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Twin polaritons in semiconductor microcavities

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Abstract

The quantum correlations between the beams generated by polariton pair scattering in a semiconductor microcavity above the parametric oscillation threshold are computed analytically. The influence of various parameters like the cavity-exciton detuning, the intensity mismatch between the signal and idler beams and the amount of spurious noise is analyzed. We show that very strong quantum correlations between the signal and idler polaritons can be achieved. The quantum effects on the outgoing light fields are strongly reduced due to the large mismatch in the coupling of the signal and idler polaritons to the external photons.

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INTRODUCTION

Exciton polaritons are the normal modes of the strong light-matter coupling in semiconductor microcavities [1]. These half-exciton, half-photon particles present large optical nonlinearities coming from the Coulomb interactions between the exciton components. Under resonant pumping, this leads to a parametric process where a pair of pump polaritons scatter into nondegenerate signal and idler modes while conserving energy and momentum. The scattering is particularly strong in microcavities because the unusual shape of the polariton dispersion makes it possible for the pump, signal and idler to be on resonance at the same time (see Fig. 1). Moreover, the relationship between the in-plane momentum of each polariton mode and the direction of the external photon to which it couples [2] enables to investigate the parametric scattering using measurements at different angles to access the various modes.

The first demonstration of parametric processes in semiconductor microcavities was performed by Savvidis *et al.* [3] using ultrafast pump-probe measurements. He observed parametric amplification, where the scattering is stimulated by excitation of the signal mode with a weak probe field. Parametric oscillation, where there is no probe and a coherent population in the signal and idler modes appears spontaneously, has since been observed by Stevenson *et al.* [4] and Baumberg *et al.* [5] in cw experiments. The lower polariton was pumped resonantly at the "magic" angle of about 16° . Above a threshold pump intensity, strong signal and idler beams were observed at about 0° and 35° , without any probe stimulation. The coherence of these beams was demonstrated by a significant spectral narrowing.

The large optical nonlinearity of cavity polaritons makes them very attractive for quantum optics. Noise reduction on the reflected light field has been predicted [6] and achieved experimentally [7] for a resonant pumping of the lower polariton at 0° . The parametric fluorescence was recently shown to produce strongly correlated pairs of signal and idler polaritons, yielding a two-mode squeezed state [8]. The parametric oscillation regime is also very interesting in this respect [9]. It is well known that optical parametric oscillators (OPO) can be used to generate twin beams, the fluctuations of which are correlated at the quantum level. A noise reduction of 86% was obtained by subtracting the intensities of the signal and idler beams produced by a LiNbO₃ OPO [10].

The purpose of this paper is to investigate the possibility of generating twin beams using

a semiconductor microcavity above the parametric oscillation threshold. The classical model developed by Whittaker [11] is no longer sufficient to study the quantum noise properties of the system. Thus we adapt the quantum model by Ciuti *et al.*, previously used in the context of parametric amplification [12] and parametric fluorescence [8, 13], to the parametric oscillator configuration. Furthermore we compute the field fluctuations using the input-output method [14, 15]. We also include the excess noise associated with the excitonic relaxation, which had not been done in Refs. [8, 13].

MODEL

Hamiltonian

Following Ciuti *et al.* [12][13] we write the effective Hamiltonian for the coupled exciton-photon system. The spin degree of freedom is neglected.

$$H = H_0 + H_{exc-exc} + H_{sat} \quad (1)$$

The first term is the linear Hamiltonian for excitons and cavity photons

$$H_0 = \sum_{\mathbf{k}} E_{exc}(k) b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \sum_{\mathbf{k}} E_{cav}(k) a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \sum_{\mathbf{k}} \hbar \Omega_R (a_{\mathbf{k}}^\dagger b_{\mathbf{k}} + b_{\mathbf{k}}^\dagger a_{\mathbf{k}}) \quad (2)$$

with $b_{\mathbf{k}}^\dagger$ and $a_{\mathbf{k}}^\dagger$ the creation operators respectively for excitons and photons of in-plane wave vector \mathbf{k} , which satisfy boson commutation rules. $E_{exc}(k)$ and $E_{cav}(k)$ are the energy dispersions for exciton and cavity mode. The last term represents the linear coupling between exciton and cavity photon which causes the vacuum Rabi splitting $2\hbar\Omega_R$. The fermionic nature of electrons and holes causes a deviation of the excitons from bosonic behavior, which is accounted for through an effective exciton-exciton interaction and exciton saturation. The exciton-exciton interaction term writes

$$H_{exc-exc} = \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V_q b_{\mathbf{k}+\mathbf{q}}^\dagger b_{\mathbf{k}'-\mathbf{q}}^\dagger b_{\mathbf{k}} b_{\mathbf{k}'} \quad (3)$$

where $V_q \simeq V_0 = \frac{6e^2 a_{exc}}{\epsilon_0 A}$ for $q a_{exc} \ll 1$, a_{exc} being the two-dimensional exciton Bohr radius, ϵ_0 the dielectric constant of the quantum well and A the macroscopic quantization area. The saturation term in the light-exciton coupling is

$$H_{sat} = - \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V_{sat} \left(a_{\mathbf{k}+\mathbf{q}}^\dagger b_{\mathbf{k}'-\mathbf{q}}^\dagger b_{\mathbf{k}} b_{\mathbf{k}'} + a_{\mathbf{k}+\mathbf{q}} b_{\mathbf{k}'-\mathbf{q}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}'}^\dagger \right) \quad (4)$$

where $V_{sat} = \frac{\hbar \Omega_R}{n_{sat} A}$ with $n_{sat} = 7 / (16 \pi a_{exc}^2)$ being the exciton saturation density. We consider resonant or quasi-resonant excitation of the lower polariton branch by a quasi-monochromatic laser field of frequency $\omega_L = E_L / \hbar$ and wave vector \mathbf{k}_L . If the pump intensity is not too high the resonances (i.e. the polariton states) are not modified. Then it is much more convenient to work directly in the polariton basis. It is possible to neglect nonlinear contributions related to the upper branch and consider only the lower polariton states. The polariton operators are obtained by a unitary transformation of the exciton and photon operators:

$$\begin{pmatrix} p_{\mathbf{k}} \\ q_{\mathbf{k}} \end{pmatrix} = \begin{pmatrix} -C_k & X_k \\ X_k & C_k \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}} \\ b_{\mathbf{k}} \end{pmatrix} \quad (5)$$

where X_k et C_k are positive real numbers called the Hopfield coefficients, given by

$$X_k^2 = \frac{\delta_k + \sqrt{\delta_k^2 + \Omega_R^2}}{2\sqrt{\delta_k^2 + \Omega_R^2}} \quad (6)$$

$$C_k^2 = \frac{\Omega_R^2}{2\sqrt{\delta_k^2 + \Omega_R^2} (\delta_k + \sqrt{\delta_k^2 + \Omega_R^2})} \quad (7)$$

X_k^2 and C_k^2 can be interpreted respectively as the exciton and photon fraction of the lower polariton $p_{\mathbf{k}}$. In terms of the lower polariton operators, the Hamiltonian (1) reads

$$H = H_P + H_{PP}^{eff} \quad (8)$$

H_P is the free evolution term for the lower polariton:

$$H_P = \sum_{\mathbf{k}} E_P(k) p_{\mathbf{k}}^\dagger p_{\mathbf{k}} \quad (9)$$

and H_{PP}^{eff} is an effective polariton-polariton interaction:

$$H_{PP}^{eff} = \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V_{\mathbf{k}, \mathbf{k}', \mathbf{q}}^{PP} p_{\mathbf{k}+\mathbf{q}}^\dagger p_{\mathbf{k}'-\mathbf{q}}^\dagger p_{\mathbf{k}} p_{\mathbf{k}'} \quad (10)$$

where

$$V_{\mathbf{k}, \mathbf{k}', \mathbf{q}}^{PP} = \{V_0 X_{|\mathbf{k}+\mathbf{q}|} X_{k'} + 2V_{sat} \times (C_{|\mathbf{k}+\mathbf{q}|} X_{k'} + C_{k'} X_{|\mathbf{k}+\mathbf{q}|})\} X_{|\mathbf{k}'-\mathbf{q}|} X_k \quad (11)$$

In the following we neglect the contribution of the saturation term, so that $V_{\mathbf{k}, \mathbf{k}', \mathbf{q}}^{PP} \simeq V_0 X_{|\mathbf{k}+\mathbf{q}|} X_{k'} X_{|\mathbf{k}'-\mathbf{q}|} X_k$. We also neglect multiple diffusions i.e. interaction between modes other than the pump mode. This approximation is valid only slightly above the parametric oscillation threshold [28]. It comes to considering only the terms where the pump polariton operator $p_{\mathbf{k}_L}$ appears at least twice:

$$\begin{aligned} H_{PP}^{eff} &= \frac{1}{2} V_{\mathbf{k}_L, \mathbf{k}_L, 0} p_{\mathbf{k}_L}^\dagger p_{\mathbf{k}_L}^\dagger p_{\mathbf{k}_L} p_{\mathbf{k}_L} \\ &+ \sum_{\mathbf{k} \neq \mathbf{k}_L} V_{\mathbf{k}_L, \mathbf{k}_L, \mathbf{k}_L - \mathbf{k}} \left(p_{2\mathbf{k}_L - \mathbf{k}}^\dagger p_{\mathbf{k}}^\dagger p_{\mathbf{k}_L} p_{\mathbf{k}_L} + h.c. \right) \\ &+ 2 \sum_{\mathbf{k} \neq \mathbf{k}_L} V_{\mathbf{k}, \mathbf{k}_L, 0} p_{\mathbf{k}_L}^\dagger p_{\mathbf{k}}^\dagger p_{\mathbf{k}_L} p_{\mathbf{k}} \end{aligned} \quad (12)$$

The first term is a Kerr-like term for the polaritons in the pump mode. The second term is a "fission" process, where two polaritons of wave vector \mathbf{k}_L are converted into a "signal" polariton of wave vector \mathbf{k} and an "idler" polariton of wave vector $2\mathbf{k}_L - \mathbf{k}$. The last term corresponds to the interaction of the pump mode \mathbf{k}_L with all the other \mathbf{k} states, which results in a blueshift proportional to $|p_{\mathbf{k}_L}|^2$.

Energy conservation

The energy conservation for the fission process $\{\mathbf{k}_L, \mathbf{k}_L\} \rightarrow \{\mathbf{k}, 2\mathbf{k}_L - \mathbf{k}\}$ reads

$$\tilde{E}_P(\mathbf{k}) + \tilde{E}_P(2\mathbf{k}_L - \mathbf{k}) = 2\tilde{E}_P(\mathbf{k}_L) \quad (13)$$

where $\tilde{E}_P(\mathbf{q})$ is the energy of the polariton of wave vector \mathbf{q} , renormalized by the interaction with the pump polaritons

$$\tilde{E}_P(\mathbf{q}) = E_P(q) + 2V_{\mathbf{q},\mathbf{k}_L,0} |\langle p_{\mathbf{k}_L} \rangle|^2 \quad (14)$$

Note that the factor of 2 disappears for $\mathbf{q} = \mathbf{k}_L$. Equation (13) always has a trivial solution $\mathbf{k} = \mathbf{k}_L$. Non-trivial solutions exist provided the wave vector \mathbf{k}_L is above a critical value, or equivalently if the angle of incidence is above the so-called "magic angle" θ_c [3]. From now on we suppose that the microcavity is excited resonantly with an angle θ_c . Fig. 2 is a plot of the quantity $|E_P(\mathbf{k}) + E_P(2\mathbf{k}_L - \mathbf{k}) - 2E_P(\mathbf{k}_L)|$ as a function of $\mathbf{k} = \{k_x, k_y\}$, \mathbf{k}_L being parallel to the x axis.

This shows that energy conservation can be satisfied for a wide range of wave vectors $\{\mathbf{k}, 2\mathbf{k}_L - \mathbf{k}\}$. In recent experiments, parametric oscillation was observed in the normal direction $\mathbf{k} = 0$ [4, 16]. In this paper we consider only the parametric process $\{\mathbf{k}_L, \mathbf{k}_L\} \longrightarrow \{\mathbf{0}, 2\mathbf{k}_L\}$ assuming that the other ones remain below threshold. Then we can neglect the effect of modes other than $\mathbf{0}, \mathbf{k}_L, 2\mathbf{k}_L$. The evolution of these three modes is given by a closed set of equations.

HEISENBERG-LANGEVIN EQUATIONS

In order to study the quantum fluctuations we have to write the Heisenberg-Langevin equations including the relaxation and fluctuation terms. The relaxation of the cavity mode comes from the interaction with the external electromagnetic field through the Hamiltonian

$$H_I = i\hbar \int \frac{d\omega}{2\pi} \kappa \left(a_{\mathbf{k}}^\dagger A_\omega - A_\omega^\dagger a_{\mathbf{k}} \right) \quad (15)$$

The coupling constant is given by $\kappa = \sqrt{2\gamma_{ak}}$ where γ_{ak} is the cavity linewidth (HWHM). This leads to the following evolution equation for the cavity field:

$$\frac{da_{\mathbf{k}}}{dt}(t) = -\gamma_{ak} a_{\mathbf{k}}(t) + \sqrt{2\gamma_{ak}} A_{\mathbf{k}}^{in}(t) \quad (16)$$

where $A_{\mathbf{k}}^{in}(t)$ is the incoming external field. In this equation the normalization are not the same for the cavity field as for the external field: $n_{a_{\mathbf{k}}}(t) = \langle a_{\mathbf{k}}^\dagger(t) a_{\mathbf{k}}(t) \rangle$ is the mean number of cavity photons, while $I_{\mathbf{k}}^{in} = \langle A_{\mathbf{k}}^{in\dagger}(t) A_{\mathbf{k}}^{in}(t) \rangle$ is the mean number of incident photons per second.

Exciton relaxation is a much more complex problem. The density is assumed to be low enough to neglect the relaxation due to exciton-exciton interaction [17]. At low density and low enough temperature the main relaxation mechanism is the interaction with acoustic phonons. A given exciton mode $b_{\mathbf{k}}$ is coupled to all the other exciton modes $b_{\mathbf{k}'}$ and to all the phonon modes fulfilling the condition of energy and wavevector conservation [18]. Relaxation in microcavities in the strong coupling regime has been studied in detail [19, 20]. However, the derivation of the corresponding fluctuation terms requires additional hypotheses, under which one can replace the exciton-phonon coupling Hamiltonian by a linear coupling to a single reservoir [9]. Then, in the same way as for the photon field, the fluctuation-dissipation part in the Langevin equation for the excitons writes

$$\frac{db_{\mathbf{k}}(t)}{dt} = -\gamma_{b_{\mathbf{k}}}b_{\mathbf{k}}(t) + \sqrt{2\gamma_{b_{\mathbf{k}}}}B_{\mathbf{k}}^{in}(t) \quad (17)$$

where $\gamma_{b_{\mathbf{k}}}$ is the exciton linewidth (HWHM) and $B_{\mathbf{k}}^{in}(t)$ the input excitonic field, which is a linear combination of the reservoir modes.

Using these results we can write the Heisenberg-Langevin equations for the cavity and exciton modes of wave vectors $\mathbf{0}$, \mathbf{k}_L , $2\mathbf{k}_L$ and then for the three corresponding lower polariton modes. We define the slowly varying operators

$$\begin{aligned} \tilde{p}_{\mathbf{k}_L}(t) &= p_{\mathbf{k}_L}(t)e^{iE_L t/\hbar} \\ \tilde{p}_{\mathbf{0}}(t) &= p_{\mathbf{0}}(t)e^{iE_p(0)t/\hbar} \\ \tilde{p}_{2\mathbf{k}_L}(t) &= p_{2\mathbf{k}_L}(t)e^{iE_p(2k_L)t/\hbar} \end{aligned} \quad (18)$$

which obey the following equations:

$$\frac{d\tilde{p}_{\mathbf{0}}}{dt} = -\frac{i}{\hbar} \left(2V_{\mathbf{0},\mathbf{k}_L,\mathbf{0}}\tilde{p}_{\mathbf{k}_L}^\dagger\tilde{p}_{\mathbf{k}_L} - i\gamma_0 \right) \tilde{p}_{\mathbf{0}} \quad (19)$$

$$\begin{aligned} & -\frac{i}{\hbar} V_{\mathbf{k}_L,\mathbf{k}_L,\mathbf{k}_L}\tilde{p}_{2\mathbf{k}_L}^\dagger\tilde{p}_{\mathbf{k}_L}^2 e^{i\Delta Et/\hbar} + P_{\mathbf{0}}^{in} \\ \frac{d\tilde{p}_{2\mathbf{k}_L}}{dt} &= -\frac{i}{\hbar} \left(2V_{2\mathbf{k}_L,\mathbf{k}_L,\mathbf{0}}\tilde{p}_{\mathbf{k}_L}^\dagger\tilde{p}_{\mathbf{k}_L} - i\gamma_{2k_L} \right) \tilde{p}_{2\mathbf{k}_L} \quad (20) \\ & -\frac{i}{\hbar} V_{\mathbf{k}_L,\mathbf{k}_L,\mathbf{k}_L}\tilde{p}_{\mathbf{0}}^\dagger\tilde{p}_{\mathbf{k}_L}^2 e^{i\Delta Et/\hbar} + P_{2\mathbf{k}_L}^{in} \end{aligned}$$

$$\begin{aligned} \frac{d\tilde{p}_{\mathbf{k}_L}}{dt} &= -\frac{i}{\hbar} \left(\Delta_L + V_{\mathbf{k}_L,\mathbf{k}_L,\mathbf{0}}\tilde{p}_{\mathbf{k}_L}^\dagger\tilde{p}_{\mathbf{k}_L} - i\gamma_{k_L} \right) \tilde{p}_{\mathbf{k}_L} \quad (21) \\ & -\frac{2i}{\hbar} V_{\mathbf{k}_L,\mathbf{k}_L,\mathbf{k}_L}\tilde{p}_{\mathbf{k}_L}^\dagger\tilde{p}_{\mathbf{0}}\tilde{p}_{2\mathbf{k}_L} e^{-i\Delta Et/\hbar} + P_{\mathbf{k}_L}^{in} \end{aligned}$$

where for any given wave vector \mathbf{q} , $P_{\mathbf{q}}^{in} = -C_q\sqrt{2\gamma_{aq}}A_{\mathbf{q}}^{in} + X_q\sqrt{2\gamma_{bq}}B_{\mathbf{q}}^{in}$ is the polariton input field (which is a linear combination of the cavity and exciton input fields ; only the driving laser field $A_{\mathbf{k}_L}^{in}$ has a nonzero mean value), $\gamma_q = C_q^2\gamma_{aq} + X_q^2\gamma_{bq}$ is the polariton linewidth ; $\Delta_L = E_p(k_L) - E_L$ is the laser detuning ; $\Delta E = E_p(2k_L) + E_p(0) - 2E_L$ is the energy mismatch.

These equations extend the model developed by Ciuti *et al.* [13] above threshold. The full treatment of the field fluctuations is included as well as the equation of motion of the pumped mode accounting for the pump depletion. They are valid only slightly above threshold, because far above threshold when we can no longer neglect multiple scattering [21, 22].

This set of equations is similar to the evolution equations of a non-degenerate optical parametric oscillator (OPO) [23]. The non-linearity is of type $\chi^{(3)}$ while in most OPOs it is of type $\chi^{(2)}$. OPOs based on four-wave mixing have already been demonstrated [24]. However, let us stress that here the parametric process involves the excitations of a semiconductor material (i.e. polaritons) instead of photons. In the following we evaluate the potential applications of this new type of OPO in quantum optics. The hybrid nature of polaritons makes the treatment of quantum fluctuations more complicated, since we have to consider additional sources of noise (i.e. the luminescence of excitons).

MEAN FIELDS ABOVE THRESHOLD

To start with we have to compute the stationary state of the system. This comes to the calculation done by Whittaker in Ref. [11]. We neglect the renormalization effects due to the interaction with the pump mode, which allows to get analytical expressions. Moreover, we suppose that the angle of incidence is adjusted in order to satisfy the resonance condition $\Delta E = 0$ and that the pump laser is perfectly resonant ($\Delta_L=0$). Equations (19)-(21) now write

$$\frac{d\tilde{p}_0}{dt} = -\gamma_0\tilde{p}_0 - iE_{int}\tilde{p}_{2\mathbf{k}_L}^\dagger\tilde{p}_{\mathbf{k}_L}^2 + P_0^{in} \quad (22)$$

$$\frac{d\tilde{p}_{2\mathbf{k}_L}}{dt} = -\gamma_{2\mathbf{k}_L}\tilde{p}_{2\mathbf{k}_L} - iE_{int}\tilde{p}_0^\dagger\tilde{p}_{\mathbf{k}_L}^2 + P_{2\mathbf{k}_L}^{in} \quad (23)$$

$$\frac{d\tilde{p}_{\mathbf{k}_L}}{dt} = -\gamma_{\mathbf{k}_L}\tilde{p}_{\mathbf{k}_L} - 2iE_{int}\tilde{p}_{\mathbf{k}_L}^\dagger\tilde{p}_0\tilde{p}_{2\mathbf{k}_L} + P_{\mathbf{k}_L}^{in} \quad (24)$$

where $E_{int} = V_{\mathbf{k}_L, \mathbf{k}_L, \mathbf{k}_L} / \hbar$. Let us recall that among the polariton input fields, only the photon part of $P_{\mathbf{k}_L}^{in}$ corresponding to the pump laser field has a nonzero mean value. The excitonic input fields $B_{\mathbf{q}}^{in}$ correspond to the luminescence of the exciton modes and are incoherent fields with zero mean value. The stationary state is given by

$$-\gamma_{k_L} \bar{p}_{\mathbf{k}_L} - 2iE_{int} \bar{p}_{\mathbf{k}_L}^* \bar{p}_0 \bar{p}_{2\mathbf{k}_L} = C_{k_L} \sqrt{2\gamma_a} \bar{A}_{\mathbf{k}_L}^{in} \quad (25)$$

$$-\gamma_0 \bar{p}_0 - iE_{int} \bar{p}_{2\mathbf{k}_L}^* \bar{p}_{\mathbf{k}_L}^2 = 0 \quad (26)$$

$$-\gamma_{2k_L} \bar{p}_{2\mathbf{k}_L}^* + iE_{int} \bar{p}_0 \bar{p}_{\mathbf{k}_L}^{*2} = 0 \quad (27)$$

For a non trivial solution to exist, the determinant of the last two equations must be zero:

$$E_{int}^2 |\bar{p}_{\mathbf{k}_L}|^4 - \gamma_0 \gamma_{2k_L} = 0 \quad (28)$$

which gives the pump polariton population threshold

$$|\bar{p}_{\mathbf{k}_L}|^2 = \frac{\sqrt{\gamma_0 \gamma_{2k_L}}}{E_{int}} \quad (29)$$

and the pump intensity threshold

$$I_{\mathbf{k}_L, S}^{in} = \left| \bar{A}_{\mathbf{k}_L, S}^{in} \right|^2 = \frac{\gamma_{k_L}^2 (\gamma_0 \gamma_{2k_L})^{1/2}}{2\gamma_a C_{k_L}^2 E_{int}} \quad (30)$$

The signal and idler polariton populations are easily derived:

$$|\bar{p}_0|^2 = \frac{\gamma_{k_L}}{2E_{int}} \sqrt{\frac{\gamma_{2k_L}}{\gamma_0}} (\sigma - 1) \quad (31)$$

$$|\bar{p}_{2\mathbf{k}_L}|^2 = \frac{\gamma_{k_L}}{2E_{int}} \sqrt{\frac{\gamma_0}{\gamma_{2k_L}}} (\sigma - 1) \quad (32)$$

where $\sigma = \sqrt{I_{\mathbf{k}_L}^{in} / I_{\mathbf{k}_L, S}^{in}}$ is the pump parameter. We finally get the intensities of the signal and idler output light fields

$$I_{\mathbf{0}}^{out} = 2\gamma_a C_0^2 |\bar{p}_0|^2 = \frac{\gamma_a \gamma_{k_L} C_0^2}{E_{int}} \sqrt{\frac{\gamma_{2k_L}}{\gamma_0}} (\sigma - 1) \quad (33)$$

$$I_{2\mathbf{k}_L}^{out} = 2\gamma_a C_{2k_L}^2 |\bar{p}_{2\mathbf{k}_L}|^2 = \frac{\gamma_a \gamma_{k_L} C_{2k_L}^2}{E_{int}} \sqrt{\frac{\gamma_0}{\gamma_{2k_L}}} (\sigma - 1)$$

Above threshold, all the polaritons created by the pump are transferred to the signal and idler modes, so that the number of pump polaritons is clamped to a fixed value. This phenomenon called pump depletion is well-known in triply resonant OPOs. The signal and idler intensities grow like $\sqrt{I_{\mathbf{k}_L}^{in}}$. These results are in agreement with those of Ref. [11].

Finally we study the ratio of the signal and idler output intensities, which is an important parameter in view of the analysis of the correlations between these two beams. It is given by the simple equation

$$\frac{I_{\mathbf{0}}^{out}}{I_{2\mathbf{k}_L}^{out}} = \frac{\gamma_{2k_L} C_0^2}{\gamma_0 C_{2k_L}^2} \quad (34)$$

We consider a typical III-V microcavity sample containing one quantum well, with a Rabi splitting $2\hbar\Omega_R = 2.8$ meV. At zero cavity-exciton detuning, one finds $k_L = 1.15 \cdot 10^4$ cm⁻¹. The photon fractions of the signal and idler modes are respectively $C_0^2 = 0.5$ and $C_{2k_L}^2 \simeq 0.053$. Assuming that they have equal linewidths the signal beam power should be about ten times that of the idler beam. It is possible to reduce this ratio a bit by increasing the cavity-exciton detuning, as can be seen in Fig 3. However, the oscillation threshold goes up. In the following, all the results will be given at zero detuning.

FLUCTUATIONS

Linearized evolution equations

For any operator $O(t)$ we define a fluctuation operator $\delta O(t) = O(t) - \langle O(t) \rangle$. In order to compute the fluctuations, we use the "semiclassical" linear input-output method, which consists in studying the transformation of the incident fluctuations by the system [15]. It has been shown to be equivalent to a full quantum treatment. We linearize equations (22)-(24) in the vicinity of the working point p_0 computed in the previous section. We obtain the following set of equations:

$$\frac{d\delta p_{\mathbf{k}_L}}{dt} = -\gamma_{\mathbf{k}_L} \delta p_{\mathbf{k}_L} - 2iE_{int} \left(\bar{p}_0 \bar{p}_{2\mathbf{k}_L} \delta p_{\mathbf{k}_L}^\dagger + \bar{p}_{\mathbf{k}_L}^* \bar{p}_{2\mathbf{k}_L} \delta p_0 + \bar{p}_{\mathbf{k}_L}^* \bar{p}_0 \delta p_{2\mathbf{k}_L} \right) + \delta P_{\mathbf{k}_L}^{in} \quad (35)$$

$$\frac{d\delta p_0}{dt} = -\gamma_0 \delta p_0 - iE_{int} \left(2\bar{p}_{2\mathbf{k}_L}^* \bar{p}_{\mathbf{k}_L} \delta p_{\mathbf{k}_L} \right) \quad (36)$$

$$\frac{d\delta p_{2\mathbf{k}_L}}{dt} = -\gamma_{2k_L} \delta p_{2\mathbf{k}_L} - iE_{int} \left(2\bar{p}_0^* \bar{p}_{\mathbf{k}_L} \delta p_{\mathbf{k}_L} \right. \\ \left. \bar{p}_{\mathbf{k}_L}^2 \delta p_0^\dagger \right) + \delta P_{\mathbf{0}}^{in} \quad (37)$$

We can now inject the mean values of the fields $p_{\mathbf{k}_L}$, p_0 et $p_{2\mathbf{k}_L}$ that we have computed in the preceding section (equations (29), (31) and (32)).

First we have to choose the phases of the fields (this choice has no influence on the physics of the problem). We set the phase of the pump field $A_{\mathbf{k}_L}^{in}$ to zero. Then $\bar{p}_{\mathbf{k}_L}$ is a positive real number. The equations (26) and (27) impose the same relationship between the signal φ_0 and idler $\varphi_{2\mathbf{k}_L}$ phases:

$$\varphi_0 + \varphi_{2\mathbf{k}_L} = -\frac{\pi}{2} \quad (38)$$

whereas the relative phase $\varphi_0 - \varphi_{2\mathbf{k}_L}$ is a free parameter. We set \bar{p}_0 to be a real positive number (again, this choice is of no consequence regarding the physics of the problem). Then $\bar{p}_{2\mathbf{k}_L}$ is a pure imaginary number. With these choices of phase, the evolution equations write

$$\frac{d\delta p_{\mathbf{k}_L}}{dt} = -\gamma_{k_L} \left(\delta p_{\mathbf{k}_L} + (\sigma - 1) \delta p_{\mathbf{k}_L}^\dagger \right) \\ - \sqrt{2\gamma_{k_L} \gamma_0 (\sigma - 1)} \delta p_0 \\ - i\sqrt{2\gamma_{k_L} \gamma_{2k_L} (\sigma - 1)} \delta p_{2\mathbf{k}_L} + \delta P_{\mathbf{k}_L}^{in} \quad (39)$$

$$\frac{d\delta p_0}{dt} = -\gamma_0 \delta p_0 + \sqrt{2\gamma_{k_L} \gamma_0 (\sigma - 1)} \delta p_{\mathbf{k}_L} \\ - i\sqrt{\gamma_0 \gamma_{2k_L}} \delta p_{2\mathbf{k}_L}^\dagger + \delta P_{\mathbf{0}}^{in} \quad (40)$$

$$\frac{d\delta p_{2\mathbf{k}_L}}{dt} = -\gamma_{2k_L} \delta p_{2\mathbf{k}_L} - i\sqrt{2\gamma_{k_L} \gamma_{2k_L} (\sigma - 1)} \delta p_{\mathbf{k}_L} \\ - i\sqrt{\gamma_0 \gamma_{2k_L}} \delta p_{\mathbf{0}}^\dagger + \delta P_{2\mathbf{k}_L}^{in} \quad (41)$$

Thanks to these three equation and their conjugate equations we can calculate the output fluctuations of the pump, signal and idler fields as a function of the input fluctuations.

Amplitude fluctuations

In this paper we are mostly interested in the amplitude correlations between signal and idler. We will see that in the simple case where we neglect the renormalization effects it is

enough to solve a system of three equations. We define the real and imaginary parts of the polariton, photon and exciton fields

$$\begin{aligned}
\alpha_{\mathbf{q}} &= \delta p_{\mathbf{q}} + \delta p_{\mathbf{q}}^\dagger \\
\beta_{\mathbf{q}} &= -i (\delta p_{\mathbf{q}} - \delta p_{\mathbf{q}}^\dagger) \\
\alpha_{\mathbf{q}}^{in(out)} &= \delta P_{\mathbf{q}}^{in(out)} + \delta P_{\mathbf{q}}^{in(out)\dagger} \\
\beta_{\mathbf{q}}^{in(out)} &= -i (\delta P_{\mathbf{q}}^{in(out)} - \delta P_{\mathbf{q}}^{in(out)\dagger}) \\
\alpha_{\mathbf{q}}^{A,in(out)} &= \delta A_{\mathbf{q}}^{in(out)} + \delta A_{\mathbf{q}}^{in(out)\dagger} \\
\beta_{\mathbf{q}}^{A,in(out)} &= -i (\delta A_{\mathbf{q}}^{in(out)} - \delta A_{\mathbf{q}}^{in(out)\dagger}) \\
\alpha_{\mathbf{q}}^{B,in(out)} &= \delta B_{\mathbf{q}}^{in(out)} + \delta B_{\mathbf{q}}^{in(out)\dagger} \\
\beta_{\mathbf{q}}^{B,in(out)} &= -i (\delta B_{\mathbf{q}}^{in(out)} - \delta B_{\mathbf{q}}^{in(out)\dagger})
\end{aligned} \tag{42}$$

The mean fields $p_{\mathbf{k}_L}$ and p_0 are real positive numbers, therefore α corresponds to amplitude fluctuations and β to phase fluctuations. The mean field $p_{2\mathbf{k}_L}$ is a pure imaginary number, therefore $-\beta$ corresponds to amplitude fluctuations and α to phase fluctuations. The evolution equations for the amplitude fluctuations write

$$\begin{aligned}
\frac{d\alpha_{\mathbf{k}_L}}{dt} &= -\gamma_{k_L} \sigma \alpha_{\mathbf{k}_L} - \sqrt{2\gamma_{k_L} \gamma_0 (\sigma - 1)} \alpha_0 \\
&\quad + \sqrt{2\gamma_{k_L} \gamma_{2k_L} (\sigma - 1)} \beta_{2\mathbf{k}_L} + \alpha_{\mathbf{k}_L}^{in}
\end{aligned} \tag{43}$$

$$\begin{aligned}
\frac{d\alpha_0}{dt} &= -\gamma_0 \alpha_0 + \sqrt{2\gamma_{k_L} \gamma_0 (\sigma - 1)} \alpha_{\mathbf{k}_L} \\
&\quad - \sqrt{\gamma_0 \gamma_{2k_L}} \beta_{2\mathbf{k}_L} + \alpha_0^{in}
\end{aligned} \tag{44}$$

$$\begin{aligned}
\frac{d\beta_{2\mathbf{k}_L}}{dt} &= -\gamma_{2k_L} \beta_{2\mathbf{k}_L} - \sqrt{2\gamma_{k_L} \gamma_{2k_L} (\sigma - 1)} \alpha_{\mathbf{k}_L} \\
&\quad - \sqrt{\gamma_0 \gamma_{2k_L}} \alpha_0 + \beta_{2\mathbf{k}_L}^{in}
\end{aligned} \tag{45}$$

We get a set of three linear differential equations. Taking the Fourier transform we obtain in matrix notation

$$\begin{pmatrix}
\gamma_{k_L} \sigma - i\Omega & \sqrt{2\gamma_{k_L} \gamma_0 (\sigma - 1)} & -\sqrt{2\gamma_{k_L} \gamma_{2k_L} (\sigma - 1)} \\
-\sqrt{2\gamma_{k_L} \gamma_0 (\sigma - 1)} & \gamma_0 - i\Omega & \sqrt{\gamma_0 \gamma_{2k_L}} \\
\sqrt{2\gamma_{k_L} \gamma_{2k_L} (\sigma - 1)} & \sqrt{\gamma_0 \gamma_{2k_L}} & \gamma_{2k_L} - i\Omega
\end{pmatrix} \times \begin{pmatrix} \alpha_{\mathbf{k}_L}(\Omega) \\ \alpha_0(\Omega) \\ \beta_{2\mathbf{k}_L}(\Omega) \end{pmatrix} = \begin{pmatrix} \alpha_{\mathbf{k}_L}^{in} \\ \alpha_0^{in} \\ \beta_{2\mathbf{k}_L}^{in} \end{pmatrix} \tag{46}$$

The inversion of the 3×3 matrix provides the amplitude fluctuations of the fields $p_{\mathbf{k}_L}$, $p_{\mathbf{0}}$ et $p_{2\mathbf{k}_L}$ as a function of the input fluctuations. It is easy to deduce the amplitude fluctuations $\alpha_{\mathbf{q}}^{A, out}$ of the output light fields thanks to the input-output relationship for the cavity mirror $A_{\mathbf{q}}^{out} = \sqrt{2\gamma_{aq}}a_{\mathbf{q}} - A_{\mathbf{q}}^{in}$ and the relationship between the photon and polariton fields $a_{\mathbf{q}} = -C_q p_{\mathbf{q}}$.

$$\alpha_{\mathbf{q}}^{A, out} = -C_q \sqrt{2\gamma_{aq}} \alpha_{\mathbf{q}} - \alpha_{\mathbf{q}}^{A, in} \quad (47)$$

Input fluctuations

In this paragraph we study the noise sources in our system. $A_{\mathbf{k}_L}^{in}$ is the coherent pump laser field $A_{\mathbf{0}}^{in}$ and both other input fields $A_{2\mathbf{k}_L}^{in}$ are equal to the vacuum field. Therefore, the amplitude fluctuations of these three fields are equal to the vacuum fluctuations. The treatment of excitonic fluctuation is more complex. The amplitude noise spectra (normalized to the vacuum noise) of the three excitonic fields $B_{\mathbf{k}_L}^{in}$, $B_{\mathbf{0}}^{in}$ et $B_{2\mathbf{k}_L}^{in}$ are given by

$$S_{\alpha_{\mathbf{q}}}^{B, in}(\Omega) = 1 + 2n_{\mathbf{q}} \text{ for } \mathbf{q} = \mathbf{0}, \mathbf{k}_L, 2\mathbf{k}_L \quad (48)$$

where $n_{\mathbf{q}}$ is the mean number of excitations in the reservoir which depends on the temperature and on the pump intensity. Since the reservoir is populated through phonon-assisted relaxation from the pump mode it is a reasonable assumption to take the reservoir occupation as proportional to the mean number of excitons in the pump mode:

$$n_{\mathbf{q}} = \beta |b_{\mathbf{q}}|^2 = \beta X_q^2 |p_{\mathbf{q}}|^2 \quad (49)$$

where β is a dimensionless constant which characterizes the efficacy of the relaxation process. This simple model accounts for the excess noise of the reflected light at low excitation intensity in a satisfactory way [9].

Noise spectra

In fluctuation measurements the measured quantity is the noise spectrum. The noise spectrum $S_O(\Omega)$ of an operator O is defined as the Fourier transform of the autocorrelation function $C_O(t, t')$:

$$S_O(\Omega) = \int C_O(\tau) e^{i\Omega\tau} d\tau \quad (50)$$

with

$$C_O(t, t') = \langle O(t)O(t') \rangle - \langle O(t) \rangle \langle O(t') \rangle = \langle \delta O(t) \delta O(t') \rangle \quad (51)$$

The noise spectrum is related to the Fourier transform $\delta O(\Omega)$ of the fluctuations $\delta O(t)$ by the Wiener-Kinchine theorem:

$$\langle \delta O(\Omega) \delta O(\Omega') \rangle = 2\pi \delta(\Omega + \Omega') S_O(\Omega) \quad (52)$$

In the same way the correlation spectrum $S_{O O'}(\Omega)$ of two operators O, O' is defined as the Fourier transform of the correlation function:

$$C_{O O'}(t, t') = \langle O(t)O'(t') \rangle - \langle O(t) \rangle \langle O'(t') \rangle \quad (53)$$

The correlation spectrum is also related to the Fourier components of the fluctuations:

$$\langle \delta O(\Omega) \delta O'(\Omega') \rangle = 2\pi \delta(\Omega + \Omega') S_{O O'}(\Omega) \quad (54)$$

The relevant quantity is the normalized correlation spectrum

$$C_{O O'}(\Omega) = \frac{S_{O O'}(\Omega)}{\sqrt{S_O(\Omega) S_{O'}(\Omega)}} \quad (55)$$

One has always $|C| \leq 1$. A nonzero value of $C_{O O'}(\Omega)$ indicates some level of correlation between the two measurements.

RESULTS

Fluctuations of the intracavity polariton fields

First, in order to shed some light on the above-mentioned analogy with an OPO, we assume that all three polariton modes have the same linewidths. This is the case if the cavity and exciton linewidths are equal ($\gamma_{ak} = \gamma_{bk}$) and do not depend on k . We set $\gamma = \gamma_{k_L} = \gamma_0 = \gamma_{2k_L} = \gamma_a = \gamma_b$.

After some straightforward algebra we get the amplitude fluctuations of the polariton fields

$$\alpha_{\mathbf{k}_L}(\Omega) = \frac{1}{D(\Omega)} \left(-\gamma(\Omega + 2i\gamma) \alpha_{\mathbf{k}_L}^{in} - \gamma\sqrt{2(\sigma - 1)}(2\gamma - i\Omega) \alpha_{\mathbf{0}}^{in} + \gamma\sqrt{2(\sigma - 1)}(2\gamma - i\Omega) \beta_{2\mathbf{k}_L}^{in} \right) \quad (56)$$

$$\alpha_{\mathbf{0}}(\Omega) = \frac{1}{D(\Omega)} \left(\gamma\sqrt{2(\sigma - 1)}(2\gamma - i\Omega) \alpha_{\mathbf{k}_L}^{in} + (\gamma^2(3\sigma - 2) - \Omega^2 - i\gamma\Omega(\sigma + 1)) \alpha_{\mathbf{0}}^{in} + \gamma(\gamma(\sigma - 2) + i\Omega) \beta_{2\mathbf{k}_L}^{in} \right) \quad (57)$$

$$\beta_{2\mathbf{k}_L}(\Omega) = \frac{1}{D(\Omega)} \left(-\gamma\sqrt{2(\sigma - 1)}(2\gamma - i\Omega) \alpha_{\mathbf{k}_L}^{in} + \gamma(\gamma(\sigma - 2) + i\Omega) \alpha_{\mathbf{0}}^{in} + (\gamma^2(3\sigma - 2) - \Omega^2 - i\gamma\Omega(\sigma + 1)) \beta_{2\mathbf{k}_L}^{in} \right) \quad (58)$$

with

$$D(\Omega) = \gamma [8\gamma^2(\sigma - 1) - \Omega^2(\sigma + 2)] + i\Omega [\gamma^2(4 - 6\sigma) + \Omega^2] \quad (59)$$

Twin polaritons

Let us now calculate the fluctuations of the difference of the signal and idler amplitudes. Let r be the normalized quantity

$$r = \frac{1}{\sqrt{2}} (\alpha_{\mathbf{0}} + \beta_{2\mathbf{k}_L}) \quad (60)$$

We find

$$r(\Omega) = (4\gamma^2(\sigma - 1) - \Omega^2 - i\Omega\gamma\sigma) r^{in} \quad (61)$$

$$\text{avec } r^{in} = \frac{1}{\sqrt{2}} (\alpha_{\mathbf{0}}^{in} + \beta_{2\mathbf{k}_L}^{in})$$

It is important to notice that r does not depend on the pump fluctuations, which cancel out when we make the difference. This property is at the origin of twin beams generation in OPOs. We get perfect noise suppression for $\Omega = 0$ and $\sigma \rightarrow 1$.

In a degenerate or quasi-degenerate OPO the symmetry between signal and idler is conserved outside the cavity, because the two fields have the same frequency and are coupled in the same way to the external field through the losses of the cavity mirrors. In such systems the "twinity" of the signal and idler fields can be shown directly by measuring the fluctuations of the difference of the output signal and idler field intensities.

In our case the signal and idler polaritons do not have the same photon fraction and are not coupled in the same way to the external field. Clearly, this should lead to a significant reduction of the correlations between the signal and idler output light fields.

Fluctuations of the output light fields

Let us first comment on the relevant analysis frequency of the noise. The noise spectra vary typically over a range of the order of the polariton linewidth. In noise measurements the experimentalists have access to very small analysis frequencies (generally a few tens of MHz, i.e. a fraction of μeV) with respect to the polariton linewidths (a few hundreds of μeV). Therefore the noise at zero frequency is the relevant quantity.

The general expressions of the amplitude noises of the three modes and of the signal-idler amplitude correlation can be found in the Appendix.

In expressions (57-59) we have taken equal linewidths for the pump, signal and idler polaritons ($\gamma_{k_L} = \gamma_0 = \gamma_{2k_L}$). This assumption is not correct in most microcavity samples. Indeed the energy of the polaritons of wave vector $2\mathbf{k}_L$ is close to the energy of the nonradiative excitons ; diffusion toward these states is enhanced by their large density of states. Moreover, the idler energy is closer to the electron-hole continuum. As a result, the *excitonic* linewidth of the idler γ_{b2k_L} is larger than that of the signal γ_{b0} and pump γ_{bk_L} modes. The assumption that the *cavity* linewidth γ_{ak} does not depend on k is correct provided the three wave vectors of interest are within the stop-band of the Bragg reflectors. In recent experiments, the idler beam has been found to be about 50 times weaker than the signal beam (see e.g. Ref. [5]), which is consistent with a linewidth ratio $\gamma_{2k_L}/\gamma_0 = 5$.

We will first give the results in the ideal case (with equal linewidths and an input noise equal to the standard quantum noise), and then study the influence of the imbalance between signal and idler and the input excitonic noise.

Ideal case

The amplitude noises of the pump, signal and idler beams as well as the signal-idler amplitude correlation are drawn in Fig. 4 as a function of the pump parameter $\sigma = \sqrt{I_{\mathbf{k}_L}^{in}/I_{\mathbf{k}_L,S}^{in}}$ in the case of equal linewidths (and no input excess noise. Although the curves go up to $\sigma=5$ let us recall that the model is not correct too far above threshold where we can no longer neglect multiple diffusions.

Let us first observe that the signal and idler noise spectra have exactly the same shape. However the idler noise is drawn towards the standard quantum level due to its low photon fraction which causes important losses at the output of the cavity. It is easy to show that the ratio of the noise signals $S - 1$ is equal to the ratio of the photon fractions:

$$\frac{S_{\alpha_0}^{A, out}(\Omega) - 1}{S_{\beta_{2\mathbf{k}_L}}^{A, out}(\Omega) - 1} = \frac{C_0^2}{C_{2k_L}^2} \quad (62)$$

The signal and idler amplitude fluctuations diverge close to the threshold (for $\sigma \rightarrow 1^+$). Noise reduction is obtained above $\sigma = 1.55$. It grows with the pump intensity and saturates at a value The amplitudes of the signal and idler beams are very strongly correlated slightly above threshold. The correlation tends to one in the vicinity of the threshold ($\sigma \rightarrow 1^+$) and vanishes rapidly when increasing the pump intensity. All these results are similar to those obtained in nondegenerate OPOs [25].

Influence of the signal-idler imbalance

In this paragraphe we still suppose that there is no input excess noise ($n_0 = n_{\mathbf{k}_L} = n_{2\mathbf{k}_L} = 0$). Let us compare the results with different linewidths to those of the "balanced" case ($\gamma = \gamma_{k_L} = \gamma_0 = \gamma_{2k_L} = \gamma_a = \gamma_b$) in equations (70)-(74). It is easy to show that the excess $S - 1$ noises of the pump, signal and idler beams are respectively multiplied by γ_a/γ_{k_L} , γ_a/γ_0 and γ_a/γ_{2k_L} . The signal-idler correlation (without normalization) is multiplied by $\gamma_a/\sqrt{\gamma_0\gamma_{2k_L}}$.

As an example the case $\gamma_0 = \gamma_{k_L} = \gamma_{2k_L}/5 = \gamma_a$ is shown in Fig. 5. The amplitude noises of the pump and signal beams have not been represented since they are unchanged. The excess noise and the noise reduction are strongly reduced on the idler beam due to its larger losses (Fig. 5 (a)). The signal-idler correlation remains strong close to threshold but

decreases more rapidly with increasing pump intensity (Fig. 5 (b)).

Influence of input excess noise

We have assumed that the biggest source of noise for a given polariton mode is the luminescence of an exciton reservoir which is populated by the polariton mode itself. The input noise for a given mode is then proportional to the mean exciton number in this mode. The efficacy of this process is given by the β coefficient introduced above ; here we will assume that β has the same value for the three modes. Slightly above the oscillation threshold, the pump mode is much more populated than the signal and idler population ; then the input noise is much greater for the pump than for the signal and idler.

Fig. 6 shows an example in the "balanced" case for a noise parameter $\beta=5.10^{-5}$, evaluated from noise measurements on the light reflected by a microcavity sample [9]. The input excess noise cuts down the noise reduction. Its influence increases with the pump intensity since it is proportional to the mean exciton population. However the correlation is actually enhanced by the excess noise. It is due to the fact that the pump input noise is distributed equally between signal and idler and contributes to the correlations.

The quantum domain

Our model predicts strong correlations between the signal and idler light fields. When can we say that these beams are quantum correlated ? We will use two different criteria, one of "quantum twinity" and one associated with QND measurement.

Quantum twinity

In degenerate or quasi degenerate OPOs, the signal and idler output beams have the same mean field values and the same noise properties. Quantum correlations between them are evidenced by measuring the noise of the difference between signal and idler intensities and comparing it to the standard quantum level. The idea behind this is to compare the fields under consideration to a classical production of twin beams, which can be achieved by using a 50% beamsplitter.

In our case, one beam is much more intense than the other one (the ratio of the intensities is of the order of 10 for equal signal and idler linewidths). What happens if the two light fields A_1 and A_2 under consideration have different mean values and different noises S_1 and S_2 ? To produce classically twin beams of unequal intensities, one can use an unequal beamsplitter. The field fluctuations at the output of such a beamsplitter write

$$\delta A_1 = t\delta A_{in} + r\delta A_v \quad (63)$$

$$\delta A_2 = r\delta A_{in} - t\delta A_v \quad (64)$$

with $t \neq r$, where A_{in} is the input field and δA_v the vacuum fluctuations entering through the other port of the beamsplitter. Now the difference $\delta A_- = \delta A_1 - \delta A_2$ is not helpful, since it does not give a quantity which is independent of δA_{in} . However, the correlation between the two beams is independent of δA_{in} :

$$\langle \delta A_1 \delta A_2 \rangle_{classicaltwins}^2 = (\langle \delta A_1^2 \rangle - 1) (\langle \delta A_2^2 \rangle - 1) \quad (65)$$

which can also be written as:

$$(C_{classicaltwins})^2 = \left(1 - \frac{1}{S_1}\right) \left(1 - \frac{1}{S_2}\right) \quad (66)$$

We can evaluate the "twinty" of the beams by using the quantity

$$G = \frac{1 - C}{1 - \sqrt{\left(1 - \frac{1}{S_1}\right) \left(1 - \frac{1}{S_2}\right)}} \quad (67)$$

which is a generalization of the usual squeezing factor on the intensity difference. G smaller than 1 means that one has been able to produce two fields which are more identical than the copies from a beamsplitter. Moreover, it is possible to show that this criterium does not depend on the way by which the two classical twins are produced [27].

Experimentally, one can measure separately C , S_1 and S_2 and compute G from (67). One can also amplify in a different way the two photocurrents in order to measure the quantity $\delta A_a = a\delta A_1 - \delta A_2/a$. When $a^2 = \sqrt{S_2/S_1}$ then:

$$G = \frac{\langle \delta A_a^2 \rangle}{2} \frac{1}{\sqrt{S_1 S_2} - \sqrt{(S_1 - 1)(S_2 - 1)}} \quad (68)$$

G is proportional to the photocurrent fluctuations when the gains are adjusted so that the noise levels are identical in the two channels. The denominator in (68) can be evaluated from the excess noises of each field.

QND correlation

A further level of correlation is achieved when the information extracted from the measurement of one field provides a QND measurement of the other, so that it is possible, using the information on one field, to correct the other from a part of its quantum fluctuations and transform it into a squeezed state. This criterium is widely used in the field of QND measurement [26]. It can be expressed in terms of the conditional variance

$$V_{1|2} = S_1(1 - C^2) \quad (69)$$

Note that when the two beams have different noises ($S_1 \neq S_2$) one has two conditional variances and therefore two possible criteria. This shows that the QND criterium evaluates the correlation from the point of view of one beam, and is not an evaluation of the quantum correlation between the two fields. One possibility is to state that the two fields are QND correlated if one has $V_{1|2} < 1$ and $V_{2|1} < 1$. This criterium is stronger than the previous one [27].

Discussion

We first investigate the "QND criterium". The conditional variances are shown in Fig. 7 in the case of equal linewidths and zero input excess noise. From the point of view of the idler beam, the conditional variance is always lower than 1, if only by a few percent. From the point of view of the signal beam, the quantum domain is very small: it begins at $\sigma = 1.53$, very close to the point where it begins to be squeezed. It is only between $\sigma=1.53$ and $\sigma=1.55$ that we get "QND correlations" between beams that individually have excess noise. For $\sigma > 1.55$ the QND correlation criterium is satisfied, although the correlation is quite small, because both beams are squeezed. In conclusion, no significant "QND correlations" can be observed on the signal and idler output beams.

We now investigate the behavior of the quantity G evaluating the "twinty" of the signal

and idler beams. It is drawn in Fig. 8 as a function of the pump parameter in various cases. In the case of equal linewidths and zero input excess noise, G goes down to 0.85 which indicates the "quantum twin" character of the two beams. If we take the nonradiative losses of the idler polaritons into account (we set again $\gamma_{2k_L} = 5\gamma_0$) G only goes under 1 by a 7 percent. However the input excess noise (corresponding to the resonant luminescence of the three polariton modes) has little effect on the quantum correlations. As explained above this comes from the fact that the pump input noise (which is the strongest slightly above threshold, when the pump polariton population is much larger than the signal and idler populations) is equally distributed between the signal and idler modes and helps building up correlations.

In conclusion, in present-day microcavity samples the "quantum twinity" criterium is overcome by only a few percent. This is due to the fact that only the polariton fields are perfectly correlated, and we can only observe their photonic parts. A simple image is the following: we observe the polariton system through a beamsplitter which amplitude transmission coefficient is equal to the Hopfield coefficient C_0 , which leads to losses that destroy the quantum effects. The correlations are further reduced by the imbalance between signal and idler. The photonic part of the idler is very small (of the order of 0.05) which corresponds to large losses.

CONCLUSION

We have presented a novel quantum model allowing to calculate the quantum fluctuations of the beams produced by a semiconductor microcavity in the regime of parametric oscillation. It extends the model developed by C. Ciuti et al. above threshold and includes the noise coming from the exciton part of the polaritons.

We show that some quantum correlation exists between the signal and idler beams in the vicinity of threshold. Taking the parameters of microcavity samples which have been shown to work in the parametric oscillation regime, it can be seen that the correlation overcomes the quantum limit by a few percent. The measurement of these correlations would be of great interest, since quantum correlations between the output beams, however small, are an indication of much bigger correlations between the intracavity polariton fields. For example, in the ideal case at threshold (see Fig. ??), if we measure a gemellity $G=0.91$ this corresponds

to perfect correlations inside the cavity.

In order to observe better quantum correlations between the output beams, it is very important that the signal and idler linewidths should be made as equal as possible. A simple solution would be to use a low finesse cavity. Then the nonradiative losses would be less important with respect to the radiative losses and the ratio of the signal and idler linewidths would be smaller. A compromise has to be found because the oscillation threshold would also be higher.

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APPENDIX : NOISE AND SIGNAL-IDLER CORRELATION

In this paragraph we give the general expressions for the amplitude noises of the signal, pump and idler output light fields at zero frequency (denoted by $S_{\alpha_0}^{A, out}$, $S_{\alpha_{\mathbf{k}_L}}^{A, out}$ and $S_{\beta_{2\mathbf{k}_L}}^{A, out}$ respectively), and the signal-idler amplitude correlation at zero frequency (denoted by $S_{\alpha_0 \beta_{2\mathbf{k}_L}}^{A, out}$).

$$S_{\alpha_{\mathbf{k}_L}}^{A, out} = 1 + C_{k_L}^2 \frac{\gamma_a}{\gamma_{k_L}} \frac{1}{\sigma - 1} \times \left[1 + \frac{X_0^2 n_{\mathbf{0}} \gamma_{b0} \gamma_{2k_L} + X_{2k_L}^2 n_{2\mathbf{k}_L} \gamma_{b2k_L} \gamma_0}{\gamma_0 \gamma_{2k_L}} \right] \quad (70)$$

$$S_{\alpha_0}^{A, out} = 1 + C_0^2 \frac{\gamma_a}{\gamma_0} \frac{1}{8(\sigma - 1)^2} \times \left[-7\sigma^2 + 16\sigma - 8 + \frac{1}{\gamma_{k_L} \gamma_0 \gamma_{2k_L}} \times (8(\sigma - 1) X_{k_L}^2 n_{\mathbf{k}_L} \gamma_{bk_L} \gamma_0 \gamma_{2k_L} + (3\sigma - 2)^2 X_0^2 n_{\mathbf{0}} \gamma_{b0} \gamma_{k_L} \gamma_{2k_L} + (\sigma - 2)^2 X_{2k_L}^2 n_{2\mathbf{k}_L} \gamma_{b2k_L} \gamma_{k_L} \gamma_0) \right] \quad (71)$$

$$S_{\beta_{2\mathbf{k}_L}}^{A, out} = 1 + C_{2k_L}^2 \frac{\gamma_a}{\gamma_{2k_L}} \frac{1}{8(\sigma - 1)^2} \times \left[-7\sigma^2 + 16\sigma - 8 + \frac{1}{\gamma_{k_L} \gamma_0 \gamma_{2k_L}} \times (8(\sigma - 1) X_{k_L}^2 n_{\mathbf{k}_L} \gamma_{bk_L} \gamma_0 \gamma_{2k_L} + (\sigma - 2)^2 X_0^2 n_{\mathbf{0}} \gamma_{b0} \gamma_{k_L} \gamma_{2k_L} + (3\sigma - 2)^2 X_{2k_L}^2 n_{2\mathbf{k}_L} \gamma_{b2k_L} \gamma_{k_L} \gamma_0) \right] \quad (72)$$

$$S_{\alpha_0 - \beta_{2\mathbf{k}_L}}^{A, out} = C_0 C_{2k_L} \frac{\gamma_a}{\sqrt{\gamma_0 \gamma_{2k_L}}} \frac{1}{8(\sigma - 1)^2} \times \left[\sigma^2 - \right.$$

$$\frac{1}{\gamma_{k_L} \gamma_0 \gamma_{2k_L}} \times [(\sigma - 2)(3\sigma - 2)(X_0^2 n_0 \gamma_{b0} \gamma_{k_L} \gamma_{2k_L} + X_{2k_L}^2 n_{2\mathbf{k}_L} \gamma_{b2k_L} \gamma_{k_L} \gamma_0) - 8(\sigma - 1) X_{k_L}^2 n_{\mathbf{k}_L} \gamma_{bk_L} \gamma_0 \gamma_{2k_L}] \quad (73)$$

where n_0 , $n_{\mathbf{k}_L}$ and $n_{2\mathbf{k}_L}$ are the input excitonic noises. From these expressions, it is easy to calculate the normalized signal-idler correlation at zero frequency $C_{\alpha_0 \beta_{2\mathbf{k}_L}}^{A, out}(\Omega)$, using definition (55).

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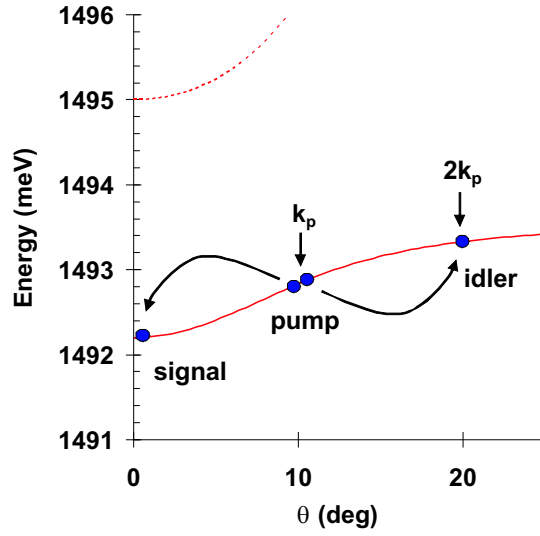


FIG. 1: Energy dispersion of the two polariton branches for a microcavity sample having a Rabi splitting of 2.8 meV, at zero cavity-exciton detuning. The arrows show the parametric conversion of the pump polaritons ($\simeq 10^\circ$) into signal (0°) and idler ($\simeq 20^\circ$) polaritons.

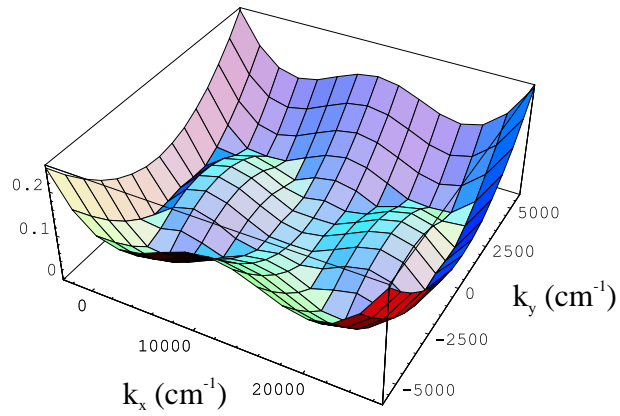


FIG. 2: Plot of the quantity $|E_P(\mathbf{k}) + E_P(2\mathbf{k}_L - \mathbf{k}) - 2E_P(\mathbf{k}_L)|$ (in meV) as a function of k_x and k_y (in cm^{-1}), for the parameters of Fig. 1.

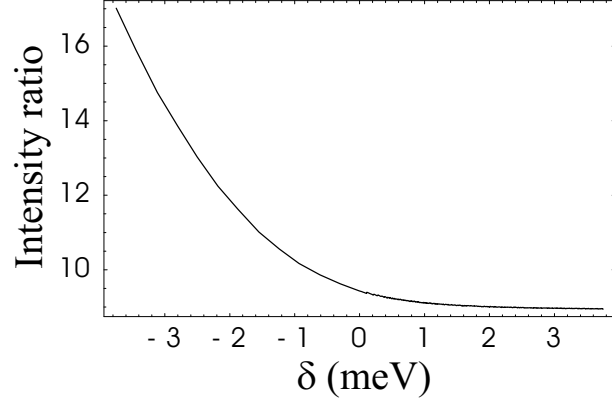


FIG. 3: Ratio of the photonic fractions of the signal and idler polaritons as a function of the cavity-exciton detuning δ . The Rabi splitting is 2.8 meV.

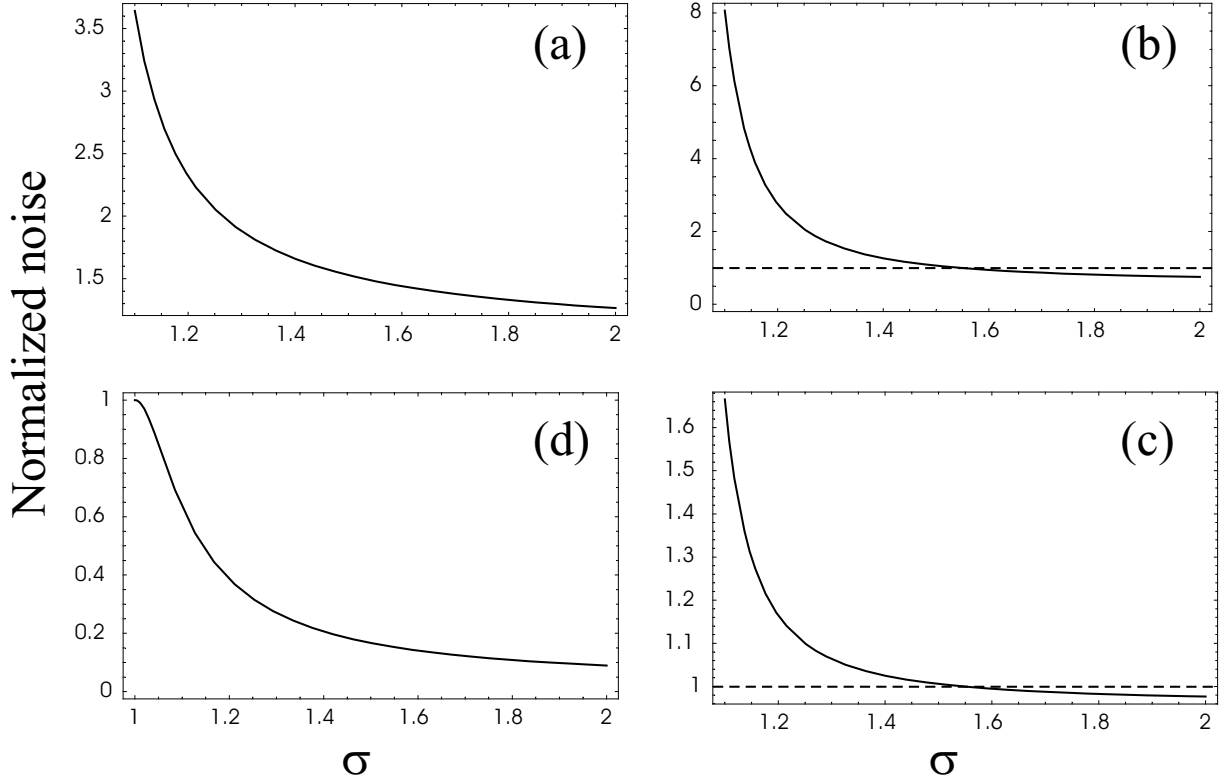


FIG. 4: (a) through (c) : amplitude noises at zero frequency of the pump, signal and idler beams respectively. (d) : signal-idler amplitude correlation at zero frequency. The three modes are assumed to have the same linewidths, and the input noise is set as equal to the standard quantum noise.

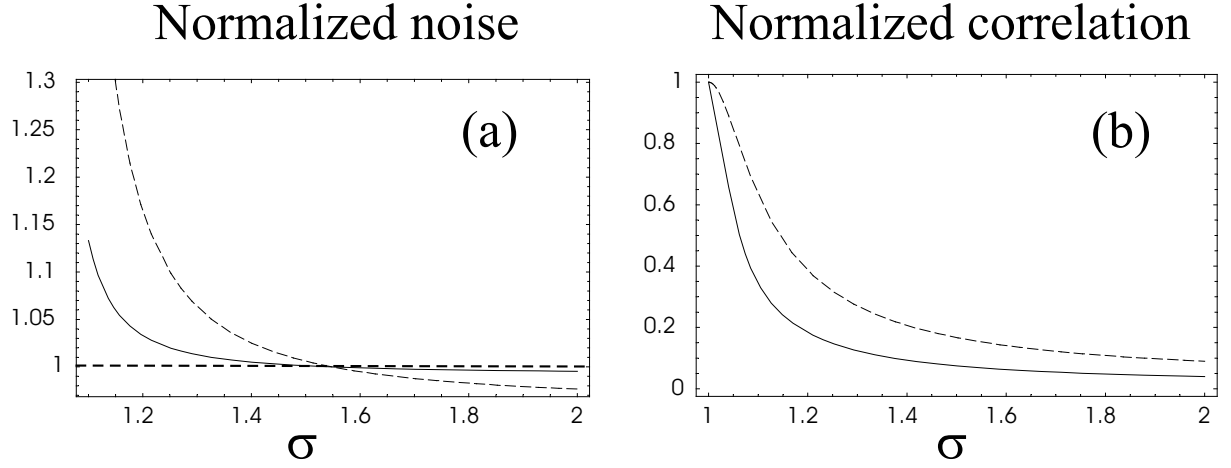


FIG. 5: (a) Amplitude noise of the idler beam and (b) signal-idler correlation at zero frequency as a function of pump intensity for $\gamma_{2k_L} = 5\gamma_0$. On both plots, the curve in dashed line is the "balanced" case $\gamma_{2k_L} = \gamma_0$.

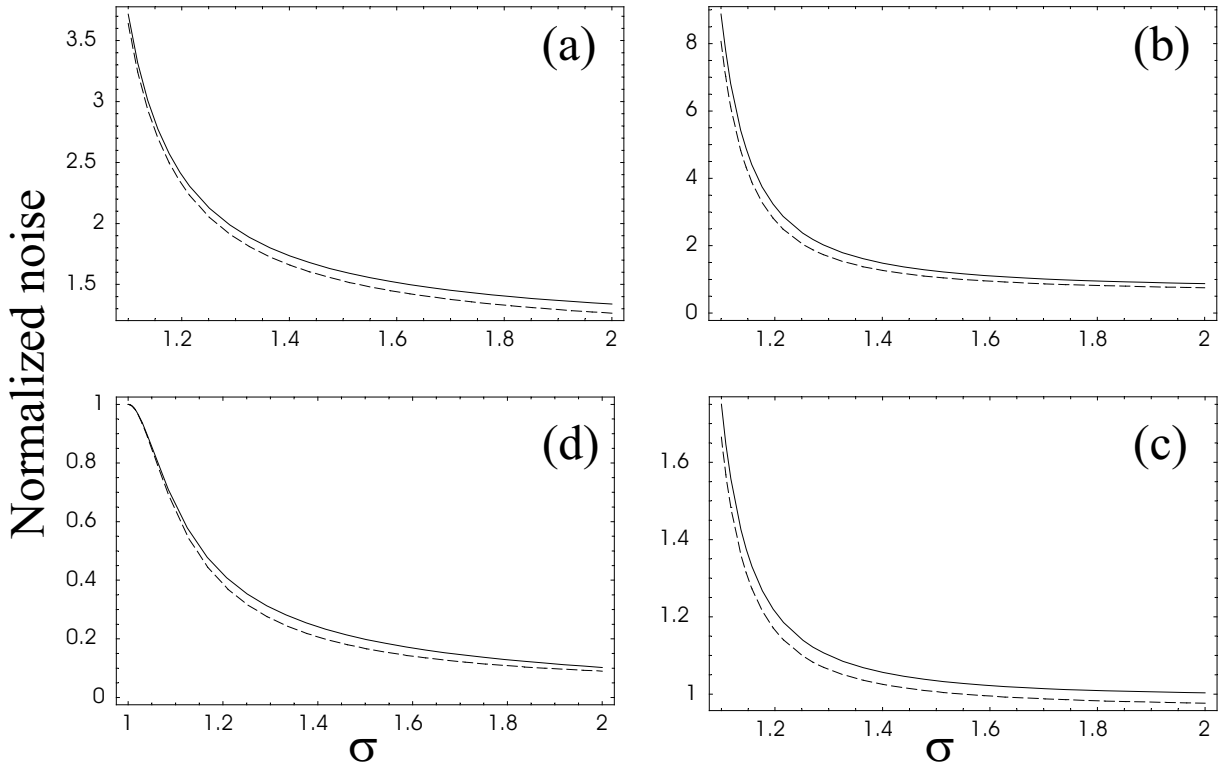


FIG. 6: Noises at zero frequency in the case of equal linewidths, with an input excess noise given by $\beta = \beta_c/2$. The ideal case $\beta = 0$ is represented on each curve as a dashed line. (a) pump beam amplitude noise ; (b) signal beam amplitude noise ; (c) idler beam amplitude noise ; (d) signal-idler amplitude correlation.

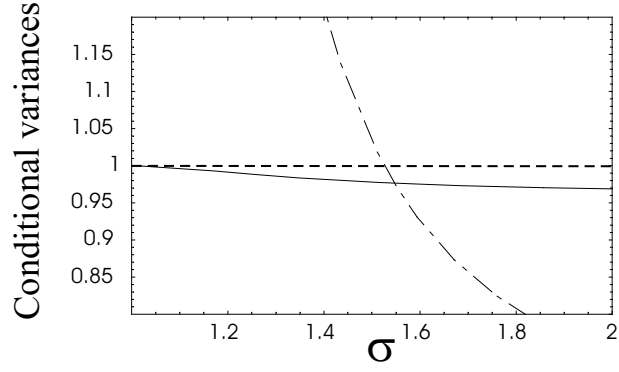


FIG. 7: (Dash-dotted line : conditional variance of the signal intensity fluctuations, knowing those of the idler ; solid line : conditional variance of the idler intensity fluctuations, knowing those of the signal. The dashed line is the standard quantum level

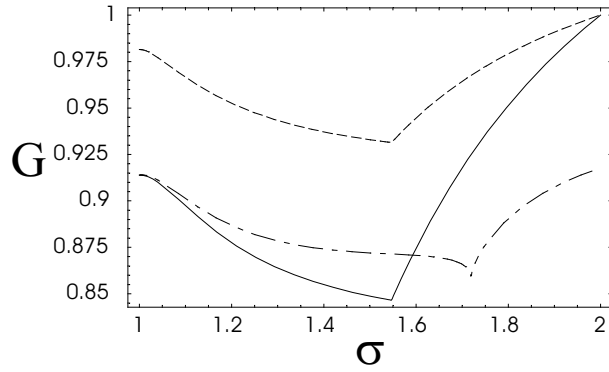


FIG. 8: Value of the "gemellity" G as a function of the pump parameter, in three different cases. (a) solid line : ideal case where all linewidths are equal and there is no excess noise. (b) dashed line : different linewidths for the signal and idler modes $\gamma_{2k_L} = 5 \gamma_0$, and no excess noise. (c) dashed-dotted line : all linewidths equal, and some excess noise given by $\beta = \beta_c/2$.