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Solitonic transmission of Bose-Einstein matter waves

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We consider a continuous atom laser propagating through a wave guide with a constriction. Two different types of transmitted stationary flow coexist. The first one coincides, at low incident current, with the non-interacting flow. As the incident flux increases, the repulsive interactions decrease the corresponding transmission coefficient. The second type only occurs for sufficiently large incident currents and has a solitonic structure. Remarkably, for any chemical potential there always exists a value of the incident flux at which the solitonic flow is perfectly transmitted.

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The transport properties of matter confined to small structures display distinct quantum effects qualitatively different from those observed at macroscopic scales. These are grounded on global phase coherence throughout the sample and can be, in many cases, understood within single particle pictures, without referring to any specific details of the system. As a result, they arise in many different fields (electronic systems, atomic physics, electromagnetism, acoustics) [1–3]. Some examples are (weak and strong) localization, Bloch oscillations and conductance quantization.

Recent experimental developments in the physics of Bose-Einstein condensation (BEC) of dilute vapor (in particular the microchip guiding technique) open up the prospect of studying coherent transport phenomena using guided atom lasers [4]. Besides, because of the extraordinary control over these systems, they offer a unique opportunity to go beyond the single particle behavior, and to study specific effects induced by interaction. In the present Letter we focus on a simple situation, where a BEC matter wave propagates through a guide with a constriction [5]. By an adiabatic approximation, the three-dimensional flow is reduced to one dimension, where the atoms now feel, due to the constriction, a longitudinal step-like potential of height V_0 . In the absence of interaction, the transmission T does not depend on the incident current but only on the beam's energy; T is always lower than unity and tends to this limit when the energy of the beam is large compared to V_0 . In the following we consider atoms with a repulsive effective interaction characterized by a scattering length $a_{sc} > 0$. The most salient features of the flow are all at variance with respect to the non-interacting case: (i) the transmission coefficient depends on the current, (ii) at given chemical potential, there exists a maximum transmitted current above which no stationary flow exists, (iii) several distinct stationary solutions with different T may coexist and (iv) for any chemical potential larger than V_0 , there exists a particular value of the incident current which induces total transmission.

Consider a continuous atom laser incident on a constriction of the waveguide. In the lowest channel, the

constriction acts as a longitudinal potential (within the adiabatic approximation [6]), whose magnitude is fixed by the ground state energy of the transverse Hamiltonian. For a BEC system, the adiabatic approximation implies that the condensate wave function can be cast in the form $\Psi(\vec{r}, t) = \psi(x, t)\phi(\vec{r}_\perp; n; x)$ (see Ref. [7]), where $\psi(x, t)$ describes the motion along the axis of the laser (the beam is flowing along the positive x direction). ϕ is the equilibrium wave function (normalized to unity) in the transverse (\vec{r}_\perp) direction. It depends parametrically on the longitudinal density $n(x, t) = \int d^2r_\perp |\Psi|^2 = |\psi(x, t)|^2$. The beam is confined in the transverse direction by a trapping potential $V_\perp(\vec{r}_\perp; x)$, which is x -dependent in the region of the constriction. Then, the longitudinal wave equation reads [7,8] (in units where $\hbar = m = 1$)

$$-\frac{1}{2}\partial_{xx}^2\psi + \left\{V_\parallel(x) + \epsilon[n(x, t); x]\right\}\psi = i\partial_t\psi. \quad (1)$$

In (1), $V_\parallel(x)$ represents an effective longitudinal potential due to the constriction. If, to be specific, we consider a transverse harmonic confinement with pulsation $\omega_\perp(x)$, then $V_\parallel(x) = \omega_\perp(x) - \omega_\perp(-\infty)$ (energy is measured with respect of the ground state energy of the non interacting transverse Hamiltonian far before the constriction). $\epsilon(n; x)$ is a nonlinear term describing the mean field interaction averaged over a transverse slice of the beam. One has $\epsilon(n; x) = 2\omega_\perp(x)na_{sc}$ in the low density regime ($na_{sc} \ll 1$), and $\epsilon(n; x) = 2\omega_\perp(x)\sqrt{na_{sc}}$ in the high density regime ($na_{sc} \gg 1$).

Our purpose is to determine the transmission of steady state solutions of (1) where $\psi(x, t) = \exp\{-i\mu t\}A(x)\exp\{iS(x)\}$, with A and S real functions. The density is $n = A^2$ and the local velocity is $v = dS/dx$. From Eq. (1) one obtains (i) flux conservation: $n(x)v(x)$ is a constant that we denote J_∞ , and (ii) a Schrödinger-like equation for the amplitude:

$$-\frac{1}{2}\frac{d^2A}{dx^2} + \left\{V_\parallel(x) + \epsilon[n(x); x] + \frac{J_\infty^2}{2n^2(x)}\right\}A = \mu A. \quad (2)$$

To define the scattering problem one needs to study the asymptotic behavior of the flow far from the constriction. Far upstream $V_\parallel(x \rightarrow -\infty) = 0$ and the non-linear term

in (1) loses its explicit x dependence, taking the simpler form $\epsilon[n(x, t)]$. Thus, in this region, (2) admits a first integral of the form [8] :

$$\frac{1}{2} \left(\frac{dA}{dx} \right)^2 + W[n(x)] = E_{cl} \quad (3)$$

$$\text{with } W(n) = -\varepsilon(n) + \mu n + \frac{J_\infty^2}{2n},$$

where $\varepsilon(n) = \int_0^n \epsilon(\rho) d\rho$ and E_{cl} is an integration constant. Eq. (3) has a simple interpretation in terms of classical dynamics. It expresses the energy conservation of a fictitious classical particle with “position” A and “time” x , moving in a potential W ; E_{cl} being the total energy of this particle. Eq. (3) is thus integrable by quadrature, and the density profile can be deduced from the plot of Fig. 1. Small values of $E_{cl} - W(n_1)$ correspond to small density oscillations, whereas the highest acceptable value is $E_{cl} = W(n_2)$, corresponding to a grey soliton.

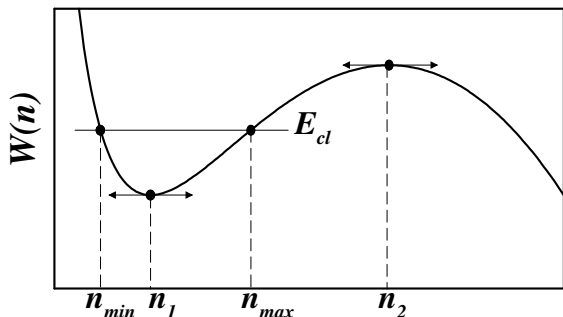


FIG. 1. W as a function of n . n_1 and n_2 are the zeros of dW/dn . At a given E_{cl} , the up-stream density (or equivalently the “position” of the fictitious particle) oscillates between n_{min} and n_{max} defined by $W(n_{min}) = W(n_{max}) = E_{cl}$.

In the far down-stream region, $V_{\parallel}(x \rightarrow +\infty)$ also takes a constant value $V_0 = \omega_{\perp}(+\infty) - \omega_{\perp}(-\infty) > 0$. Hence, (2) admits, in this region, a first integral analogous of (3) where, due to the change in ω_{\perp} , $\epsilon(n)$ (resp. $\varepsilon(n)$) takes a different form which we denote $\epsilon_0(n)$ (resp. $\varepsilon_0(n)$). The new form of $W(n)$ is denoted $W_0(n)$ and the new constant of integration is E_{cl}^0 :

$$\frac{1}{2} \left(\frac{dA}{dx} \right)^2 + W_0[n(x)] = E_{cl}^0 \quad (4)$$

$$\text{with } W_0(n) = -\varepsilon_0(n) + (\mu - V_0)n + \frac{J_\infty^2}{2n}.$$

It follows from general arguments on the dispersion of elementary excitations of Eq. (1) that the physically acceptable boundary conditions of Eq. (2) correspond to a constant far down-stream density (see [8]). The asymptotic $x \rightarrow +\infty$ density should thus be equal either to $n_{1,0}$ (we denote this as “case A”), or to $n_{2,0}$ (case B); $n_{1,0}$ and $n_{2,0}$ – being the analogous of n_1 and n_2 of Fig. 1 – are extrema of $W_0(n)$. They are solutions of

$$\mu = \epsilon_0(n) + V_0 + \frac{J_\infty^2}{2n^2}. \quad (5)$$

In the non-interacting case the term $\varepsilon_0(n)$ is absent from $W_0(n)$ which has only one minimum (n_{10}). Case A is therefore the only possible solution in non-interacting systems. Case B describes new non-perturbative effects related to interaction. It corresponds to an asymptotic down-stream density which is part of a soliton.

There exists a maximum value J_∞^{max} of J_∞ above which Eq. (5) admits no solution: as J_∞ is increased (keeping μ and V_0 fixed), the two extrema of $W_0(n)$ move toward each other, until they coalesce and disappear. This marks the onset of a time-dependent flow. If, to be specific, we consider the case $\epsilon_0(n) = g_0 n^{\nu_0}$, then

$$J_\infty^{max} = \left[\frac{2}{\nu_0 + 2} (\mu - V_0) \right]^{\frac{\nu_0 + 1}{\nu_0}} \sqrt{\nu_0} g_0^{-1/\nu_0}. \quad (6)$$

The scattering process is now well defined. It corresponds to the matching between two asymptotic densities described by the classical motion of a particle of energy E_{cl} in a potential $W(n)$ at $x \rightarrow -\infty$, and of energy E_{cl}^0 in a potential $W_0(n)$ at $x \rightarrow +\infty$ (with E_{cl}^0 either equal to $W_0(n_{1,0})$ or to $W_0(n_{2,0})$). Eq. (2) being non-linear, an important question is how to properly define a transmission and a reflection coefficient; i.e., is it possible to disentangle an incident and a reflected wave in the upstream flow? We follow here an approach closely related to usual experimental set-ups, and choose to work with an incident and a reflected beam which can be approximated by plane waves. This corresponds to a regime where Eqs. (2,3) can be linearized in the far upstream region. In this regime, for $x \rightarrow -\infty$, we write $n(x) = n_1 + \delta n(x)$ and expand $nW(n)$ to second order in δn . Then Eq. (3) leads to

$$\left(\frac{d\delta n}{dx} \right)^2 + \kappa_1^2 \delta n^2 = 8(n_1 + \delta n)[E_{cl} - W(n_1)], \quad (7)$$

where $\kappa_1^2 = 4(v_1^2 - c_1^2)$, $v_1 = J_\infty/n_1$ being the average velocity of the up-stream beam and $c_1 = [n_1(d\epsilon/dn)_{n_1}]^{1/2}$ the sound velocity of a beam with constant density n_1 . The linearization (7) is valid provided $|\delta n(x)/n_1| \ll \kappa_1^2/c_1^2$. In this regime, if one further imposes $v_1 \gg c_1$, the up-stream density oscillations can be analyzed in term of incident and reflected *particles* (and not quasi-particles). This allows to unambiguously define the incident, reflected, and transmitted current as $J_i = (n_1 + \delta n_1/2)v_1$, $J_r = \delta n_1 v_1/2$ and $J_t = n_1 v_1 = J_\infty$ (where $\delta n_1 = 4[E_{cl} - W(n_1)]/\kappa_1^2$). Hence, once E_{cl} is known, the transmission at given incident current J_i is determined through

$$E_{cl} = W(n_1) + \frac{\kappa_1^2}{4} \delta n_1 = W(n_1) + \frac{\kappa_1^2}{2v_1} J_i (1 - T). \quad (8)$$

The linearization procedure explained so far is valid in the case of small upstream interaction (this is the essence

of the condition $v_1 \gg c_1$). However, all interaction effects are fully taken into account in the down-stream region, where they are indeed more important (the constriction acts as a barrier which lowers the velocity of the down-stream flow and, by flux conservation, increases its density [9]). In the following we solve the exact non-linear equation (2) and use the linearization procedure only to define the transmission coefficient T . Thus, the results presented below are of very general validity, but their analysis in term of transmission coefficient is only correct in so far as the linearization procedure is valid.

The method is now the following: for a given μ and J_i , assume a particular value of T . This determines $J_\infty = T J_i$, fixes the form of the function $W_0(n)$, the value of $n(+\infty)$ (it is equal to $n_{1,0}$ in case A and to $n_{2,0}$ in case B) and of $E_{cl}^0 = W_0[n(+\infty)]$. Integrating (2) backwards from $x = +\infty$ to $x = -\infty$ yields E_{cl} , which should be compatible with (8). If not, the value of T has to be modified until self-consistency is achieved.

To understand the physical picture we consider an abrupt step-like constriction. In this geometry, numerical integration of (2) can be bypassed because E_{cl} is simply expressed in terms of E_{cl}^0 (see Eq. (9)). We have numerically checked that the adiabatic approximation is essentially exact for non-interacting atoms in the regime studied below. The wide range of validity of the adiabatic approximation is well established for linear waves (see, e.g., [10]), but much less is known in the nonlinear case [11].

We thus consider the case $V_{\parallel}(x < 0) = 0$, $\epsilon(n; x < 0) = \epsilon(n)$ and $V_{\parallel}(x > 0) = V_0$, $\epsilon(n; x > 0) = \epsilon_0(n)$. Eq. (2) admits the first integral (3) for all $x \leq 0$ and (4) for all $x \geq 0$. E_{cl} is determined through E_{cl}^0 by imposing continuity of A and A' at $x = 0$:

$$E_{cl} - W[n(0)] = E_{cl}^0 - W_0[n(0)]. \quad (9)$$

Let's consider case A first. The asymptotic down-stream density is $n_{1,0}$ and thus one has, for all $x \geq 0$, $n(x) = n_{1,0}$ (the fictitious classical particle remains at the bottom of the potential well W_0). In particular, $n(0) = n_{1,0}$ and the matching (9) determines E_{cl} uniquely. The value of T is denoted T^A in this case.

Case B is more interesting because the structure of the down-stream solution is richer: $n(x \geq 0)$ being part of the profile of a grey soliton, $n(0)$ can be varied continuously provided the matching (9) is fulfilled at a value acceptable for Eq. (3). Effectively, the only restriction imposed is that $n_{min} < n(0) < n_{max}$ (n_{min} and n_{max} are defined in Fig. 2). As a result, for fixed μ and J_i , E_{cl} is not uniquely determined by $n(+\infty)$, and the transmission T^B varies between 0 and a value that we denote as T_{max}^B .

To illustrate the different possible stationary flows, we consider a continuous beam of ^{23}Na atoms propagating through a guide with a transverse confinement $\omega_{\perp}(x < 0) = 2\pi \times 1$ kHz, to which we impose a narrowing with $\omega_{\perp}(x > 0) = 2\pi \times 5$ kHz. This represents

a barrier of height $V_0 = 192$ nK. We take $\mu = 220$ nK, which corresponds to the kinetic energy of atoms having a velocity of 1.3 cm/s. In the region $x < 0$, $\epsilon(n) = gn$ with $g = 2a_{sc}\omega_{\perp}(x < 0) = 264$ nK.nm. In the region $x > 0$, the transverse frequency of the guide is multiplied by 5, and thus $\epsilon_0(n) = g_0n$ with $g_0 = 5 \times g$. An important parameter of the system is the maximum transmitted current J_{∞}^{max} above which no stationary flow can exist in the guide. From Eq. (6) (with $\nu_0 = 1$) one obtains $J_{\infty}^{max} = 3.7 \times 10^4$ atom/s.

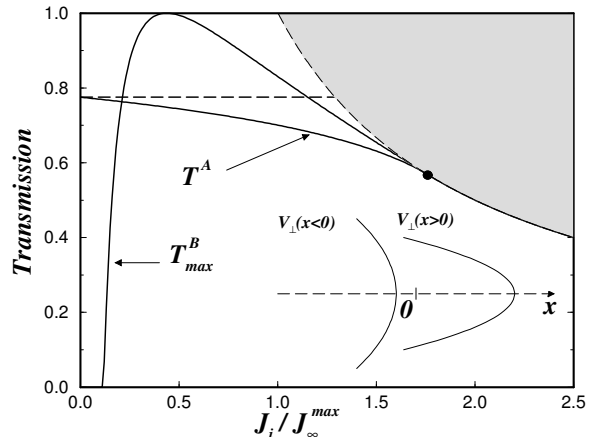


FIG. 2. T^A and T_{max}^B as a function of J_i/J_{∞}^{max} (for fixed μ). The horizontal dashed line is the transmission $T^{(0)}$ of non-interacting atoms. The grey zone above the dashed hyperbola (of equation $T = J_{\infty}^{max}/J_i$) is a region where no stationary flow exists. Inset: Schematic of the constriction's geometry.

Fig. 2 summarizes the results obtained. The linearization condition $v_1 \gg c_1$ is extremely well satisfied in the whole range of incident currents considered (the less favorable case occurs at large J_i , where $v_1 \sim 20 c_1$). The horizontal dashed line is the value of the transmission coefficient of non-interacting atoms: $T^{(0)} = 4[\mu(\mu - V_0)]^{1/2}(\sqrt{\mu} + \sqrt{\mu - V_0})^{-2} = 0.78$. It is current independent. When the current is increased from zero, T^A decreases from this value down to $T^A = 0.57$. At this point (located with a black spot on the figure), $J_{\infty} (= T^A J_i)$ is equal to J_{∞}^{max} , and a stationary flow of type A is no longer permitted (actually it bifurcates to a type B solution). The prominent feature of the behavior of T^A as a function of J_i is its decrease compared to the non-interacting value $T^{(0)}$. The physical reason behind this phenomenon is simple: the available kinetic energy necessary to step over the barrier is reduced when the interaction energy increases, i.e., when the incident current increases. This picture is supported by a perturbative treatment which accurately describes the flow at low incident current ($J_i \ll J_{\infty}^{max}$) and confirms that the decrease of T^A corresponds to an increased fraction of the interaction energy in the chemical potential.

Case B being mediated via interaction, does not exist

for low current; it exists only above a current $(2g_0)^{-1}(\mu - V_0)^2(8\mu)^{-1/2} = 4.3 \times 10^3$ atom/s = $0.11 J_\infty^{max}$. From this point, T_{max}^B increases rapidly up to 1 (reached at $J_i = 0.45 J_\infty^{max}$ in the case of Fig. 2), and then decreases down to a point where one can show that it exactly meets the end point of T^A . From there on, the value of T^B is limited by the condition that the flow should be stationary, and one has $T_{max}^B = J_\infty^{max}/J_i$, which coincides with the dashed hyperbola in Fig. 2.

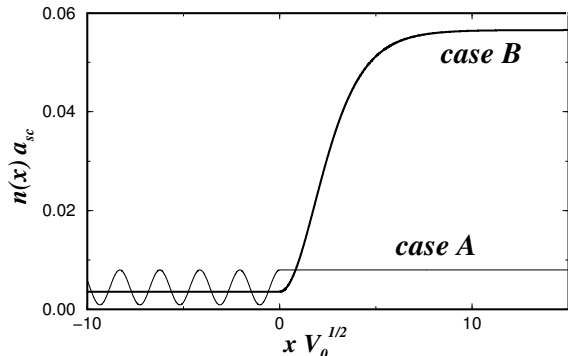


FIG. 3. Density profile (in dimensionless units) for the constriction defined in the text, at incident current $J_i = 0.44 J_\infty^{max}$ (flowing from left to right). The thick solid line corresponds to a perfectly transmitted solitonic flow (case B) that coexists with a type-A flow with $T^A = 0.75$ (thin solid line).

The nonlinear transport induced by the repulsive two-body interaction has therefore a non-trivial consequence: new soliton-like solutions emerge which allow for an increased transmission. This contrasts with the behavior of case B where the transmission is lowered by the interaction. One can show that, for any value of $\mu > V_0$, there always exists a value of J_i such that complete transmission exists in case B: the profile for $T_{max}^B = 1$ consists of a constant up-stream density $n(x \leq 0) = n_1$ connected at $x = 0$ to half a soliton. Fig. 3 displays the density profiles of the two stationary flows at an incident current corresponding to the perfect transmission in case B (see Fig. 2). Notice the significant difference in the height of the down-stream densities, a purely nonlinear effect mediated by a solitonic profile. We have performed numerical computations that show that the same type of solution also exists for smooth constrictions and that they are dynamically stable, as confirmed by a Bogoliubov analysis. The interactions can thus have two different and, in some sense, opposite consequences on the transport properties of a condensate flow. They diminish the transmission in some instances (case A), but also allow for new stationary flows that can be perfectly transmitted (case B) [12].

The effect of the constriction is to induce a step-like potential in the longitudinal motion of the condensate. There are other ways to experimentally realize the same

effect. For instance, by applying a blue-detuned laser beam on the region $x > 0$ of a condensate propagating along a guide of constant diameter. In this case, the characteristics of the flow and the barrier should be easily controlled by modifying the laser's frequency, intensity and waist, thus allowing for a neater experimental observation of the above predicted transport phenomena.

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