



**HAL**  
open science

## Continuous variable entanglement using cold atoms

Vincent Josse, Aurelien Dantan, Alberto Bramati, Michel Pinard, Elisabeth Giacobino

► **To cite this version:**

Vincent Josse, Aurelien Dantan, Alberto Bramati, Michel Pinard, Elisabeth Giacobino. Continuous variable entanglement using cold atoms. 2003. hal-00000437v1

**HAL Id: hal-00000437**

**<https://hal.science/hal-00000437v1>**

Preprint submitted on 23 Jun 2003 (v1), last revised 1 Dec 2003 (v3)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Continuous variable entanglement using cold atoms

V. Josse, A. Dantan, A. Bramati, M. Pinard, and E. Giacobino

Laboratoire Kastler Brossel, Université Pierre et Marie Curie, Ecole Normale Supérieure et CNRS,  
Case 74, 4 place Jussieu, 75252 Paris Cedex 05, France

(Dated: June 23, 2003)

We present experimental demonstration of *quadrature* and *polarization entanglement* generated via the interaction between a coherent linearly polarized field and cold atoms in a high finesse optical cavity. The non linear atom-field interaction produces two squeezed modes with orthogonal polarizations which are used to generate a pair of non separable beams, the entanglement of which is measured using the inseparability criterion for continuous variables recently derived by Duan *et al.* [Phys. Rev. Lett. **84**, 2722 (2000)].

PACS numbers: 42.50.Dv, 42.50.Ct, 03.67.Hk

Entanglement in the continuous variable regime has attracted a lot of attention in the quantum optics and quantum information fields in connection with quantum teleportation, cryptography, quantum computing and dense coding [1]. Since the first realization of quadrature entangled beams by Ou *et al.* [2], various methods, such as  $\chi^{(2)}$  process in OPA [3] or Kerr effect in optical fibers [4], have been used to generate non separable beams. Recently, the concept of *polarization entanglement*, i.e. entanglement of Stokes operators between two beams, has been investigated by Korolkova *et al.* [5], and first demonstrated experimentally by Bowen *et al.* [6] by mixing two squeezed beams issued from independent OPAs. The Kerr non linearity of fibers was also exploited by Glöckl *et al.* to generate a pulsed source of polarization entanglement [7]. Very recently, we reported on the generation of *polarization squeezing* using the interaction between a linearly polarized coherent field and a cloud of cold cesium atoms placed in a high finesse optical cavity [8]. We have demonstrated that both the mean field mode and the vacuum mode with orthogonal polarization exiting the cavity can be squeezed. In this paper we focus on the continuous variable entanglement generated in this experiment and use the *inseparability criterion* proposed by Duan *et al.* and Simon [9] to quantify the entanglement. The paper is organized as follows: first, we put forward a method for determining the maximally entangled modes in our system and show how a direct measurement of the quadrature entanglement of the beam exiting the cavity can be performed using two balanced homodyne detections. Secondly, we present the experimental set-up and results. In the last part of the paper, we produce two non separable beams through mixing with a local oscillator (LO) and achieve polarization entanglement by locking the relative phase between the LO and the field exiting the cavity.

First, let us consider two orthogonally polarized modes  $A_a$  and  $A_b$  of the electromagnetic field satisfying the usual bosonic commutation rules  $[A_\alpha, A_\beta^\dagger] = \delta_{\alpha\beta}$ . If  $X_\alpha(\theta) = (A_\alpha^\dagger e^{i\theta} + A_\alpha e^{-i\theta})$  and  $Y_\alpha(\theta) = X_\alpha(\theta + \pi/2)$  are the usual quadrature operators,  $X_a + X_b$  and  $Y_a - Y_b$  are

the continuous variable analogous of the EPR-type operators first introduced by Einstein, Podolsky and Rosen [10]. The inseparability criterion of [9] sets a limit on the sum of these EPR operators variances

$$\mathcal{I}_{a,b}(\theta) = \Delta^2(X_a + X_b)(\theta) + \Delta^2(Y_a - Y_b)(\theta) < 2 \quad (1)$$

For states with Gaussian statistics,  $\mathcal{I}_{a,b} < 2$  is a necessary and sufficient condition for entanglement and has already been used to quantify continuous variable entanglement [5, 6, 7].

In our system, an  $x$ -polarized beam interact with a cloud of cold cesium atoms in an optical cavity. The experimental set-up, also detailed in [8] is shown in Fig. 2. The cavity coupling mirror has a transmission coefficient of 10%, the rear mirror is highly reflecting. Hence, the cavity is close to the bad cavity limit for which the cavity linewidth (5 MHz) is larger than the atomic linewidth (2.6 MHz). We probe the atoms with a linearly polarized laser beam detuned of about 50 MHz in the red of the  $6S_{1/2}$ ,  $F=4$  to  $6P_{3/2}$ ,  $F=5$  transition. The optical power of the probe beam ranges from 5 to 15  $\mu$ W. After exiting the cavity, both the mean field mode  $A_x$  and the orthogonally polarized vacuum mode  $A_y$  are squeezed for frequencies ranging between 3 and 12 MHz. An interpretation of these results can be provided by modelling the complicated  $6S_{1/2}$ ,  $F=4$  to  $6P_{3/2}$ ,  $F=5$  transition by an X-like four-level atomic structure as in Fig. 1(a). For times shorter than the optical pumping times, this system is equivalent to two independent two-level systems [11]. The circularly polarized components of the light,  $A_\pm$ , are then uncorrelated. When both transitions are saturated, the atoms behave as a Kerr-like medium for  $A_\pm$  [12], both of which may be squeezed. Because of the symmetry of the system, they are squeezed for the same quadrature. It is then straightforward to see that the  $x, y$  modes are also squeezed on account of cross-Kerr effect, but for *orthogonal* quadratures [8].

Our goal is to retrieve the two modes with orthogonal polarizations which are maximally entangled in the sense of the inseparability criterion (1). One has to minimize the

ccsd-00000437 (version 1) : 23 Jun 2003

quantity  $\mathcal{I}_{a,b}(\theta)$  with respect to  $a, b$  and  $\theta$ . Expanding (1) yields

$$\mathcal{I}_{a,b}(\theta) = \langle \delta A_a^\dagger \delta A_a + \delta A_a \delta A_a^\dagger + \delta A_b \delta A_b^\dagger + \delta A_b^\dagger \delta A_b \rangle + 2[e^{-2i\theta} \langle \delta A_a \delta A_b \rangle + e^{2i\theta} \langle \delta A_a^\dagger \delta A_b^\dagger \rangle] \quad (2)$$

The right hand side of the first line in (2) being independent of the polarization basis, one has to maximize  $|\langle \delta A_a \delta A_b \rangle|$  with respect to  $a, b$ . It can be shown that there always exists a polarization basis  $u, v$  such that  $\langle \delta A_u \delta A_v \rangle = 0$  and the  $u, v$  modes quadrature variances are minimal for the same value of  $\theta$  [13]. In principle, these modes are not *a stricto sensu* uncorrelated, since  $\langle \delta A_u \delta A_v^\dagger \rangle$  can be non zero. However, the fundamental result is that they are "uncorrelated" in the sense of the inseparability criterion: one can then think of the correlation properties as being created by the mixing of two independent beams [2, 4, 6]. The optimal entanglement produced in the system is directly related to the quantum noise properties of these modes  $u, v$ . Indeed, the maximally entangled modes  $a^*, b^*$  are

$$A_{a^*} = (A_u + iA_v)/\sqrt{2}, \quad A_{b^*} = (A_u - iA_v)/\sqrt{2} \quad (3)$$

and the minimum value of  $\mathcal{I}_{a,b}$  is then given by the sum of the  $u, v$  modes minimal noises

$$\mathcal{I}_{a^*,b^*} = \min_{a,b,\theta} \mathcal{I}_{a,b}(\theta) = \min_{\theta} [\Delta^2 X_u(\theta) + \Delta^2 X_v(\theta)] \quad (4)$$

Conversely, if one or two of the  $u, v$  modes are squeezed the value  $\mathcal{I}_{a^*,b^*}$  of the optimal entanglement is equal to the best sum of squeezing that can be obtained in the system. This analysis paves the way to the experimental determination of the best entanglement produced in a system exhibiting quantum correlations: one has to look for the "uncorrelated" modes  $u, v$ , an experimental signature being that  $\mathcal{I}_{u,v}(\theta)$  does not depend on  $\theta$  [see (2)], and measure their squeezing (if any). The maximally entangled modes are then given by (3) and their entanglement by (4). This method could be of use for experimental schemes in which the correlations are not produced by mixing independent beams [14, 15].

In our system,  $A_x$  and  $A_y$  are uncorrelated and squeezed for orthogonal quadratures, so that one can set  $A_u = A_x$  and  $A_v = iA_y$ , and using (3), one can see that the maximally entangled modes are the  $\pm 45^\circ$  modes to the  $x, y$  basis. Equivalently, one could also set  $A_u = A_+$  and  $A_v = A_-$ , which are uncorrelated and squeezed for the same quadratures, and reach the same conclusion.

These results may be summarized using the classical Poincaré sphere representation [5]. This representation relies on the Stokes parameters which characterize the polarization state of a light beam

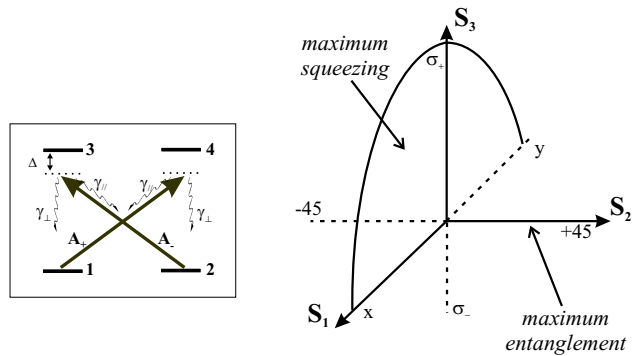


FIG. 1: *Left*: X-like atomic level structure. *Right*: Poincaré sphere for the Stokes parameters. While the sum of squeezing is maximal in the  $(S_1, S_3)$  plane, entanglement is optimal along the  $S_2$ -axis.

$$\begin{aligned} S_0 &= A_x^* A_x + A_y^* A_y & S_1 &= A_x^* A_x - A_y^* A_y \\ S_2 &= A_x^* A_y + A_y^* A_x & S_3 &= i(A_y^* A_x - A_x^* A_y) \end{aligned}$$

In the Poincaré sphere [see Fig. 1(b)], the previous reasoning tells us that the sum of squeezing is maximal in the meridional plane of the uncorrelated modes, defined by  $S_1$  and  $S_3$ , while the optimal entanglement axis is that of the  $\pm 45^\circ$  modes. This gives us the relevant quantity -  $\mathcal{I}_{+45,-45}(\theta)$  - which has to be measured experimentally. Indeed, using  $A_{\pm 45} = (A_x \pm A_y)/\sqrt{2}$ , the inseparability criterion for the  $\pm 45^\circ$  modes can be expressed directly in terms of the  $x, y$  modes variances

$$\mathcal{I}_{+45,-45}(\theta) = \Delta^2 X_x(\theta) + \Delta^2 Y_y(\theta) < 2 \quad (5)$$

When  $\theta$  corresponds to the angle of the squeezed quadrature of  $A_x$ , both variances are below unity, and  $\mathcal{I}_{\pm 45} < 2$ .

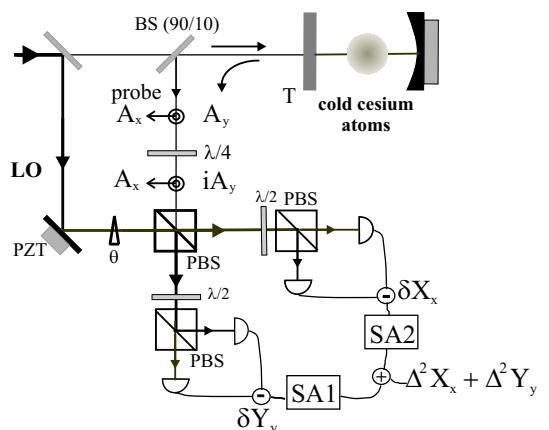


FIG. 2: Experimental set-up: PBS: polarizing beam splitter;  $\lambda/2$ : half-wave plate; PZT: piezo-electric ceramic.

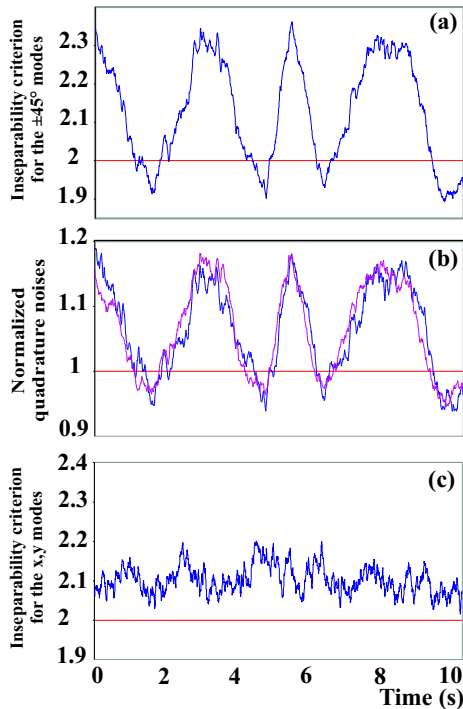


FIG. 3: (a) Direct measurement of  $\mathcal{I}_{+45,-45}$ , at a frequency of 5 MHz, when the relative phase  $\theta$  between the LO and the mean field mode is varied. (b) Corresponding quadrature noise spectra for  $A_x$  and  $iA_y$ . (c) Measurement of  $\mathcal{I}_{x,y}(\theta)$  at the same frequency.

In order to experimentally measure the inseparability criterion (5), we need to simultaneously measure the fluctuations of both  $A_x$  and  $iA_y$ . After exiting the cavity, the beam is mixed on a beamsplitter with a local oscillator (LO), and the two resulting beams are sent through two balanced homodyne detections. We insert a quarter-wave plate that dephases  $A_y$  by  $\pi/2$  and simultaneously measure the quadrature noise of each beam for the same quadrature. The sum of these two signals directly gives the sought quantity  $\mathcal{I}_{+45,-45}(\theta)$ . In Fig. 3(a), we plot a typical measurement of the entanglement as a function of  $\theta$ , for an analysis frequency of 5 MHz. Its minimal value is about 1.9 and corresponds to a case for which  $A_x$  and  $iA_y$  are both squeezed by about 5% [Fig. 3(b)]. As expected,  $\mathcal{I}_{x,y}$  is almost independent of  $\theta$ , since these modes are uncorrelated. *Quadrature entanglement* is thus achieved in a frequency range given by the cavity bandwidth (3 to 12 MHz). In good agreement with the theory, we find that the best entanglement is found around 5 MHz and retrieve the same squeezing on  $A_x$  and  $iA_y$  in the high frequency limit [8].

Last, we show that this quadrature entangled beam allows to generate *polarization entanglement*. The entangled modes  $A_{\pm 45}$  are simply obtained from the  $x, y$  modes using a half-wave plate and then mixed with the LO on a PBS. One retrieves two spatially separated beams  $\alpha$  and

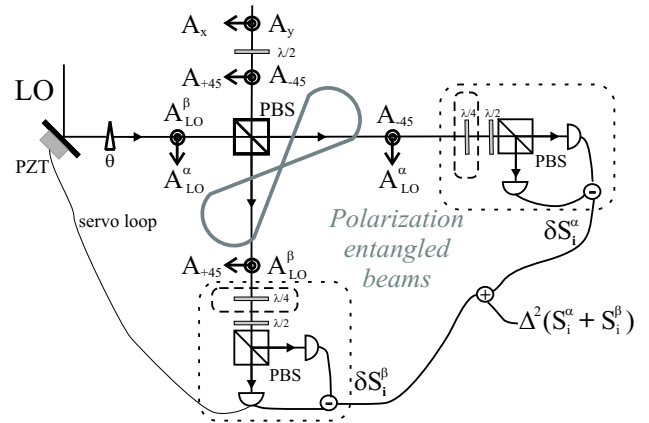


FIG. 4: Set-up for non separable beams generation. Inserting the quarter-wave plates (or not) allows for measuring the fluctuations of  $S_3^\alpha + S_3^\beta$  (or  $S_2^\alpha + S_2^\beta$ ). The servo loop is used to lock the LO phase to the squeezed quadrature angle.

$\beta$ , as represented on Fig. 4. Defining two sets of quantum Stokes operators [5] for the new beams,  $S_i^\alpha$  (from  $A_{LO}^\alpha$  and  $A_{-45}$ ) and  $S_i^\beta$  (from  $A_{LO}^\beta$  and  $A_{+45}$ ) and denoting by  $\alpha_{LO}$  the LO amplitude ( $|\alpha_{LO}| \gg |\alpha_{\pm 45}|$ ), one has  $\langle S_1^\alpha \rangle = -\langle S_1^\beta \rangle \simeq |\alpha_{LO}|$ , so that the analogous of the EPR-type operators are now  $S_2^\alpha + S_2^\beta$  and  $S_3^\alpha + S_3^\beta$

$$\langle [S_2^\alpha + S_2^\beta, S_3^\alpha + S_3^\beta] \rangle = 2i\langle S_1^\alpha + S_1^\beta \rangle \simeq 0$$

A similar criterion to (1) may be written for these conjugate variables by introducing the quantity [5]

$$\mathcal{I}_{\alpha,\beta}^S = \Delta^2(S_2^\alpha + S_2^\beta) + \Delta^2(S_3^\alpha + S_3^\beta) \quad (6)$$

Since the commutator between  $S_2$  and  $S_3$  depends on  $S_1$ , the inseparability condition is more complicated in this case. Polarization entanglement [5] is then achieved when

$$\mathcal{I}_{\alpha,\beta}^S < 2[|\langle S_1^\alpha \rangle| + |\langle S_1^\beta \rangle|] = 4|\alpha_{LO}|^2 \quad (7)$$

Bearing in mind that  $|\alpha_{LO}| \gg |\alpha_{\pm 45}|$ , the Stokes parameters fluctuations are related to those of the  $\pm 45^\circ$  modes [8]

$$\begin{aligned} \delta S_2^\alpha &= \alpha_{LO} \delta X_{-45}, & \delta S_2^\beta &= \alpha_{LO} \delta X_{+45} \\ \delta S_3^\alpha &= \alpha_{LO} \delta Y_{-45}, & \delta S_3^\beta &= -\alpha_{LO} \delta Y_{+45} \end{aligned}$$

which straightforwardly leads to

$$\mathcal{I}_{\alpha,\beta}^S = 2|\alpha_{LO}|^2 \mathcal{I}_{+45,-45}$$

The polarization entanglement condition  $\mathcal{I}_{\alpha,\beta}^S < 4|\alpha_{LO}|^2$  is thus directly equivalent to the

inseparability criterion (5) for the  $\pm 45^\circ$  modes. Therefore, quadrature entanglement can be mapped into a polarization basis and lead to polarization entanglement [6]. To prove this equivalence and check the consistency with the direct measurement we use the set-up shown in Fig. 4 and lock the phase of the LO with the squeezed quadrature angle. We then alternatively measure the sum of the  $S_2$  or  $S_3$  Stokes operator noises by inserting the quarter-wave plates or not. In Fig. 5, we present the normalized quadrature noises of  $S_2^\alpha + S_2^\beta$  and  $S_3^\alpha + S_3^\beta$  for an analysis frequency of 5 MHz. Consistently with the direct measurement, we observe polarization entanglement on a 3 MHz-10 MHz frequency range. This entanglement between the beams corresponds to a reduction by approximately 5% in the noise of each conjugate variables,  $\mathcal{I}_{\alpha,\beta}^S/2|\alpha_{LO}|^2 = 1.9$ . Moreover, when both conjugate variables are simultaneously squeezed, that is, when both  $A_x$  and  $iA_y$  are squeezed, we even achieve *squeezed-state entanglement*, as defined in [5]. This entanglement is in principle richer since the variances of both conjugate variables in (6) have to drop below 1, whereas the polarization entanglement condition can be satisfied with only one Stokes operator squeezed and the other in a coherent state for instance.

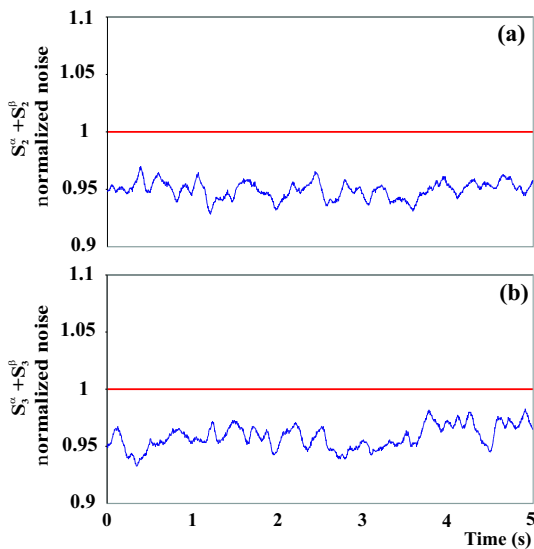


FIG. 5: Normalized noises at 5 MHz of  $S_2^\alpha + S_2^\beta$  (a) and  $S_3^\alpha + S_3^\beta$  (b), the LO phase  $\theta$  being locked with the value of squeezed quadrature angle.

To conclude, we have reported on the generation of continuous variable entanglement via the interaction with cold atoms in cavity. First, we have shown that the two

squeezed modes can be used to perform a direct measurement of the inseparability criterion. A pair of non separable beams was then generated, the entanglement of which was obtained via the measurement of the polarization noise of the spatially separated beams. Polarization entanglement was achieved after mapping the quadrature entanglement into two spatially separated beams.

This work was supported by the QIPC European Project No. IST-1999-13071 (QUICOV).

- 
- [1] C.H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993); S.L. Braunstein, H.J. Kimble, Phys. Rev. A **61**, 042321 (2000); D.P. DiVincenzo, Science **270**, 255 (1995), B. Julsgaard, A. Koshekin, E.S. Polzik, Nature **413**, 400 (2001).
  - [2] Z.Y. Ou, S.F. Pereira, H.J. Kimble, K.C. Peng, Phys. Rev. Lett. **68**, 3663 (1992).
  - [3] Y. Zhang, H. Wang, X. Li, J. Jing, C. Xie, K. Peng, Phys. Rev. A **62**, 023813 (2000).
  - [4] Ch. Silberhorn, P.K. Lam, O. Weiß, F. Köning, N. Korolkova, G. Leuchs, Phys. Rev. Lett. **86**, 4267 (2001).
  - [5] N. Korolkova, G. Leuchs, R. Loudon, T.C. Ralph, Ch. Silberhorn, Phys. Rev. A, **65**, 052306 (2002); N. Korolkova, Ch. Silberhorn, O. Glöckl, S. Lorenz, Ch. Marquardt, G. Leuchs, Eur. Phys. J. D **18**, 229 (2002); N. Korolkova, R. Loudon, quant-ph/0303135.
  - [6] W.P. Bowen, N. Treps, R. Schnabel, P.K. Lam, Phys. Rev. Lett. **89**, 253601 (2002); W.P. Bowen, R. Schnabel, P.K. Lam, T.C. Ralph, Phys. Rev. Lett. **90**, 043601 (2003).
  - [7] O. Glöckl, J. Heersink, N. Korolkova, G. Leuchs, S. Lorenz, quant-ph/0302083; O. Glöckl, S. Lorenz, C. Marquardt, J. Heersink, M. Brownnutt, Ch. Silberhorn, Q. Pan, P. van Loock, N. Korolkova, G. Leuchs, quant-ph/0302068.
  - [8] V. Josse, A. Dantan, L. Vernac, A. Bramati, M. Pinard, E. Giacobino, Phys. Rev. Lett., quant-ph/0304022.
  - [9] L.M. Duan, G. Giedke, J.I. Cirac, P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000); R. Simon, Phys. Rev. Lett. **84**, 2726 (2000).
  - [10] A. Einstein, B. Podolsky, R. Rosen, Phys. Rev. **47**, 777 (1935).
  - [11] V. Josse, A. Dantan, A. Bramati, M. Pinard, E. Giacobino, J. Opt. B, quant-ph/0302142.
  - [12] A. Lambrecht, E. Giacobino, J.M. Courty, Optics Comm. **115**, 199 (1995).
  - [13] V. Josse *et al.*, to be published.
  - [14] L. Longchambon, J. Laurat, T. Coudreau, C. Fabre, in preparation.
  - [15] L. Boivin, H.A. Haus, Optics Lett. **21**, 146 (1996).