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# WONG-ROSAY THEOREM IN ALMOST COMPLEX MANIFOLDS

HERVÉ GAUSSIER AND ALEXANDRE SUKHOV

ABSTRACT. We study the compactness of sequences of diffeomorphisms in almost complex manifolds in terms of the direct images of the standard integrable structure.

## INTRODUCTION

The classical Wong-Rosay theorem states that every domain in the euclidean space  $\mathbb{C}^n$  or more generally in a complex manifold of dimension  $n$ , with an automorphism orbit accumulating at a  $\mathcal{C}^2$  strictly pseudoconvex point, is biholomorphically equivalent to the unit ball in  $\mathbb{C}^n$  (see [2, 6, 7, 9]). The aim of this paper is to extend this theorem to strictly pseudoconvex domains in almost complex manifolds. Our main result can be considered as a compactness theorem for sequences of diffeomorphisms. This shows that the convergence of such sequences can be controlled in terms of the direct images of the standard complex structure. Our approach is based on the scaling method, introduced by S.Pinchuk [6] for the case of the integrable structure. In order to apply it in the almost complex case, we need substantial modifications. In particular we use lower estimates for the Kobayashi infinitesimal pseudometric on almost complex manifolds [3] and a priori estimates for  $J$ -holomorphic curves [8].

## 1. STATEMENT OF THE RESULTS

An almost complex manifold  $(M, J)$  is a smooth ( $\mathcal{C}^\infty$ ) real manifold equipped with an almost complex structure  $J$ , that is a  $\mathcal{C}^\infty$ -field of complex linear structures on the tangent bundle  $TM$  of  $M$ . Given two almost complex manifolds  $(M, J)$  and  $(M', J')$  and a smooth map  $f$  from  $M'$  to  $M$  we say that  $f$  is  $(J', J)$ -holomorphic if its differential  $df : TM' \rightarrow TM$  satisfies

$$(1) \quad df \circ J' = J \circ df$$

on  $TM$ . We denote by  $\mathcal{O}_{(J', J)}(M', M)$  the set of  $(J', J)$ -holomorphic maps from  $M'$  to  $M$  and by  $Diff_{(J', J)}(M', M)$  the set of  $(J', J)$ -holomorphic diffeomorphisms from  $M'$  to  $M$ . Since equation (1) has generically no solution the set  $Diff_{(J', J)}(M', M)$  is generically empty. However given a diffeomorphism  $f$  from  $M'$  to  $M$  and an almost complex structure  $J'$  on  $M'$  then  $f \in Diff_{(J, J')}(M, M')$  for the almost complex structure  $J = df \circ J' \circ df^{-1}$  naturally associated with  $f$ . If  $M$  and  $M'$  are two domains in  $\mathbb{C}^n$  and  $f \in Diff_{(J_0, J_0)}(M', M)$  we simply say that  $f$  is a biholomorphism from  $M'$  to  $M$ , or that  $M$  is biholomorphic to  $M'$ .

We consider the following situation:

- $D$  is a bounded domain in  $\mathbb{C}^n$ ,

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- $p$  is a point in an almost complex manifold  $(M, J)$ ,  $U$  is a relatively compact neighborhood of  $p$  in  $M$ ,  $r$  is a  $\mathcal{C}^2$  strictly  $J$ -plurisubharmonic function in a neighborhood of  $\bar{U}$  satisfying  $r(p) = 0$  and  $dr \neq 0$  on  $U$ ,
- $(r_\nu)_\nu$  is a sequence of  $\mathcal{C}^2$  functions in a neighborhood of  $\bar{U}$  such that  $\lim_{\nu \rightarrow \infty} \|r_\nu - r\|_{\mathcal{C}^2(\bar{U})} = 0$ ,
- for every  $\nu$ ,  $f_\nu$  is a diffeomorphism from  $D$  to  $G^\nu \subset M$  with  $G^\nu \cap U = \{x \in U : r_\nu(x) < 0\}$
- for every  $\nu$ , the almost complex structure  $J_\nu := df_\nu \circ J_0 \circ df_\nu^{-1}$  extends smoothly to  $\bar{U}$ .

Then we have:

**Theorem 1.** *Assume that  $\lim_{\nu \rightarrow \infty} \|J_\nu - J\|_{\mathcal{C}^2(\bar{U})} = 0$ . If there is a point  $x^0 \in D$  such that  $\lim_{\nu \rightarrow \infty} f_\nu(x^0) = p$ , then  $D$  is biholomorphic to the unit ball  $\mathbb{B}_n$  in  $\mathbb{C}^n$ .*

Theorem 1 implies the following compactness result:

**Corollary 1.** *Let  $D$  be a bounded domain in  $\mathbb{C}^n$ , not biholomorphic to the unit ball  $\mathbb{B}_n$  and let  $G = \{x \in M : r(x) < 0\}$  be a relatively compact domain in  $(M, J)$ , where  $r$  is a  $\mathcal{C}^2$  strictly  $J$ -plurisubharmonic function on  $M$ . Let  $(f_\nu)_\nu$  be a sequence of diffeomorphisms from  $D$  to  $G$ , defined in a neighborhood of  $\bar{D}$ . If  $\|J_\nu - J\|_{\mathcal{C}^2(\bar{D})} \rightarrow_{\nu \rightarrow \infty} 0$  then  $(f_\nu)_\nu$  is compact.*

Finally we have an analogue of the Wong-Rosay theorem in almost complex manifolds:

**Theorem 2.** *Let  $D = \{x \in M : r(x) < 0\}$  be a relatively compact domain in an almost complex manifold  $(M, J)$ , where  $r$  is a  $\mathcal{C}^2$  strictly plurisubharmonic function on  $M$ . Assume that the set  $\text{Diff}_{(J, J_0)}(D, \mathbb{B}_n)$  is empty. Then the set  $\text{Diff}_{(J, J)}(D, D)$  is compact for the compact-open topology.*

## 2. PRELIMINARIES

**2.1. Plurisubharmonic functions.** Let  $(M, J)$  be an almost complex manifold. We denote by  $TM$  the real tangent bundle of  $M$  and by  $T_{\mathbb{C}}M$  its complexification. If :

$$T^{(1,0)}M := \{X \in T_{\mathbb{C}}M : JX = iX\} = \{\zeta - iJ\zeta, \zeta \in TM\},$$

$$T^{(0,1)}M := \{X \in T_{\mathbb{C}}M : JX = -iX\} = \{\zeta + iJ\zeta, \zeta \in TM\},$$

then  $T_{\mathbb{C}}M = T^{(1,0)}M \oplus T^{(0,1)}M$ . The set of complex forms of type  $(1,0)$  on  $M$  is defined by :

$$T_{(1,0)}M = \{w \in T_{\mathbb{C}}^*M : w(X) = 0, \forall X \in T^{(0,1)}M\}$$

and the set of complex forms of type  $(0,1)$  on  $M$  by :

$$T_{(0,1)}M = \{w \in T_{\mathbb{C}}^*M : w(X) = 0, \forall X \in T^{(1,0)}M\}.$$

Then  $T_{\mathbb{C}}^*M = T_{(1,0)}M \oplus T_{(0,1)}M$ , meaning that every complex one form  $w$  on  $M$  may be uniquely decomposed as  $w = w_{(1,0)} + w_{(0,1)}$ , where  $w_{(1,0)} \in T_{(1,0)}M$  and  $w_{(0,1)} \in T_{(0,1)}M$ . Then the operators  $\partial_J$  and  $\bar{\partial}_J$  are defined on the space of smooth functions on  $M$  by  $\partial_J u = du_{(1,0)}$  and  $\bar{\partial}_J u = du_{(0,1)}$ , for every complex smooth function  $u$  on  $M$ .

Let  $r$  be a  $\mathcal{C}^2$  function on  $M$ . The Levi form of  $r$  is defined on  $TM$  by

$$\mathcal{L}^J(r)(X) := -d(J^*dr)(X, JX).$$

We recall that an upper semicontinuous function  $u$  on  $(M, J)$  is called  $J$ -plurisubharmonic on  $M$  if the composition  $u \circ f$  is subharmonic on  $\Delta$  for every  $f \in \mathcal{O}_J(\Delta, M)$ . Then we have the following characterization of  $J$ -plurisubharmonic functions (see [3]):

**Proposition 1.** *Let  $u$  be a  $\mathcal{C}^2$  real valued function on  $M$ . Then  $u$  is  $J$ -plurisubharmonic on  $M$  if and only if  $\mathcal{L}^J(u)(X) \geq 0$  for every  $X \in TM$ .*

Hence, following [3], we say that a  $\mathcal{C}^2$  real valued function  $u$  on  $M$  is *strictly  $J$ -plurisubharmonic* on  $M$  if  $\mathcal{L}^J(u)$  is positive definite on  $TM$ .

We point out that the strict  $J$ -plurisubharmonicity is stable with respect to small  $\mathcal{C}^2$  deformations of the almost complex structure and of the function.

**2.2. Kobayashi pseudometric.** We denote by  $\Delta$  the unit disc in  $\mathbb{C}$ . Let  $(M, J)$  be an almost complex manifold. According to [5] for every  $p \in M$  there is a neighborhood  $\mathcal{V}$  of 0 in  $T_p M$  such that for every  $v \in \mathcal{V}$  there exists  $f \in \mathcal{O}_J(\Delta, M)$  satisfying  $f(0) = p$ ,  $df(0)(\partial/\partial x) = v$ . We may therefore define the Kobayashi-Royden infinitesimal pseudometric  $K_{(M, J)}$ :

**Definition 1.** *For  $p \in M$  and  $v \in T_p M$ ,  $K_{(M, J)}(p, v)$  is the infimum of the set of positive  $\alpha$  such that there exists a  $J$ -holomorphic disc  $f : \Delta \rightarrow M$  satisfying  $f(0) = p$  and  $df(0)(\partial/\partial x) = v/\alpha$ .*

Since for every  $f \in \mathcal{O}_{(J', J)}(M', M)$  and every  $\varphi \in \mathcal{O}_J(\Delta, M')$  the composition  $f \circ \varphi$  is in  $\mathcal{O}_J(\Delta, M)$  we have :

**Proposition 2.** *Let  $f : (M', J') \rightarrow (M, J)$  be a  $(J', J)$ -holomorphic map. Then for every  $p' \in M'$  and  $v' \in T_{p'} M'$  :*

$$(2) \quad K_{(M, J)}(f(p'), df(p')(v')) \leq K_{(M', J')}(p', v').$$

The upper semicontinuity of  $K_{(M, J)}$  on the tangent bundle  $TM$  of  $M$ , proved in [4], enables to define the *integrated pseudodistance*  $d_{(M, J)}^K$  on  $M \times M$ :

**Definition 2.** *Let  $p, q \in M$ . Denote by  $\Gamma_{p, q}$  the set of all  $\mathcal{C}^1$ -paths  $\gamma : [0, 1] \rightarrow M$  satisfying  $\gamma(0) = p$ ,  $\gamma(1) = q$ . Then :*

$$d_{(M, J)}^K(p, q) = \inf_{\gamma \in \Gamma_{p, q}} \int_0^1 K_{(D, J)}(\gamma(t), \gamma'(t)) dt.$$

If  $(M', J')$  and  $(M, J)$  are almost complex manifolds and  $f : M' \rightarrow M$  is  $(J', J)$ -holomorphic, then Proposition 2 implies that for every  $p', q' \in M'$  we have :

$$(3) \quad d_{(M, J)}^K(f(p'), f(q')) \leq d_{(M', J')}^K(p', q').$$

Let  $f \in \text{Diff}_{(J', J)}(M', M)$  be a diffeomorphism from  $M'$  to  $M$ . Then the inverse map  $f^{-1} : M \rightarrow M'$  is in  $\text{Diff}_{(J, J')}(M, M')$  and, from the inequalities (2) and (3), we have for every  $p', q' \in M'$  and every  $v' \in T_{p'} M'$ :

$$(4) \quad K_{(M, J)}(f(p'), df(p')(v')) = K_{(M', J')}(p', v')$$

and

$$(5) \quad d_{(M, J)}^K(f(p'), f(q')) = d_{(M', J')}^K(p', q').$$

The following results are proved in [3] (Theorem 6 and Corollary 2):

**Theorem A** *Let  $D = \{r < 0\}$  be a relatively compact domain in an almost complex manifold  $(M, J)$ , where  $r$  is a  $\mathcal{C}^2$  defining function of  $D$ , strictly  $J$ -plurisubharmonic*

in a neighborhood of  $\bar{D}$ . Then there exist positive constants  $c$  and  $\lambda_0$  such that for every almost complex structure  $J'$  defined in a neighborhood of  $\bar{D}$  and such that  $\|J' - J\|_{\mathcal{C}^2(\bar{D})} \leq \lambda_0$  we have :

$$(6) \quad K_{(D,J')}(p, v) \geq c \left[ \frac{|\partial_{J'} r(p)(v - iJ(p)v)|^2}{|r(p)|^2} + \frac{\|v\|^2}{|r(p)|} \right]^{1/2},$$

for every  $p \in D$  and every  $v \in T_p M$ .

As an application of equation (6) we obtain the boundary behaviour of the Kobayashi pseudodistance in a strictly  $J$ -pseudoconvex domain.

**Corollary B** *Let  $D$  be a domain in an almost complex manifold  $(M, J)$  and let  $p \in \partial D$ . Assume that there is a neighborhood  $U$  of  $p$  such that  $D \cap U = \{x \in U : r(x) < 0\}$ , where  $r$  is a  $\mathcal{C}^2$  strictly  $J$ -plurisubharmonic function in a neighborhood of  $\bar{U}$ . Then for every  $q \in D$  we have :*

$$\lim_{\substack{x \in D \\ x \rightarrow p}} d_{(D,J)}^K(x, q) = +\infty.$$

We note that Corollary B depends only on the strict  $J$ -plurisubharmonicity of  $r$ , namely on the constant  $c$  introduced in theorem A and on a positive constant  $M$  such that  $-M \leq r$  on  $\bar{D}$ . This uniformity will be used in Section 3.

### 3. PROOF OF THEOREM 1

In this Section we assume that the assumptions of Theorem 1 are satisfied.

**3.1. Attraction property.** The following Lemma is a direct application of Corollary B.

**Lemma 1.** *For every  $K \subset\subset D$  we have :  $\lim_{\nu \rightarrow \infty} f_\nu(K) = p$ .*

*Proof of Lemma 1.* Let  $K \subset\subset D$  be such that  $x^0 \in K$ . Since the function  $x \mapsto d_D^K(x^0, x)$  is bounded from above by a constant  $C$  on  $K$ , it follows from the decreasing property of the Kobayashi pseudodistance that

$$(7) \quad d_{(G^\nu, J_\nu)}^K(f_\nu(x^0), f_\nu(x)) \leq C$$

for every  $\nu$  and every  $x \in K$ . Moreover, since  $r$  is strictly  $J$ -plurisubharmonic in a neighborhood of  $\bar{U}$  and since the sequence  $(r_\nu)_\nu$  converges to  $r$  in the  $\mathcal{C}^2(\bar{U})$  convergence, there is a positive constant  $c$  and a positive integer  $\nu_0$  such that the function  $r_\nu - c|z|^2$  is  $J$ -plurisubharmonic on  $\bar{U}$  for every  $\nu \geq \nu_0$ . Hence we may apply Corollary B to  $G^\nu$  uniformly with respect to  $\nu$ . This implies that for every  $V \subset\subset U$ , containing  $p$ , we have:

$$(8) \quad \lim_{\nu \rightarrow \infty} d_{(G^\nu, J_\nu)}^K(f_\nu(x^0), G^\nu \cap \partial V) = +\infty.$$

It follows from conditions (7) and (8) that  $f_\nu(K) \subset V$  for every sufficiently large  $\nu$ . This gives the statement.  $\square$

The almost complex structure  $J$  may be represented locally at  $p$  as a small  $\mathcal{C}^2$  deformation of the standard structure. More precisely, we have the following Lemma (see [3]):

**Lemma 2.** *For every  $\lambda_0 > 0$  there exist a neighborhood  $U_0$  of  $p$  and a coordinate diffeomorphism  $z : U_0 \rightarrow \mathbb{B}_n$  such that  $z(p) = 0$ ,  $dz(p) \circ J(p) \circ dz^{-1}(0) = J_0$  and the direct image  $\hat{J} = z^*(J)$  satisfies  $\|\hat{J} - J_0\|_{\mathcal{C}^2(\mathbb{B}_n)} \leq \lambda_0$ .*

*Proof.* Shrinking  $U$  if necessary there exists a diffeomorphism  $z$  from  $U$  onto  $\mathbb{B}_n$  satisfying  $z(p) = 0$  and  $dz(p) \circ J(p) \circ dz^{-1}(0) = J_0$ . For  $\lambda > 0$  consider the dilation  $d_\lambda : t \mapsto \lambda^{-1}t$  in  $\mathbb{C}^n$  and the composition  $z_\lambda = d_\lambda \circ z$ . Then  $\lim_{\lambda \rightarrow 0} \|z_\lambda^*(J) - J_0\|_{\mathcal{C}^2(\mathbb{B}_n)} = 0$ . Setting  $U = z_\lambda^{-1}(\mathbb{B}_n)$  for  $\lambda > 0$  small enough, we obtain the desired statement.  $\square$

We denote  $z := (z_1, \dots, z_n)$  and  $z_j := x_j + iy_j$  for  $j = 1, \dots, n$ . Since  $r$  is strictly  $J$ -plurisubharmonic in a neighborhood of  $\bar{U}$  we may assume, shrinking  $U$  if necessary and using Lemma 2, that the expansion of  $r$  is given in the  $z$ -coordinates by:

$$r = x_n + \sum_{1 \leq k \leq n} z_k \bar{z}_k + \mathcal{O}(|z_n||z'| + |z'|^3).$$

Since the sequence  $(J_\nu)_\nu$  converges to  $J$  in the  $\mathcal{C}^2(\bar{U})$  convergence, there exist coordinates  $z^\nu := (z_1^\nu, \dots, z_n^\nu) : U \rightarrow \mathbb{B}_n$ , centered at  $f_\nu(x^0)$ , where  $z_j^\nu = x_j^\nu + iy_j^\nu$  for every  $1 \leq j \leq n$ , converging to  $z$  in the  $\mathcal{C}^2(\bar{U})$ -convergence. Moreover, we may assume that the expansion of  $r_\nu$  in the  $(z^\nu, \bar{z}^\nu)$ -coordinates is given by :

$$\begin{aligned} r_\nu &= r_\nu(f_\nu(x^0), \overline{f_\nu(x^0)}) + \lambda_n^\nu x_n^\nu \\ &+ \operatorname{Re}(\sum_{k,l=1}^n \lambda_{k,l}^\nu z_k^\nu \bar{z}_l^\nu) + \sum_{k,l=1}^n \lambda_{k,\bar{l}}^\nu z_k^\nu \bar{z}_l^\nu + \mathcal{O}(|z_1||z'| + |z'|^3) \end{aligned}$$

where the condition  $\mathcal{O}(|z_1||z'| + |z'|^3)$  is uniform with respect to  $\nu$ . Finally it follows from the convergence of the sequence of coordinates  $(z^\nu)_\nu$  and of the sequence of functions  $(r^\nu)_\nu$  that:

$$\left\{ \begin{array}{l} \lim_{\nu \rightarrow \infty} r_\nu(f_\nu(x^0), \overline{f_\nu(x^0)}) = 0, \\ \lim_{\nu \rightarrow \infty} \lambda_n^\nu = 1, \\ \lim_{\nu \rightarrow \infty} \lambda_{k,l}^\nu = 0 \quad \text{for every } 1 \leq k, l \leq n, \\ \lim_{\nu \rightarrow \infty} \lambda_{k,\bar{k}}^\nu = 1 \quad \text{for } 1 \leq k \leq n, \\ \lim_{\nu \rightarrow \infty} \lambda_{k,\bar{l}}^\nu = 0 \quad \text{for } k \neq l. \end{array} \right.$$

For every  $1 \leq j \leq n$  and every  $\nu \geq 1$ , we consider

$$L_j^\nu := \{x \in U : z_k^\nu(x) = 0, 1 \leq k \leq n, k \neq j\}$$

and we define

$$\tau_j^\nu := \operatorname{dist}(f_\nu(x^0), \partial G^\nu \cap L_j^\nu).$$

We define the dilation  $\Lambda^\nu : U \rightarrow \mathbb{E}_\nu := \{w \in \mathbb{C}^n : \sum_{j=1}^n |\tau_j^\nu w_j|^2 < 1\}$  by

$$\Lambda^\nu(x) = (z_1^\nu(x)/\tau_1^\nu, \dots, z_n^\nu(x)/\tau_n^\nu).$$

and the almost complex structure  $\tilde{J}_\nu$  on  $\Lambda^\nu(U)$  by  $\tilde{J}_\nu := \Lambda^\nu \circ dz^\nu \circ J_\nu \circ d(z^\nu)^{-1} \circ (\Lambda^\nu)^{-1}$ . Since  $z^\nu$  is  $(J_\nu, dz^\nu \circ J_\nu \circ d(z^\nu)^{-1})$ -holomorphic,  $\Lambda^\nu$  is  $(J_\nu, \tilde{J}_\nu)$ -holomorphic. Finally, we consider the domain

$$\tilde{G}^\nu := \Lambda^\nu(G^\nu \cap U) = \{z \in \Lambda^\nu(U) : r_\nu((\Lambda^\nu)^{-1}(z), \overline{(\Lambda^\nu)^{-1}(z)}) < 0\}.$$

The next Lemma gives the limit behaviour of the domains  $\tilde{G}^\nu$  and of the almost complex structures  $\tilde{J}_\nu$ .

**Lemma 3.** *The following conditions are satisfied :*

- (i)  $\forall j \leq n-1, \tau_j^\nu \simeq (\tau_1^\nu)^{1/2}$ ,
- (ii)  $\lim_{\nu \rightarrow \infty} \tilde{G}^\nu = \mathbb{H} = \{z \in \mathbb{C}^n : \text{Im}(z_n) < |z_1|^2 + \dots + |z_{n-1}|^2\}$ ,
- (iii)  $\forall q \in \mathbb{H}, \lim_{\nu \rightarrow \infty} \tilde{J}_\nu(q) = J_0$ .

*Proof of Lemma 3.* The convergence in statement (ii) is the Hausdorff convergence on compact subsets. Conditions (i) and (ii) are direct consequences of the convergence of  $(r_\nu)_\nu$  to  $r$  combined with the form of  $r$  written above and is analogue to the usual complex case (see [6]).

Let  $K$  be relatively compact in  $\mathbb{H}$ . It follows from condition (ii) that  $K$  is contained in  $\tilde{G}^\nu$  for sufficiently large  $\nu$ , implying that  $(\Lambda^\nu \circ z^\nu)^{-1}(K)$  is contained in  $G^\nu$  for sufficiently large  $\nu$ . Let  $q \in K$  and let  $v \in \mathbb{C}^n$ . By definition we have:

$$\tilde{J}_\nu(q)(v) = (\Lambda^\nu \circ dz^\nu \circ J_\nu)((\Lambda^\nu \circ z^\nu)^{-1}(q))(d(z^\nu)^{-1}((\Lambda^\nu)^{-1}(q))((\Lambda^\nu)^{-1}(v))).$$

It follows from Lemma 2 that there is a neighborhood  $W$  of  $p$  contained in  $U$  and a positive constant  $C$  such that

$$dz \circ J \circ dz^{-1} = J_0 + J_1$$

on  $z(W)$ , with  $|J_1(w)| \leq C|w|$  on  $z(W)$ . Since  $(J_\nu)_\nu$  converges to  $J$  in the  $\mathcal{C}^2(\bar{U})$  convergence, we may also write  $dz^\nu \circ J_\nu \circ d(z^\nu)^{-1} = J_0 + J_{\nu,1}$  on  $z(W)$ , with  $|J_{\nu,1}(w)| \leq C|w|$  on  $z(W)$  after an unessential change of coordinates and changing the value of the positive constant  $C$ , uniformly with respect to  $\nu$ , if necessary. Hence :

$$\begin{aligned} dz^\nu \circ J_\nu((\Lambda^\nu \circ z^\nu)^{-1}(q))(d(z^\nu)^{-1}((\Lambda^\nu)^{-1}(q))((\Lambda^\nu)^{-1}(v))) &= J_0((\Lambda^\nu)^{-1}(v)) \\ &+ \mathcal{O}(|(\Lambda^\nu \circ z^\nu)^{-1}(q)| \|(\Lambda^\nu)^{-1}(v)\|), \end{aligned}$$

and thus, since  $\Lambda^\nu$  is  $J_0$ -holomorphic :

$$\tilde{J}_\nu(q)(v) = J_0(v) + \mathcal{O}(\|v\| |(\Lambda^\nu \circ z^\nu)^{-1}(q)|).$$

Since  $\lim_{\nu \rightarrow \infty} (\Lambda^\nu)^{-1} = 0$  uniformly on compact subsets of  $\mathbb{C}^n$ , this gives condition (iii).  $\square$

For every  $\nu$ , let  $F^\nu := \Lambda^\nu \circ f_\nu$ . Then  $F^\nu$  is a  $(J_0, \tilde{J}_\nu)$ -holomorphic map from  $f_\nu^{-1}(G^\nu \cap V)$  to  $\tilde{G}^\nu$  and we have :

**Lemma 4.** *The following conditions are satisfied :*

- (i) *We may extract from  $(F^\nu)_\nu$  a subsequence converging, uniformly on compact subsets of  $D$ , to a map  $F : D \longrightarrow \bar{\mathbb{H}}$ ,*
- (ii) *We may extract from  $((F^\nu)^{-1})_\nu$  a subsequence converging, uniformly on compact subsets of  $\mathbb{H}$ , to a map  $\tilde{F} : \mathbb{H} \longrightarrow \bar{D}$ ,*
- (iii)  *$F$  is a biholomorphism from  $D$  to  $\mathbb{H}$  with  $F^{-1} = \tilde{F}$ .*

Statement (iii) is the conclusion of Theorem 1 since  $\mathbb{H}$  is biholomorphic to the unit ball in  $\mathbb{C}^n$  by a Cayley transform.

*Proof of Lemma 4.* We note first that by Lemma 1 :  $\lim_{\nu \rightarrow \infty} f_\nu^{-1}(G^\nu \cap V) = D$ . Condition (i) is based on the estimates of the Kobayashi-Royden infinitesimal pseudometric given in Theorem A. Indeed, there is a neighborhood  $U_0$  of  $p$  and a positive

constant  $c$  such that for every  $q \in U_0$ , every  $v^\nu \in T_q M$  and every sufficiently large  $\nu$  we have :

$$K_{(G^\nu, J_\nu)}(q, v^\nu) \geq c \left[ \frac{|\partial_{J_\nu} r_\nu(q)(v^\nu - iJ_\nu(q))|^2}{|r_\nu(q)|^2} + \frac{\|v^\nu\|^2}{|r_\nu(p)|} \right]^{1/2}.$$

According to Lemma 1, there exists an integer  $\nu_0$  such that for every  $\nu \geq \nu_0$  and every  $K \subset\subset D$ , with  $\text{int}(K) \neq \emptyset$ , we have :  $K \subset\subset f_\nu^{-1}(G^\nu \cap V)$  and  $f_\nu(K) \subset U_0$ .

Let  $x \in K$ ,  $v \in \mathbb{C}^n$  and let  $v^\nu = df_\nu(x)(v)$  in the coordinates  $z^\nu$ . Then there exists  $c' > 0$  such that :

$$K_{(G^\nu, J_\nu)}(f_\nu(x), v^\nu) \geq c' \left[ \frac{|(df_\nu(x)(v))_1|^2}{\text{dist}(f_\nu(x), \partial G^\nu \cap L_1^\nu)^2} + \frac{|(df_\nu(x)(v))'|^2}{\text{dist}(f_\nu(x), \partial G^\nu \cap L_1^\nu)} \right]^{1/2}.$$

However  $K_{(G^\nu, J_\nu)}(f_\nu(x), v^\nu) = K_{(D, J_0)}(x, v)$  for every  $\nu$ . Since  $K$  is relatively compact in  $D$  there exists a positive constant  $C$  such that  $K_{(D, J_0)}(x, v) \leq C$  for every  $x \in K$ ,  $v \in \mathbb{C}^n$ . In particular, since  $\text{dist}(f_\nu(x), \partial G^\nu \cap L_1^\nu) = \tau_1^\nu$ , we obtain

$$\begin{cases} (df_\nu(x)(v))_1 \simeq \tau_1^\nu, \\ (df_\nu(x)(v))_j \simeq (\tau_1^\nu)^{1/2}, \quad \text{for every } 2 \leq j \leq n. \end{cases}$$

Thus it follows from condition (i) of Lemma 3 that there exists a positive constant  $C$  such that  $\|dF^\nu(x)(v)\| \leq C\|v\|$  for every  $x \in K$ ,  $v \in \mathbb{C}^n$ . In particular, the sequence  $(F^\nu)_\nu$  is equicontinuous on  $K$ . Since  $(F^\nu(x^0))_\nu$  is bounded, we know by Ascoli theorem that we may extract a subsequence from  $(F^\nu)_\nu$ , still denoted  $(F^\nu)_\nu$ , which converges uniformly on  $K$  to a map  $F_K$ . The holomorphicity of  $F_K$  relies on the more general result :

**Lemma 5.** *Let  $D$  (resp.  $D'$ ) be a relatively compact domain in a smooth manifold  $M$  (resp.  $M'$ ). Let  $(J_\nu)_\nu$  (resp.  $(J'_\nu)_\nu$ ) be a sequence of almost complex structures on  $M$  (resp.  $M'$ ). converging in the  $C^\infty(\bar{D})$  convergence (resp.  $C^\infty(\bar{D}')$  convergence) to an almost complex structure  $J$  (resp.  $J'$ ). For every  $\nu$ , let  $F^\nu \in \mathcal{O}_{(J_\nu, J_\nu)}(D', D)$ . Assume that  $(F^\nu)_\nu$  converges uniformly on compact subsets of  $D$  to a map  $F$ . Then  $F$  belongs to  $\mathcal{O}_{(J', J)}(D', \bar{D})$ .*

*Proof of Lemma 5.* Let  $p' \in D'$ . There is a neighborhood  $U'$  of  $p'$  in  $M'$  and coordinates  $(x'_1, \dots, x'_{2n}) : U' \rightarrow \mathbb{B} \subset \mathbb{R}^{2n}$ , centered at  $p'$ , such that  $\partial/\partial x'_{n+j} = J'(0)(\partial/\partial x'_j)$ , for every  $1 \leq j \leq n$ . There is a neighborhood  $U$  of  $F(p')$  and coordinates  $(x_1, \dots, x_{2n}) : U \rightarrow \mathbb{B} \subset \mathbb{R}^{2n}$ , centered at  $F(p')$ , such that  $\partial/\partial x_{n+j} = J(0)(\partial/\partial x_j)$ , for every  $1 \leq j \leq n$ . Let  $J_\nu = (J_{j,k}^\nu)_{1 \leq j, k \leq 2n}$  in the  $(x_1, \dots, x_{2n})$  coordinates,  $J'_\nu = (J'_{j,k}^\nu)_{1 \leq j, k \leq 2n}$  in the  $(x'_1, \dots, x'_{2n})$  coordinates and  $F^\nu = (F_1^\nu, \dots, F_{2n}^\nu)$ . The condition  $F^\nu \in \mathcal{O}_{(J'_\nu, J_\nu)}(D', D)$  is written in  $U'$  :

$$(9) \quad \sum_{k=1}^{2n} \frac{\partial F_m^\nu}{\partial x'_k}(x) J'_{k,l}^\nu(x) = \sum_{k=1}^{2n} J_{m,k}^\nu(F^\nu(x)) \frac{\partial F_k^\nu}{\partial x'_l}(x)$$

for every  $x \in U'$ ,  $1 \leq l, m \leq 2n$ . Since the maps  $F_m^\nu$  are locally bounded by assumption, they define distributions, converging as distributions to  $F$ . In particular, we may define all the derivatives  $\partial F_m / \partial x'_k$  as distributions. Moreover, since the sequences  $(J_\nu)_\nu$  and  $(J'_\nu)_\nu$  converge smoothly respectively to  $J$  and  $J'$ , the derivatives



of  $F$  satisfy the following identity as distributions :

$$\sum_{k=1}^{2n} \frac{\partial F_m}{\partial x'_k}(x)' J_{k,l}(x) = \sum_{k=1}^{2n} J_{m,k}(F(x)) \frac{\partial F_k}{\partial x'_l}(x)$$

for  $x \in U'$ ,  $1 \leq l, m \leq 2n$ , or equivalently :  $dF \circ J' = J \circ dF$ . In particular,  $F$  is solution of an elliptic operator, and so  $F$  is a smooth map. Consequently,  $F \in \mathcal{O}_{(J',J)}(D', \bar{D})$ .  $\square$

Consequently, according to Lemma 5 and condition (iii) of Lemma 3,  $F_K$  is holomorphic from  $\text{int}(K)$  to  $\mathbb{H}$ . By considering a compact exhaustion of  $D$  we obtain condition (i).

Proof of (ii). Let  $0 < r < 1$ ,  $B(0, r) = \{z \in \mathbb{C}^n : \|z\| < r\}$  and let  $C_a$  be a Cayley transform from  $\mathbb{H}$  to  $\mathbb{B}_n$  (this is a biholomorphism, for the standard structure  $J_0$  on  $\mathbb{H}$  and  $\mathbb{B}_n$ ). According to Lemma 3 (ii) we have, for sufficiently large  $\nu$ , the inclusion  $C_a^{-1}(B(0, r)) \subset \tilde{G}^\nu$ . Since the sequence  $(\tilde{J}_\nu)_\nu$  converges uniformly on compact subsets of  $\mathbb{H}$  to  $J_0$  by Lemma 3 (iii), the sequence  $(J'_\nu := dC_a \circ \tilde{J}_\nu \circ dC_a^{-1})_\nu$  converges to  $J_0$  uniformly on  $\bar{\mathbb{B}}_n$ . For every  $0 < r < 1$  and every  $z \in B(0, r/2)$  one can fill the ball  $B(0, r)$  by  $J_0$ -holomorphic discs centered at  $z$ , with derivatives at the origin bounded from below by a positive constant, uniformly with respect to  $r$  close to 1 and to  $z \in B(0, r/2)$ . According to Section 5.4 a of [5] (where the stability of  $J$ -holomorphic discs under small smooth deformations of a given almost complex structure  $J$  is proved) there exists a positive constant  $C$ , uniform with respect to  $r$  close to 1 and to sufficiently large  $\nu$ , such that for every  $z \in B(0, r/2)$  and for every vector  $v \in \mathbb{C}^n$  with  $\|v\| = 1$ , there is a  $J'_\nu$ -holomorphic disc  $f_{J'_\nu, z} : \Delta \rightarrow B(0, r)$ , centered at  $z$ , satisfying

$$(10) \quad d(f_{J'_\nu, z})(\partial/\partial x) = \alpha v \quad \text{with } \alpha \geq C.$$

Since for sufficiently large  $\nu$  the holomorphic disc  $\tilde{f}_{J'_\nu, z} := (C_a \circ F^\nu)^{-1} \circ f_{J'_\nu, z}$  satisfies the inclusion  $\tilde{f}_{J'_\nu, z}(\Delta) \subset D \subset \subset \mathbb{C}^n$ , there exists a positive constant  $C'$  such that

$$(11) \quad |(\tilde{f}_{J'_\nu, z})'(0)| \leq C'$$

for sufficiently large  $\nu$ .

It follows from conditions (10) and (11) that the first derivative of  $(C_a \circ F^\nu)^{-1}$  is bounded from below by a positive constant, uniformly on  $B(0, r/2)$  with respect to  $\nu \gg 1$  and to  $r$  close to 1. By Ascoli Theorem we may extract from  $((F^\nu)^{-1})_\nu$  a subsequence that converges uniformly to a map  $\tilde{F} : \mathbb{H} \rightarrow \bar{D}$ . It follows from Lemma 5 that  $\tilde{F}$  is holomorphic. This proves condition (ii).

*Proof of (iii).* We know that  $F(x^0) = 0 \in \mathbb{H}$ . Assume now that there is  $x \in D$  such that  $F(x) \in \partial\mathbb{H}$  and let  $\gamma$  be a  $\mathcal{C}^1$ -path in  $D$  such that  $\gamma(0) = x^0$ ,  $\gamma(1) = x$ . We consider  $t_0 \leq 1$  such that  $F(\gamma(0, t_0]) \subset \mathbb{H}$  and  $F(\gamma(t_0)) \in \partial\mathbb{H}$ . Since  $\mathbb{H}$  is complete hyperbolic ( $\mathbb{H}$  is biholomorphic to the unit ball in  $\mathbb{C}^n$ ), we obtain that  $\lim_{t \rightarrow t_0} d_{\mathbb{H}}^K(0, F(\gamma(t))) = \infty$ . However, for every  $t < 1$  :

$$d_{\mathbb{H}}^K(0, F(\gamma(t))) \leq \sup_{s \in [0, 1]} d_D^K(x^0, \gamma(s)) < \infty$$

by the compactness of  $\gamma([0, 1])$  in  $D$ . This is a contradiction, implying that  $F(D) \subset \mathbb{H}$ . We prove now that  $\tilde{F}(\mathbb{H}) \subset D$ . Since  $D$  is bounded in  $\mathbb{C}^n$ ,  $D$  is hyperbolic for the standard structure  $J_0$ . Let  $r > 0$  and consider the Kobayashi ball  $B_{(D, J_0)}^K(x^0, r)$ . Since  $f_\nu$  is a biholomorphism from  $(D, J_0)$  to  $(G^\nu, J_\nu)$ , we have  $f_\nu(B_{(D, J_0)}^K(x^0, r)) = B_{(G^\nu, J_\nu)}(f_\nu(x^0), r)$  by the decreasing property of the Kobayashi pseudodistance (see

equality (5)). Since the sequence of points  $(f_\nu(x^0))_\nu$  converges to  $p$  we obtain from Corollary B that  $B_{(G^\nu, J_\nu)}(f_\nu(x^0), r) \subset\subset G^\nu \cap U$  for sufficiently large  $\nu$ . In particular, since  $(f_\nu)^{-1}$  is continuous on  $G^\nu$ , we have :  $B_{(D, J_0)}^K(x^0, r) \subset\subset D$ . This implies that  $D$  is complete hyperbolic. Assume that  $q \in \mathbb{H}$  is such that  $\tilde{F}(q) \in \partial D$ . Then there exists  $r > 0$  such that  $q \in B_{(\mathbb{H}, J_0)}^K(0, r)$ . Let  $\gamma : [0, 1] \rightarrow \mathbb{H}$  be a  $\mathcal{C}^1$ -path such that  $\gamma(0) = 0$ ,  $\gamma(1) = q$  and  $\gamma([0, 1]) \subset B_{(\mathbb{H}, J_0)}^K(0, 2r)$ . We define  $t_0 \leq 1$  such that  $\tilde{F}(\gamma([0, t_0])) \subset D$  and  $\tilde{F}(\gamma(t_0)) \in \partial D$ . By the complete hyperbolicity of  $D$ , we have :  $\lim_{t \rightarrow t_0} d_{(D, J_0)}^K(x^0, \tilde{F}(\gamma(t))) = \infty$ , which contradicts the condition  $d_{(D, J_0)}^K(x^0, \tilde{F}(\gamma(t))) \leq d_{(\mathbb{H}, J_0)}(0, \gamma_t) \leq 2r$  for every  $t < t_0$ . Consequently :  $\tilde{F}(\mathbb{H}) \subset D$ . Now, since for every  $\nu$ ,  $(F^\nu)^{-1} \circ F^\nu = id_D$  and  $F^\nu \circ (F^\nu)^{-1} = id_{\mathbb{H}}$  we obtain, by taking the limit when  $\nu \rightarrow \infty$ , that  $\tilde{F} \circ F = id_D$  and  $F \circ \tilde{F} = id_{\mathbb{H}}$ , meaning that  $F$  is a biholomorphism from  $D$  to  $\mathbb{H}$ . This proves Theorem 1.  $\square$

#### 4. PROOF OF THEOREM 2

We explain now how to adapt the proof of Theorem 1 to obtain Theorem 2. Hence we assume in this Section that the assumptions of Theorem 2 are satisfied.

We note first that Lemma 1 is also true under the assumptions of Theorem 2. This will be used in the sequel. Moreover we have the following

**Lemma 6.** *For every  $0 \in K \subset\subset \Delta$  we have*

$$\lim_{\nu \rightarrow \infty} (f_\nu \circ f)(K) = p,$$

*uniformly with respect to  $f \in \mathcal{O}_{(J_0, J)}(\Delta, D)$  such that  $f(0) = x^0$ .*

*Proof of Lemma 6.* The proof follows line by line the proof of Lemma 1, replacing  $f_\nu$  by  $f_\nu \circ f$  where  $f \in \mathcal{O}_{(J_0, J)}(\Delta, D)$ ,  $f(0) = x^0$ . Indeed since the function  $x \mapsto d_{(\Delta, J_0)}^K(0, x)$  is bounded from above on  $K$  by a positive constant  $C$ , we have:

$$d_{(D, J)}^K(f_\nu \circ f(0), f_\nu \circ f(x)) \leq C$$

for every  $\nu \geq 1$  and for every  $x \in K$ . Fix a neighborhood  $V \subset\subset M$  of  $p$ . Since

$$\lim_{\nu \rightarrow \infty} d_{(D, J)}^K(f_\nu(x^0), D \cap \partial V) = \infty$$

we have  $f_\nu \circ f(K) \subset V$  for sufficiently large  $\nu$ , uniformly with respect to  $f \in \mathcal{O}_{(J_0, J)}(\delta, D)$  such that  $f(0) = x^0$ .  $\square$

*Proof of Theorem 2.* Assume by contradiction that there is a sequence  $(f_\nu)_\nu$  in  $Diff_{(J, J)}(D, D)$  and points  $x^0 \in D$ ,  $p \in \partial D$  such that

$$\lim_{\nu \rightarrow \infty} f_\nu(x^0) = p.$$

Lemma 1 gives for every compact  $K$  in  $D$ :

$$\lim_{\nu \rightarrow \infty} f_\nu(K) = p.$$

Let  $U$  be a relatively compact neighborhood of  $p$  in  $M$  with diffeomorphisms  $z : U \rightarrow \mathbb{B}_n$  and  $z^\nu : U \rightarrow \mathbb{B}_n$  satisfying  $z(p) = 0$  and  $z^\nu(f_\nu(x^0)) = 0$ , defined pages 4 and 5. We consider  $G^\nu := D$ ,  $J_\nu := J$  and  $\tilde{J}_\nu := \Lambda^\nu \circ dz^\nu \circ J \circ d(z^\nu)^{-1} \circ (\Lambda^\nu)^{-1}$  for every  $\nu$ , where  $\Lambda^\nu$  is the dilation defined page 5. If we set  $\tilde{G}^\nu := \Lambda^\nu(D \circ U)$  then by Lemma 3 we have:

$$\lim_{\nu \rightarrow \infty} \tilde{G}^\nu = \mathbb{H} \quad \text{and} \quad \lim_{\nu \rightarrow \infty} \tilde{J}_\nu = J_0,$$

uniformly on compact subsets of  $\mathbb{C}^n$ . Moreover if  $F^\nu := \Lambda^\nu \circ f_\nu$  then  $F^\nu \in \text{Diff}_{(J, \tilde{J}_\nu)}(f_\nu^{-1}(D \cap V), \tilde{G}^\nu)$  and Lemma 4 gives that  $(F^\nu)_\nu$  converges after extraction to a map  $F$ . This map  $F$  is  $(J, J_0)$ -holomorphic according to lemma 5.

We will use the following quantitative version of Proposition 2.3.6 of [8]:

**Proposition 3.** *Let  $D'$  be a domain in  $\mathbb{C}^n$ . There is a positive constant  $\delta_0$  such that for every almost complex structure  $J'$  in a neighborhood of  $\overline{D'}$  satisfying  $\|J' - J_0\|_{\mathcal{C}^2(\overline{D'})} \leq \delta_0$  we have*

$$(12) \quad \|f_{J'}\|_{\mathcal{C}^1(K)} \leq C_{K, \delta_0} \|f_{J'}\|_{\mathcal{C}^0(K)},$$

for every  $f_{J'} \in \mathcal{O}_{(J_0, J')}(\Delta, D')$  and for every  $K \subset\subset \Delta$ , where  $C_{K, \delta_0}$  is a positive constant depending only on  $K$  and  $\delta_0$ .

Let now  $U' \subset\subset U$  be a neighborhood of  $p$  in  $M$  such that

$$(13) \quad \|(dz \circ J \circ dz^{-1}) - J_0\|_{\mathcal{C}^2(\overline{z(U')})} \leq \delta_0.$$

Fix  $0 < r < 1$ , sufficiently close to 1. Since  $\lim_{\nu \rightarrow \infty} \|\tilde{J}^\nu - J_0\|_{\mathcal{C}^2(\bar{B}(0, r))} = 0$ , it follows from [5] Section 4.5 that there is a covering  $\mathcal{R}_\nu$  of  $\bar{B}(0, r)$  by  $\tilde{J}_\nu$ -holomorphic discs centered at the origin for sufficiently large  $\nu$ , these discs being small deformations of the straight holomorphic discs in the ball. More precisely there exists  $0 \in K_r \subset\subset \Delta$  and there exists a positive constant  $c_r$  such that for sufficiently large  $\nu$ :

$$(14) \quad \bar{B}(0, r) \subset \cup_{f \in \mathcal{R}_\nu} f(K_r)$$

and

$$(15) \quad \inf_{f \in \mathcal{R}_\nu, \zeta \in K_r} \|df(\zeta)(\partial/\partial x)\| \geq c_r.$$

Moreover we may assume that  $C_a^{-1}(\bar{B}(0, r)) \subset G^\nu$ . For  $f \in \mathcal{R}_\nu$  consider the  $(J_0, J)$ -holomorphic map

$$g^\nu := (F^\nu)^{-1} \circ C_a^{-1} \circ f$$

from  $\Delta$  to  $D$ . Since  $g^\nu(0) = x^0$  it follows from Lemma 6 that there exists  $\nu_0 \geq 1$  such that  $f_{\nu_0} \circ g^\nu(K_r) \subset U'$ , uniformly with respect to  $\nu \gg 1$ . It follows now from condition (13) and from Proposition 3 that there exists a positive constant  $C_r$  such that for sufficiently large  $\nu$ :

$$\|f_{\nu_0} \circ g^\nu\|_{\mathcal{C}^1(K_r)} \leq C_r$$

or equivalently

$$\|g^\nu\|_{\mathcal{C}^1(K_r)} \leq C'_r$$

with  $C'_r > 0$ .

It follows from inequality (15) that

$$\|(F^\nu)^{-1} \circ C_a^{-1}\|_{\mathcal{C}^1(\bar{B}(0, r))} \leq C''_r$$

for a positive constant  $C''_r$  and finally that

$$\|(F^\nu)^{-1}\|_{\mathcal{C}^1(C_a^{-1}(\bar{B}(0, r)))} \leq \tilde{C}_r$$

for every sufficiently large  $\nu$ , where  $\tilde{C}_r$  is a positive constant.

Since  $D$  is relatively compact in  $M$ , Ascoli Theorem gives condition (ii) of Lemma 4. Lemma 5 implies that  $\tilde{F}$  is  $(J_0, J)$ -holomorphic. The end of the proof of Theorem 2 follows line by line the proof of Lemma 4 (iii). Since  $\tilde{F} \circ F = id_D$  and  $F \circ \tilde{F} = id_{\mathbb{H}}$  we obtain that  $F \in \text{Diff}_{(J, J_0)}(D, \mathbb{H})$ . This gives the contradiction.  $\square$

## REFERENCES

- [1] AUDIN, M., LAFONTAINE, J., Holomorphic curves in symplectic geometry, Birkhäuser, Progress in Mathematics, 117, 1994.
- [2] GAUSSIER, H., KIM, K.T., KRANTZ, S.G., A note on the Wong-Rosay theorem in complex manifolds, *Complex Var. Theory Appl.* **47** (2002), no. 9, 761–768.
- [3] GAUSSIER, H., SUKHOV, A., Estimates of the Kobayashi metric on almost complex manifolds, *Prépublication LATP 03-011*.
- [4] KRUGLIKOV, B.S., Existence of close pseudoholomorphic disks for almost complex manifolds and their application to the Kobayashi-Royden pseudonorm. (Russian) *Funktsional. Anal. i Prilozhen.* **33** (1999), no. 1, 46–58, 96; translation in *Funct. Anal. Appl.* **33** (1999), no. 1, 38–48.
- [5] NIJENHUIS, A., WOOLF, W., Some integration problems in almost-complex and complex manifolds, *Ann. Math.*, **77** (1963), 424-489.
- [6] PINCHUK, S., The scaling method and holomorphic mappings, *Several complex variables and complex geometry, Part 1* (Santa Cruz, CA, 1989), 151–161, *Proc. Sympos. Pure Math.*, 52, Part 1, Amer. Math. Soc., Providence, RI, 1991.
- [7] ROSAY, J.P., Sur une caractérisation de la boule parmi les domaines de  $\mathbb{C}^n$  par son groupe d'automorphismes, *Ann. Inst. Fourier*, **29** (1979), 91-97.
- [8] SIKORAV, J.C., Some properties of holomorphic curves in almost complex manifolds, pp.165-189, in Holomorphic curves in symplectic geometry, Michèle Audin, Jacques Lafontaine Editors, Birkhäuser (1994).
- [9] WONG, B., Characterization of the unit ball in  $C^n$  by its automorphism group, *Invent. Math.*, **41** (1977), no. 3, 253-257.

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