visco-thermal effects, the eigenfrequencies of the cylindrical air column cannot be exactly harmonically related. Taking into account dispersion leads to write the speed of sound with respect to frequency as follows

$$c = c_0 \left(1 - \frac{1}{R\sqrt{2k}} (\sqrt{\ell_v} + (\gamma - 1)\sqrt{\ell_h}) \right), \quad (20)$$

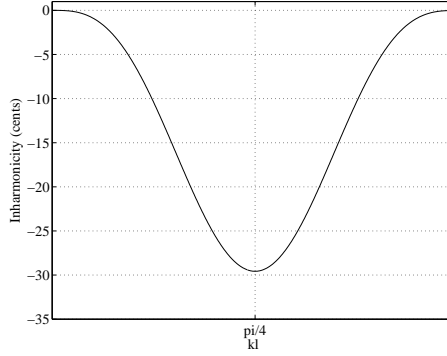


Fig. 12. *Inharmonicity between first and second resonance frequencies for a positive troncated cone as a function of $k_1 l$ ($R = 7.5$ mm, $\theta = 1.7^\circ$, $\ell_c = 5$ mm).*

where c_0 is the speed of sound into infinite space, ℓ_v and ℓ_h are the viscous and thermal characteristic lengths, R is the tube radius, $\gamma = C_p/C_v$ and $k = \omega/c_0$ is the unperturbed wavenumber of the played tone. Equation (20) implies that dispersion entails positive inharmonicity which can be evaluated by

$$IH = \frac{\Gamma_1 - \Gamma_3}{1 - \Gamma_1}, \quad (21)$$

where $\Gamma_n = \frac{1}{R\sqrt{2k_n}}(\sqrt{\ell_v} + (\gamma - 1)\sqrt{\ell_h})$ is the dispersion factor associated to the n th eigenfrequencies. As shown in figure 13, inharmonicity decreases as the inverse of the square root of the playing frequency.

Finally, since radiation of wind instruments depends on frequency, radiation is also a cause of inharmonicity. The radiation length correction given by Causse et al [19] in the case of unflanged circular pipe is

$$k\Delta\ell = \text{Arctan} \left(0.6133kR - 0.036(kR)^3 + 0.034(kR)^3 \ln kR - 0.0187(kR)^5 \right). \quad (22)$$

As a consequence, inharmonicity appears to be positive and very small (see figure 13).

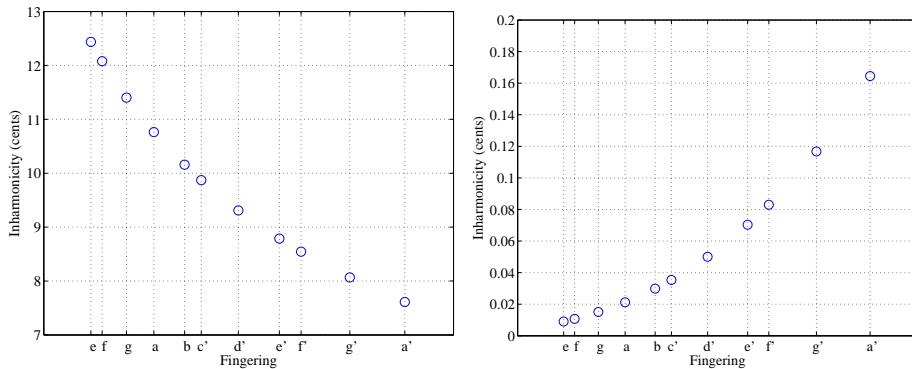


Fig. 13. *Inharmonicity associated with visco-thermal dispersion (left) and radiation (right) for a cylindrical tube of radius $R = 7.5$ mm.*

Basic perturbation	Sign of IH	Effective parameters
open side hole	< 0	s/Sh' and ℓ_d
closed-hole	$k\ell < \pi/4 \Rightarrow IH > 0$ $k\ell > \pi/4 \Rightarrow IH < 0$	v/S and ℓ
abrupt change in cross section in the upper part of the instrument	$S' > S \Rightarrow IH > 0$ $S' < S \Rightarrow IH < 0$	S'/S and ℓ
localized enlargement/contraction	> 0 or < 0	S'/S and ℓ
diverging troncated cone	< 0	ℓ/x_1 and ℓ
converging troncated cone	> 0	ℓ/x_1 and ℓ
register hole	> 0	s/Sh' and ℓ

Table 2

Inharmonicity associated with basic acoustical perturbations close to the tube input.

Remark 2 *The influence of the temperature on the inharmonicity has not been investigated in this study whereas an axial temperature drop has previously been reported [20]. Assuming the perturbation being small, the effect of this gradient may be evaluated by means of a length correction in the limit that the expression of the temperature profile in the tube is well known.*

3.5 Conclusion

The previous results concerning inharmonicity are taken up again in table 2 for clarity. This table appears to be very useful to predict the effect on the two first resonance frequencies of small change. In particular, it allows to improve the harmonic structure of a clarinet and for our purpose, it may give clue to understand the origin of the clarinet tuning shortcoming.

Figure 14 illustrates inharmonicity associated with the main kinds of perturbations encountered in a simplified clarinet except for the flaring horn at the open end. The instrument is considered perfectly cylindrical with 18 tone holes of fixed size to play a complete register and a register hole in order to achieve tones of the second register. We focus our attention on the effect associated with the set of closed or open tone holes, the register hole and dispersion. Examination of inharmonicity due to closed side holes shows the tendency to produce negative inharmonicity across the entire register: the mode ratios fall between -25 and -20 cents flat for tones in the middle of the register and

increase to zero for tones at both ends.

As expected, open holes also generate negative inharmonicity but the effect becomes significant for the highest tones only.

Concerning the effect of dispersion, positive inharmonicity of an amount to about +10 cents accross the entire register is produced.

As a conclusion, when the register hole effect is ignored, the f_2/f_1 -mode fre-

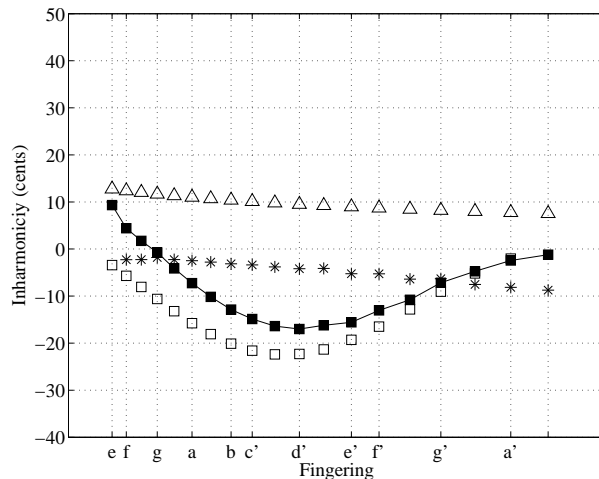


Fig. 14. Predicted inharmonicity associated with : dispersion (\triangle), closed holes (\square), open holes (\star). Black squares (\blacksquare) represent the sum of all these inharmonicitites.

quency ratios appears to be less than 3 for every fingerings except for the three lowest tones (see figure 14). An important fact to be mentioned is that calculations have been performed on a simplified clarinet without a flaring horn whose effect narrows the relationship between first and second resonance frequencies for lowest tones only. Then, considering a flaring horn would give negative inharmonicity for the lowest tones as it has been reported earlier [21] with measurements. Moreover, figure 15 shows the measured deviation of the second resonance frequency due to the register opening on a clarinet reported by Dalmont et al. [7]. Experimental results are in good agreement with the predicted values given by the only equation (10). It shows that the main explanation for the tuning problem of the clarinet lies in the register hole opening and that inharmonicities associated with open holes, closed holes and dispersion are apparently compensating troubles on a real clarinet. Finally, it leads to the conclusion that if positive inharmonicity is observed on a clarinet, this is mainly due to the effect of the register hole which tends to enlarge the f_2/f_1 ratios across the entire register. Then, in the following, the register hole is considered as the only inharmonicity source.

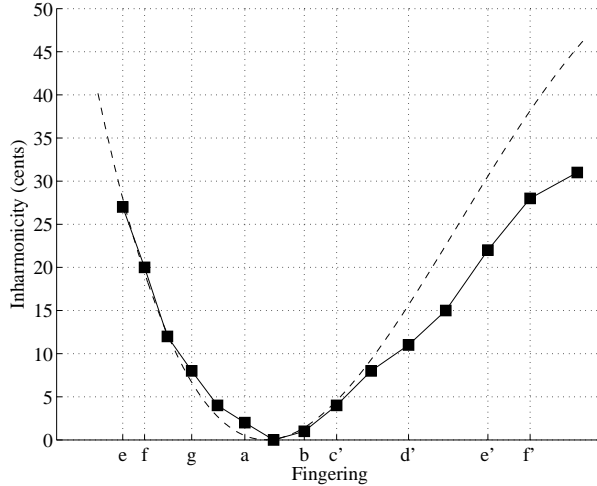


Fig. 15. Measured and theoretical deviations of the second resonance frequency due to the register hole opening for a clarinet (see equation (10) for the predicted values). The register hole location is 140 mm down the reed tip.

4 Optimization of the location of the register hole and tuning corrections : statement of the problem

4.1 Formulation of the optimization problem

Optimization techniques are used to find a set of design parameters $x^* = \{x_1^*, \dots, x_n^*\}$ that can in some way be defined as optimal. These parameters are obtained by minimizing (or maximizing) an objective function \mathcal{F} which may be subject to constraints and/or parameter boundaries.

In the present paper, we use optimization to give suggestions on how to improve the harmonic relationship between the first and the second resonance frequencies by first optimizing the location of the register hole and then by inserting perturbations in the higher part of the resonator. Thus, the design parameters are subjected to the following requirements:

- small changes location must be less than the distance between the reed tip and the first tone hole;
- dimensions of the acoustical systems must be reasonable for the realization;
- geometrical dimensions are positive.

Our optimization problem is formulated as follows:

$$\begin{cases} \min_{x \in \mathbb{R}^n} \mathcal{F}(x) & \text{where } \mathcal{F} \text{ is the objective function} \\ l_{bi} \leq x_i \leq u_{bi} & i=1, \dots, n \end{cases} \quad (23)$$

where x is the vector of design parameters ($x \in \mathbb{R}^n$) and l_{bi} and u_{bi} are the lower and upper parameter boundaries respectively, for the design parameter x_i .

4.2 Objective functions

4.2.1 Location of the register hole

The first aim of this paper is to find the optimum register hole placement on a cylindrical tube in order to play two registers accurately in tune with the same fingering. As previously shown, the presence of the register hole means that it is impossible to build a clarinet in such a way. Thus, the objective is to find the register hole location that entails the smallest frequency shift of the second resonance frequency for all fingerings. As a consequence, the first optimization is performed in order to play a 19-tone compass i.e a complete register. Then, a second optimization is performed restricting attention to fingerings from e to f' (above this fingering, clarinetists do not use the same fingering to play the fundamental and the associated twelfth).

Assuming that the height and the radius of the register hole are fixed according to the practice of instrument makers, the distance ℓ of the hole from the clarinet reed tip is the only optimization variable. The optimization problem deals with equation (10) which predicts theoretically the frequency deviation associated with the opening of the register hole. It can be formulated with one of the two following objectives:

- objective 1: to minimize the maximum of the frequency deviation;
- objective 2: to minimize the mean of the square of the derivative with respect to k of equation (10).

The first objective consists in limiting the most important tuning default of the instrument. This objective, which is very simple and intuitive for anyone who is interested in instrument design, can be written:

$$\textbf{Objective 1: } \mathcal{F}_1 = \sup \left(\frac{2}{3\pi} \frac{s}{Sh'} \frac{\cos^2 k_2 \ell}{k_2} \right). \quad (24)$$

An interesting point about objective 1 is that the solution can be approximated analytically as it is shown in section 5.1.1. On the contrary, once the maximum of the deviation is achieved for a fixed register hole location, the frequency deviations associated with other fingerings are not taken into account for the evaluation of the function.

Concerning objective 2, the objective function to minimize is written as

$$\textbf{Objective 2: } \mathcal{F}_2 = \frac{1}{2} \int_{k_{min}}^{k_{max}} \left[\frac{\partial}{\partial k_2} \left(\frac{2}{3\pi} \frac{s}{Sh'} \frac{\cos^2 k_2 \ell}{k_2} \right) \right]^2 dk_2, \quad (25)$$

where $\partial/\partial k$ is the derivative with respect to the wavenumber k , k_{min} and k_{max} being related to the playing frequency via the effective length by $k_{min} = 3\pi/2\ell_{max}$ and $k_{max} = 3\pi/2\ell_{min}$ respectively. Contrary to objective 1, this objective function takes into account the deviation associated with all fingerings for its evaluation. The global minimum of this objective function is achieved when the integrand is zero, i.e when the frequency deviation is constant for all fingerings. Nevertheless, noting that equation (10) is necessarily equal to zero for $k_2 \ell = 3\pi/2$, it appears that the register hole location given by this objective function is the one that minimizes too much important inharmonicity variations around the zero deviation and may lead to a more homogeneous register jump across the entire register.

4.2.2 Correction of the register hole effect

The second aim of this optimization is to give suggestions on how to compensate for the register hole effect. We are looking for geometrical dimensions of acoustical systems whose effects alter the resonance frequencies in order to restore the original f_2/f_1 -mode frequency ratio. Denoting IH_{reg} and IH_{pert} , inharmonicity associated with the register hole and the perturbation respectively, the objective function can be written as

$$\textbf{Objective 1: } \mathcal{F}_1 = \frac{1}{2} \int_{k_{min}}^{k_{max}} [IH_{reg} + IH_{pert}]^2 dk. \quad (26)$$

The minimum of equation (26) is reached when inharmonicity associated with the inserted perturbation is of the same magnitude as the register hole deviation and in the opposite direction.

Moreover, similarly to the study of the location of the register hole, a second objective function has also been investigated by minimizing the function defined as

$$\textbf{Objective 2: } \mathcal{F}_2 = \sup (|IH_{reg} + IH_{pert}|), \quad (27)$$

which deals with the maximum of the total inharmonicity.

5 Results and discussions

For the optimization, the radius of the main tube was taken as $R = 7.5 \text{ mm}$. The height of the register hole was 12.5 mm and its radius 1.65 mm . Concerning the upper bound for the acoustical system location, it has been set to

147 mm which is the first tone hole location according to measurements made by Nederveen. The other parameter boundaries for all perturbations have been chosen in order to make the realization possible .

5.1 Register hole location

5.1.1 Objective 1: maximum of the frequency deviation

As mentioned earlier, an optimum of objective 1 defined by equation (24) can be derived analytically. In order to achieve the solution, we rewrite the frequency shift as follows

$$IH = \frac{2}{3\pi} \frac{s}{Sh} \ell \frac{\cos^2 k_2 \ell}{k_2 \ell}, \quad (28)$$

and note that the magnitude of the deviation is proportional to ℓ and varies as the function $F(x) = \cos^2 x/x$ where $x = k_2 \ell$. We will first proof that the maximum of $F(x)$ does not depend on ℓ for a certain interval of values of $k_2 \ell$ and therefore the optimum value of IH is the minimum of ℓ on this interval.

The effective length of the tube varies between $\ell_{max} = L$, the total length of the instrument, and ℓ_{min} . If the compass of a register is one twelfth minus one semi-tone, the two extreme values of the effective length are related by $\ell_{min} = \ell_{max}/2^{18/12}$ which for simplicity we assume first to be $\ell_{min} = \ell_{max}/3$. The location of the register hole, defined by ℓ , is in the upper part of the instrument and satisfies $\ell < \ell_{min}$.

Then, for a given effective length, ℓ_{eff} lying between ℓ_{min} and ℓ_{max} , the wavenumber is defined as $k_2 = 3\pi/2\ell_{eff}$, and therefore the argument of $F(x)$ varies follows :

$$\frac{3\pi\ell}{2\ell_{max}} < k_2\ell < \frac{3\pi\ell}{2\ell_{min}}, \quad (29)$$

thus

$$\frac{3\pi}{2} \frac{\ell}{L} < k_2\ell < \frac{9\pi}{2} \frac{\ell}{L}. \quad (30)$$

Because the ratio ℓ/L is less than 1/3, the maximum value of interest for $x = k\ell$ is therefore $3\pi/2$ for which $F(x) = 0$. Figure 16 shows the variation of $F(x)$. From $\ell = L/3$, for the interval defined by inequalities (30), $F(x)$ varies between 0 and 0, with a maximum value equal to 0.327, for $x_0 = 2.975$. When ℓ decreases from $L/3$, figure 16 shows that the maximum values remains constant, equal to $F(x_0)$, except if ℓ becomes so small that the value of $F(x)$ for the minimum value of $k_2\ell$ reaches the value 0.327. This corresponds to $\ell^* = 0.205 L$. Below this value, the maximum of $F(x)$ grows rapidly.

As explained before, the optimum of IH for the objective 1 is finally the value

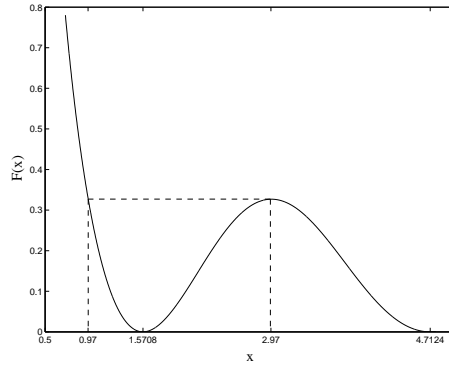


Fig. 16. Perturbation function $F(x) = \cos^2 x/x$. The same local maximum is reached for $x_0 = 2.97$ and $x^* = 0.97$.

$\ell^* = 0.205 L$. Nevertheless, this value is not so critical, because the objective function \mathcal{F}_1 for objective 1 is linear with ℓ , over the interval $[0.205 L, L/3]$, thus the variation is not strong. Moreover, it appears clearly that any other objective relative to the maximum of the frequency deviation will lead to a value within the interval, because the maximum is reached two times instead of one only.

Finally, taking into account that the interval $[\ell_{min}, \ell_{max}]$ corresponds actually to a slightly smaller interval than a twelfth i.e $2^{18/12}$, a numerical study leads to the optimal value for objective 1 slightly smaller than $\ell^* = 0.205 L$, i.e $\ell^* = 0.2041 L$. Figure 17 confirms that for objective 1, the function \mathcal{F}_1 increases linearly above this optimum value but increases strongly when ℓ decreases below it.

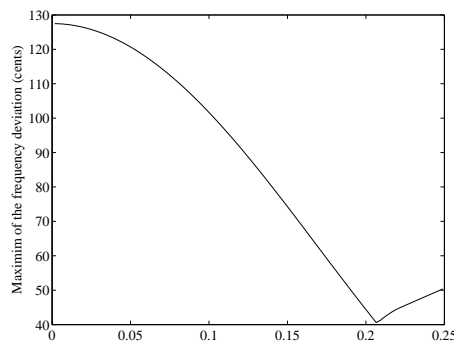


Fig. 17. Maximum of the frequency deviation as a function of ℓ/L . Above $\ell/L = 0.205$, the maximum value of the frequency shift increases linearly with the register hole location.

The register hole deviation obtained with this "optimal" position is plotted on figure 18. As expected, the maximum of deviation appears at the beginning of the register and is almost equal to a quarter tone. Moreover, this result shows that the register hole is located close to its ideal position to produce the c'/g'' transition: the register hole (i.e the pressure node) is located at one third the effective length for this fingering so that the frequency shift is zero.

5.1.2 Objective 2: mean of the square derivative of the deviation

The register hole location achieved with the minimization of (25) is found to be

$$\frac{\ell^*}{L} = 0.2481 ,$$

which is not only larger than the previous result but also larger than the one found with numerical data given by Nederveen ($\ell/L = 0.2292$). However, the frequency deviations obtained with this optimal position and the one found in literature [7,13] are very similar: the $a-e''$ transition is correct and the frequency shift at the beginning of the register is about 20 cents (see figure 18).

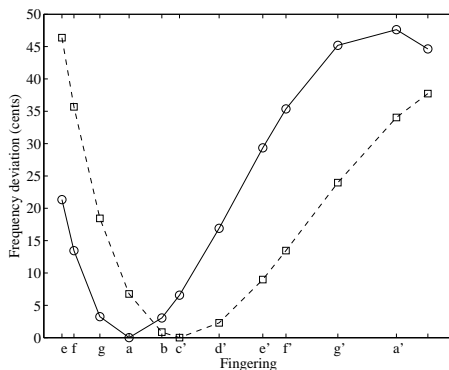


Fig. 18. *Frequency deviations associated with the register hole. The register hole is found to be located at $\ell^* = 120.45$ mm from the top end for objective 1 (\square) and $\ell^* = 146.4$ mm for objective 2 (\circ).*

5.1.3 Conclusion

Performing optimization by minimizing the objective functions defined by (24) and (25) gives different results and indicates that the final location of the register hole is the result of a compromise. However, as expected with equation (28), the magnitude of the maximum of deviation increases with the distance ℓ but very slightly for location larger than $\ell/L = 0.2041$. As a consequence, above this critical value, the location of the register hole is not so essential on this point of view. Finally, it appears that a register hole location far from the reed tip and above the critical value should be an interesting compromise in order to accurate the first twelfths of the register. Hence, we choose the location given by the optimization of objective 2 that is $\ell^* = 146.4$ mm.

Restricting now attention to fingerings from e to f' (and not to $a' \neq$) may be interesting in order to understand what instrument makers do. The result is that the optimization process converges to a position for the register hole between the two previous extreme values for the two objective functions. Moreover, in the case of the minimization of the maximum of the frequency

deviation, the distance from the clarinet reed tip is found to be $\ell = 133.7 \text{ mm}$. This result corresponds very well to the location used by Nederveen in its calculation [1] and to the one chosen by many makers. For that case, the twelfths at the bottom of the scale are still very large but the maximum of the frequency deviation, which is also obtained for the lowest tone, has noticeably fallen to 35 cents (see figure 19).

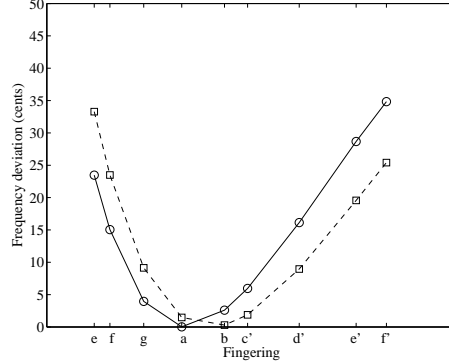


Fig. 19. Frequency deviation associated with the register hole in the case of a real clarinet *i.e* restricting our optimization to fingering from e to f' . Above f' , fingerings are modified to ensure the well tuning and the third resonance is used. Optimal positions are $\ell^* = 133.7 \text{ mm}$ (□) and $\ell^* = 145.0 \text{ mm}$ (○)

5.2 Adjustements of natural frequencies by means of small changes

5.2.1 Overview of the possibilities

As mentioned earlier, the objective of this part is to compensate for the frequency deviation due to the register hole by means of small changes. We are looking for a solution localized in the upper part of the instrument, *i.e* a solution acting for all fingerings. Looking at table 2, only three systems give inharmonicity in the right direction: an abrupt change in cross section area with $S' < S$, a change of conicity at the top end, and a localized enlargement or contraction. Concerning the case of a closed-hole, it has been shown (see figure 8) that both positive and negative inharmonicity can be generated. In order to produce negative inharmonicity, the condition $k_1 \ell > \pi/4$ which corresponds to playing frequencies larger than $f_1 = c/8\ell_{max}$ must be valid. Thus, the accuracy of the twelfths at the end of the second-register scale (from e' fingering) would be improved only.

5.2.2 Abrupt change in cross section area: $S' < S$

Fixing the radius of the tube downstream the discontinuity equal to 7.5 mm , the upstream tube radius R' and the location ℓ of the discontinuity are used

as optimization variables.

Performing optimization leads to the following results :

- objective 1 : $\ell^* = 73.9 \text{ mm}$ and $R'^* = 7.1 \text{ mm}$.
- objective 2 : $\ell^* = 69.0 \text{ mm}$ and $R'^* = 7.0 \text{ mm}$.

The deviation from the tempered scale of the first resonance frequencies and inharmonicity between the first and second resonance frequencies for each fingering are plotted on figure 20. Results show that even if "optimal" parameters are quite different, the shape and the magnitude of final inharmonicities obtained with the two objective functions are very similar. The f_2/f_1 frequency ratio is noticeably improved for all fingerings except for the twelfth associated with the lowest tone which is still 20-cent large. An interesting result is that the discontinuity in the diameter is found to be located near the barrel joint which is at a distance of $60 \pm 4 \text{ mm}$ from the closed-end on a clarinet. This result agrees with the practice of many makers: Nederveen noticed that change in cross section area are mostly found near the embouchure for reed instruments.

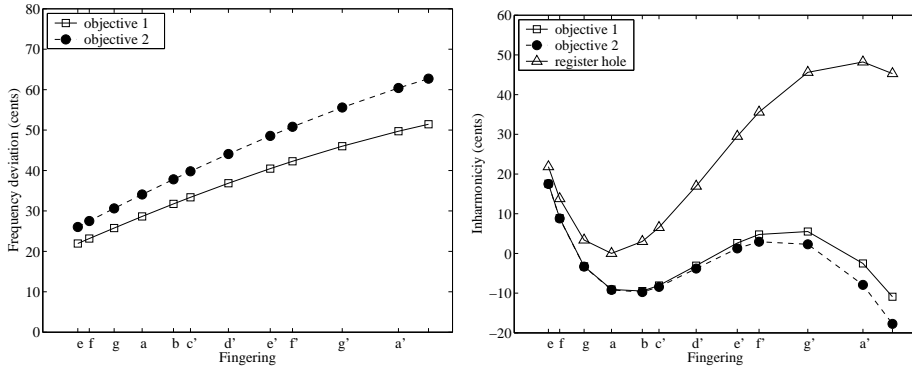


Fig. 20. Frequency deviation for the first register (left) and inharmonicity between first and second resonance frequencies : abrupt change in cross section area.

5.2.3 Localized enlargement/contraction

The optimization variables used in this case are the location ℓ of the perturbation, its length ℓ' and the radius R' as indicated on table 1. Performing optimization for the case of a localized enlargement leads to the conclusion that small enlargement does not improve the relationship between first and second resonance frequencies. Even if optimization process converges with both objective functions, final inharmonicities are not satisfactory. Figure 21 shows the tendency of the bottom notes to widen the f_2/f_1 ratio which is musically unacceptable. On the contrary, contracting the air column causes negative inharmonicity across the entire register and tends to improve the register jump (see figure 22). The optimization process converges to the following parameters

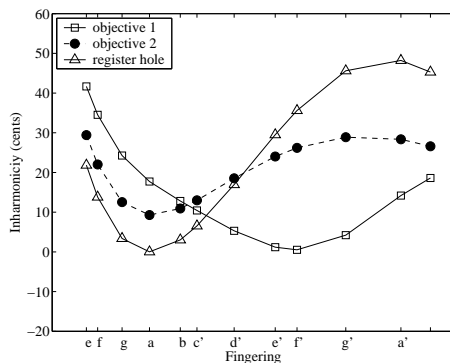


Fig. 21. Final inharmonicity between first and second resonance frequencies : localized enlargement.

:

$$\text{objective 1} \begin{cases} \ell^* = 60.0 \text{ mm} \\ \ell'^* = 22.41 \text{ mm} \\ R'^* = 7.0 \text{ mm} \end{cases} \quad \text{objective 2} \begin{cases} \ell^* = 62.74 \text{ mm} \\ \ell'^* = 36.76 \text{ mm} \\ R'^* = 7.17 \text{ mm} \end{cases}$$

When minimizing the objective function defined by equation (24), an interesting result is that the design parameters given by the optimization process are very similar to those of a clarinet barrel. As for an abrupt change in cross section, the contraction is located at a point near the barrel joint. In addition, the acoustic length of the contraction is the same as a common barrel ($\ell_{barrel} \sim 30 \text{ mm}$). When looking at what makers do, their practise varies considerably concerning the geometry of the barrel. However, some makers insert narrower barrel than the cylindrical portion of the entire instrument in order to get accurate twelfths in the upper part of the scale [22]. While the results given by the minimization of (26) generate negative inharmonicity (see figure 22), the optimization process converges to two lower boundaries ($R'^{prime*} = R'_{max}$ and $\ell^* = \ell_{min}$). When enlarging the domain where optimization variables are looking for, a new minimum (lower than the one found) is achieved. Thus, we do not consider this solution in the next. Finally, the mean value of inharmonicity is about -0.6 cents and the standard deviation of the mean value is equal to 11.9 cents.

5.2.4 Change in conicity at the input

The optimization variables used for the case of tapered perturbation (see table 1) are the radius of the upper end of our model R' , the top angle θ and

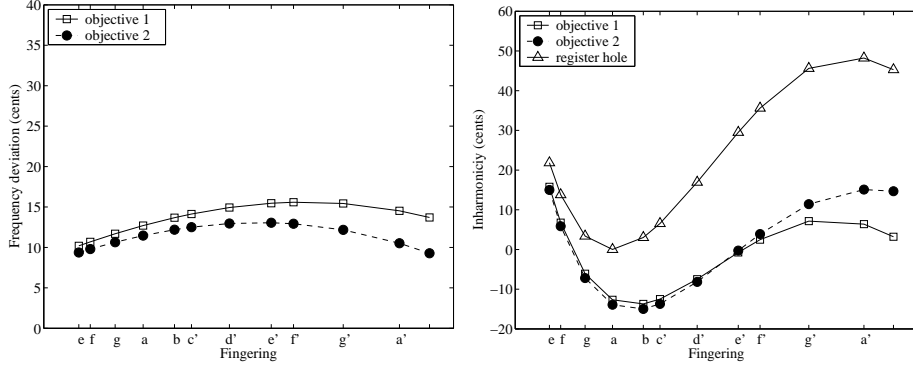


Fig. 22. Frequency deviation for the first register (left) and inharmonicity between first and second resonance frequencies : localized contraction.

the location point ℓ . For each objective function, the optimization process converges to the following design parameters :

$$\text{objective 1} \begin{cases} \ell^* = 73.05 \text{ mm} \\ \theta^* = 0.22^\circ \\ R^* = 7.37 \text{ mm} \end{cases} \quad \text{objective 2} \begin{cases} \ell^* = 98.5 \text{ mm} \\ \theta^* = 0.10^\circ \\ R^* = 7.46 \text{ mm} \end{cases}$$

The lengths of the tapered perturbations corresponding to objective 1 and objective 2 are about 44.5 mm and 46.5 mm which are both larger than the length of a classic barrel. Concerning the location point, it is near the barrel joint for objective 1 and slightly below this point for objective 2. An important thing to note is that the location achieved by the optimization process is always close to the barrel joint whatever small change is used.

Figure 23 shows final inharmonicities obtained with the minimization of the two objective functions. The f_2/f_1 frequency ratio is improved globally across the entire scale for both objective functions except for the first fingering which is deteriorated slightly for objective 1. As a consequence, only the design parameters given by the minimization of objective 2 are considered. The mean value of the total inharmonicity is found to be 3.6 cents and the standard deviation of the mean value is about 11 cents.

Finally, results indicate that the 20-cent deficiency still remains for the notes at the bottom of the scale whatever acoustic system is used in order to alter the resonance frequencies. Thus, no improvement can be done to compensate for the effect of the register hole except for tones at the end of the scale.

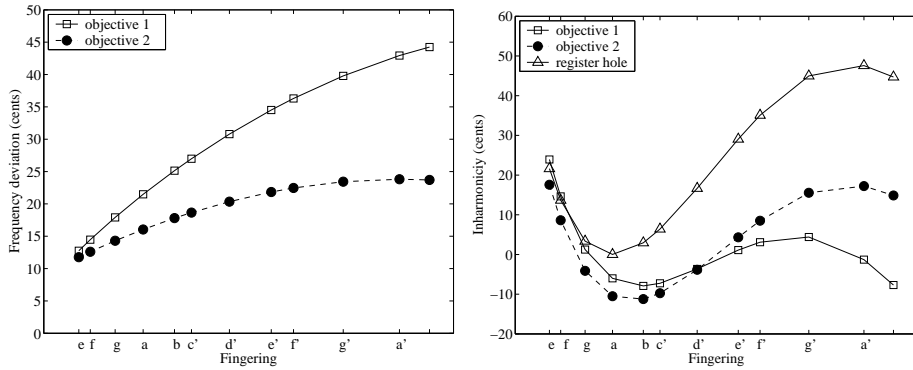


Fig. 23. *Frequency deviation for the first register (left) and inharmonicity between first and second resonance frequencies (right): positive truncated cone.*

6 Conclusion

As many deviations from the standard cylindrical tube can be observed on a woodwind instrument, the present paper aims to give some theoretical results concerning the effects of small changes of air column shape on the relationship between the two first resonance frequencies. This work has been focused on small perturbations which avoid flow problems on vibrating mechanical systems only.

Calculations of length corrections associated with small perturbations allow to analyze the origin of the tuning defaults of a wind instrument and can be used to investigate, for a clarinet, the tuning deficiency of the twelfths at the bottom of the scale.

In addition, the use of simple optimization techniques can also be interesting in order to improve instrument qualities. However, optimization is very sensitive to the definitions of the objectives, and, for our purpose, performing optimization on the accuracy of the twelfths does not impose a unique possible criterion. For instance, the use of optimization techniques with weight for the lowest tones is of course possible and would probably give other results. Instrument making is clearly an act of compromise at first.

The main results of the present work can be summarized as follows:

- designing an new instrument having a compass for the two registers of a twelfth with the same fingering except the register hole opening, is probably impossible without tuning deficiencies;
- if the aim is the improvement of a common clarinet, the use of contractions in the barrel, often used by makers, is a good solution; nevertheless, no solution located in the upper part of the instrument can ensure the perfect tuning of the twelfths corresponding to the first lowest tones. This result confirms the practice of instruments makers.

Finally, combining global and local solutions, i.e solutions acting mainly on one particular tone, is necessary in order to achieve the best tuning.

Acknowledgements

We wish to thank P.Mallet for blowing clarinets and O.Gipouloux for fruitful discussions.

Appendix

A Exact calculation of length corrections

The aim of the appendix A is to give some exact formulae for some elements often encountered on wind instruments. Exact formulae can be useful for the computation of resonance frequencies by using an iterative procedure. The geometries of the different elements are shown in table 1.

Open tone hole

In the case of an open hole, equation (6) gives the result

$$k\Delta\ell = \text{Arctan} \left(\frac{\tan k\ell_d}{1 + \frac{s}{Sh'k} \tan k\ell_d} \right), \quad (\text{A.1})$$

where s and S are the cross section area of the tone hole and of the main tube respectively and h' is the height of the hole including radiation and higher order modes length corrections.

Closed-hole

A closed-hole is considered to be an acoustic compliance C whose volume is equal to the volume of the hole. If $Y = jC\omega$ in equation (3) the following result is obtained:

$$k\Delta\ell = \text{Arctan} \left(\frac{Xk\ell \cos^2 k\ell}{1 + \frac{1}{2}Xk\ell \sin 2k\ell} \right), \quad (\text{A.2})$$

where $X = \frac{v}{S\ell}$, k is the wavenumber and ℓ is the closed-hole location.

Coupled closed-holes

Denoting ℓ_1 and ℓ_2 the location points of the closed-holes, the expression of the length correction associated with two coupled closed-holes is derived from the following system

$$\begin{cases} Y_{up1} = Y_1 + Y_{down1} & \text{at point } \ell_1, \\ Y_{up2} = Y_2 + Y_{down2} & \text{at point } \ell_2, \\ Y_{down2} = -jY_c \tan k(\ell_2 + \Delta\ell), \end{cases}$$

where Y_i is the admittance of the closed-hole i , Y_{up} and Y_{down} are the admittances upstream and downstream the discontinuity respectively. After some algebra, the length correction is given by

$$k\Delta\ell = \text{Arctan} \left(\frac{A_1 + A_2 + X_1 X_2 k^2 \ell_1 \ell_2 \cos k\ell_1 \cos k\ell_2 \sin k(\ell_1 - \ell_2)}{1 + B_1 + B_2 + X_1 X_2 k^2 \ell_1 \ell_2 \cos k\ell_1 \sin k\ell_2 \sin k(\ell_1 - \ell_2)} \right) \quad (\text{A.3})$$

where $A_i = \frac{v_i}{S\ell_i} k\ell_i \cos^2 k\ell_i$ and $B_i = \frac{1}{2} \frac{v_i}{S\ell_i} k\ell_i \sin 2k\ell_i$ is relative to the closed-hole i .

In the limit of small perturbations, it can be pointed out that this result is given by the sum of the length correction associated with each closed-hole. It is also valid in the limit that the distance between hole is much smaller than the wavelength.

Abrupt change in cross section

Looking for a equivalent system of section S , one can write at the discontinuity

$$-jY'_c \tan k\ell = -jY_c \tan k(\ell + \Delta\ell)$$

where $Y'_c = S'/\rho c$ and $Y_c = S/\rho c$. Manipulating the above expression gives result in

$$k\Delta\ell = \text{Arctan} \left(\frac{\frac{1}{2}(\alpha - 1) \sin 2k\ell}{1 + (\alpha - 1) \sin^2 k\ell} \right), \quad (\text{A.4})$$

where $\alpha = S'/S$ is the ratio of cross sections of the tubes located upstream and downstream the discontinuity.

Localized enlargement/contraction at the input of the instrument

Considering a localized enlargement or contraction of length ℓ' and cross section area S' located at point ℓ and denoting $\alpha = S'/S$ the change in diameter

where S is the cross section area of the main tube, it can be written:

$$\begin{cases} Y_{up1} = -jY_c \tan k\ell & \text{at point } \ell \\ Y_{down2} = -jY_c \tan k(\ell + \ell' + \Delta\ell) & \text{at point } \ell + \ell' . \end{cases}$$

The use of the transfer impedance formula of Y_{up1} between ℓ and $\ell + \ell'$ results in

$$k\Delta\ell = \text{Arctan} \left(\tan k\ell' \frac{TT_2 - \alpha(1 + TT_2) + \alpha^2}{-T \tan k\ell' + \alpha(1 + TT_2) + \alpha^2 \tan k\ell' T_2} \right) ,$$

where $T = \tan k\ell$ and $T_2 = \tan k(\ell + \ell')$.

Denoting $\alpha = 1 + X$, the expression of the length correction becomes

$$k\Delta\ell = \text{Arctan} \left(X \sin k\ell' \frac{\cos k(2\ell + \ell') + X \cos k\ell \cos k(\ell + \ell')}{1 + X \cos^2 k\ell' + X(X + 2) \sin k\ell' \cos k\ell \sin k(\ell + \ell')} \right) . \quad (\text{A.5})$$

In the limit that α is close to the unity, an approximation of equation (A.5) is given by equation (15).

Truncated cone at the input

This paragraph deals with the case of a perturbation in taper of length ℓ_c located at distance ℓ from the reed tip. Calculations for a positive and a negative taper are identical. The unique difference is that the sign of both quantities X and X' has to be inverted.

Assuming the quantities x_i to be positive and $x_2 > x_1$, and noting that the two inductances located at points ℓ and $\ell + \ell_c$ are positive and negative respectively, the following system is derived :

$$\begin{cases} Y_{up2} = Y_2 + Y_{down2} & \text{at point } \ell + \ell_c \ (x_2 > 0) \text{ and } L > 0 , \\ Y_{up1} = -Y_1 + Y_{down} & \text{at point } \ell \ (x_1 > 0) \text{ and } L < 0 , \\ Y_{down1} = -jY_c \tan k(\ell + \ell_c + \Delta\ell) , \end{cases}$$

where $\Delta\ell$ is the global length correction.

Denoting $X = \frac{\ell}{x_1}$ and $X' = \frac{\ell + \ell_c}{x_2}$, the result is

$$k\Delta\ell = \text{Arctan } \Lambda , \quad (\text{A.6})$$

where

$$\Lambda = \left(\frac{X \frac{\cos^2 k(\ell + \ell_c)}{k(\ell + \ell_c)} - X' \frac{\cos^2 k\ell}{k\ell} + X X' \frac{\cos k\ell \cos k(\ell + \ell_c)}{k\ell \frac{k(\ell + \ell_c)}{k(\ell + \ell_c)}} \sin k\ell_c}{1 - \frac{1}{2} X' \frac{\sin 2k\ell}{k\ell} + \frac{1}{2} X \frac{\sin 2k(\ell + \ell_c)}{k(\ell + \ell_c)} + X X' \frac{\cos k\ell \sin k(\ell + \ell_c)}{k\ell \frac{k(\ell + \ell_c)}{k(\ell + \ell_c)}} \tan k(\ell + \ell_c) \sin k\ell_c} \right) .$$

In the limit of small perturbation, the length correction for a truncated cone is the sum of the length correction associated with both changes in taper.

B Effect of the resistive term in the hole impedance on resonance frequencies

The aim of this appendix is to give an analytic expression of the playing frequency in term of the resistive part of the hole impedance. This can be interesting especially when resistive effects are large and can occur at high level when nonlinear effects appear, especially for narrow holes, like the register hole. This appendix describes the calculations which lead to the following expression for the frequency of the played tone

$$f = \frac{f_o + Af_c}{1 + A} \quad (\text{B.1})$$

where f_o and f_c are the frequencies obtained when the tube is open or closed respectively, and A is a coefficient depending on the resistive term R .

Looking at figure 4, it can be written at the discontinuity point the impedance equivalent Z_{eq} to the tone hole impedance and to the tube downstream the discontinuity as

$$Z_{eq} = Z_h // Z_{down} ,$$

where $Z_h = R + jX$ is the tone hole impedance and Z_{down} the impedance of the tube downstream the discontinuity. Then, using the transfer impedance formula, the input impedance Z_e of the system can be derived as follows:

$$Z_e = \frac{-t + jZ_h T \left(\frac{1+t^2}{T-t} \right)}{Z_h \left(\frac{1+t^2}{T-t} \right) + j} , \quad (\text{B.2})$$

where

- Z_e is the input impedance of the system ;
- $Z_h = R + jX$ is the tone hole impedance ;
- $t = \tan k\ell$;
- $T = \tan kL$.

For the resonance frequencies, the imaginary part of the input impedance vanishes, i.e

$$\text{Im}(Z_e) = 0 \iff \left(\frac{T-t}{1+t^2} + X \right) \left(t \frac{T-t}{1+t^2} + XT \right) + TR^2 = 0 . \quad (\text{B.3})$$

Denoting

$$\begin{aligned} A &= \sin k(L - 2\ell) + \sin kL + 2\chi \cos kL , \\ B &= \cos k(L - 2\ell) - \cos kL + 2\chi \sin kL , \\ C &= 2R^2 \sin 2kL , \end{aligned}$$

where $\chi = k\ell_{hole}$, equation (B.3) can be rewritten in the form

$$AB + C = 0 . \quad (\text{B.4})$$

By writing $kL = k_0L + \epsilon_1$ where k_0 is the wavenumber of the fundamental frequency of the played tone defined as $A = 0$ (when R vanishes):

$$\sin k_0(L - 2\ell) + \sin k_0L + 2\chi \cos k_0L = 0 , \quad (\text{B.5})$$

the use of Taylor's formula to the first order for the A and B quantities results in:

$$A = \epsilon_1[(L - 2\ell) \cos k_0(L - 2\ell) + L \cos k_0L + 2\ell_{hole} \cos k_0L - 2\chi L \sin k_0L] , \quad (\text{B.6})$$

$$\begin{aligned} B &= \cos k_0(L - 2\ell) - \cos k_0L + 2\chi \sin k_0L \\ &+ \epsilon_1[L \sin k_0L - (L - 2\ell) \sin k_0(L - 2\ell) + 2\ell_{hole} \sin k_0L + 2\chi L \cos k_0L] . \end{aligned} \quad (\text{B.7})$$

Neglecting terms of second order in ϵ_1 and using equation (B.5), the quantity AB becomes:

$$AB = \left(\frac{1}{\cos k_0\ell_{hole}} [L(1 + \cos 2k_0\ell) + 2\ell_{hole} \cos^2 kL - 2\ell \cos k(L - 2\ell) \cos kL] \right) \left(\frac{\cos 2k_0\ell_d - 1}{\cos k_0L} \right) . \quad (\text{B.8})$$

In the limit of $k_0\ell_d \ll 1$ (i.e. $\frac{\ell_d}{\ell}$ and $\frac{\Delta\ell}{\ell}$ are smaller than unity) and noting $k_0 = \frac{\pi}{2(\ell + \Delta\ell)}$, we have:

$$\begin{aligned} \cos k_0L &= -\frac{\pi}{2} \frac{\ell_d^2}{\ell(\ell_{hole} + \ell)} + o\left(\frac{\ell_d}{\ell}\right) \\ \cos 2k_0\ell_d &= 1 + o\left(\frac{\ell_d}{\ell}\right) \\ \cos k_0(L - 2\ell) &= \sin \frac{\pi}{2} \frac{\Delta\ell + \ell_d}{2} = \frac{\pi}{2} \frac{\Delta\ell + \ell_d}{2} + o\left(\frac{\ell_d}{\ell}\right) \\ \cos 2k_0\ell &= -1 + o\left(\frac{\ell_d}{\ell}\right) . \end{aligned}$$

Substituting these results in equation (B.4) leads to the following equation

$$\left(\pi^2 \ell_{hole} \frac{\ell_d + \Delta \ell}{\ell} \right) \epsilon_1 + 2R^2 \sin 2kL = 0 .$$

Noting $kL = k_c L + \epsilon_2$ where $k_c L = \frac{\pi}{2}$ (k_c is the wavenumber when the hole is closed), it can be written :

$$\pi^2 \ell_{hole} \frac{\ell_d + \Delta \ell}{\ell} (k - k_0) + 4R^2 L (k - k_c) = 0 , \quad (\text{B.9})$$

therefore,

$$f = \frac{f_o + A f_c}{1 + A} , \quad (\text{B.10})$$

where

- $A = \left(\frac{2R}{\pi} \right)^2 \frac{1}{\ell_{hole}} \frac{L \ell}{\ell_d + \Delta \ell}$;
- f_o is the frequency of the tone with the open hole ;
- f_c is the frequency of the tone with the hole being close ;
- c is the speed of sound .

A careful limiting process shows that all is in order in the two extreme cases i.e when $R \rightarrow 0$ and $R \rightarrow \infty$. In the limit that R goes to zero, equation (B.10) gives the frequency to tend to f_o , the frequency of the tone when the hole is open. In the opposite limit when R tends to infinity, equation (B.10) gives the frequency to be f_c . Figure ?? shows that the playing frequencies given by equation (B.10) coincide quite well with the playing frequencies obtained by the zero values of the imaginary part of the input admittance.

Moreover, in the limit that R is much smaller than unity, it follows

$$f = f_o + A(f_c - f_o) . \quad (\text{B.11})$$

With the use of experimental results obtained by Dalmont et al. [14] about the non linear behavior of an open side hole, it appears that the resistive term of the shunt impedance has small effects in the determination of the playing frequency. For instance, a 5-cent difference is obtained with $R/Z_c = 0$ and $R/Z_c = 0.05$.

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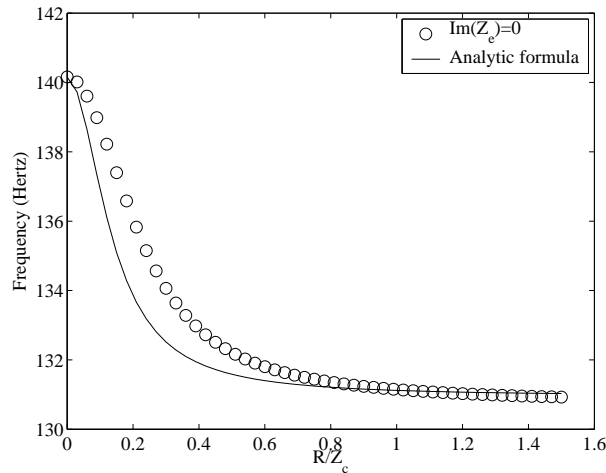


Fig. B.1. *Playing frequency as a function of the real part of the tone hole impedance ($R = 10 \text{ mm}$, $r = 7 \text{ mm}$, $h = 9 \text{ mm}$). In the limit that $R \rightarrow 0$, the frequency of the played tone tends to f_o , the frequency obtained when the hole is open and on the other hand, in the limit that $R \rightarrow \infty$, the frequency of the played tone approaches f_c , the frequency when the hole being closed.*

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