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# Spinodal in asymmetric nuclear matter

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## Abstract

The phase diagram of nuclear matter is quite rich - it shows such phenomena as phase-transitions, formation of condensates, clustering, etc. From the analysis of the spinodal instability, one can learn about the region of liquid-gas coexistence in nuclear matter at low densities and finite isospin asymmetries. In a recent paper, we have shown that asymmetric nuclear matter at sub-nuclear densities should undergo only one type of instability. The associated order parameter is dominated by the isoscalar density and so the transition is of liquid-gas type. The instability goes in the direction of a restoration of the isospin symmetry leading to a fractionation phenomenon.

The nuclear interaction is very similar to the Van der Waals potential which acts between molecules. For this reason, below saturation density, the nuclear interaction is also expected to lead to a liquid-gas phase transition [1]. Recently, a converging ensemble of experimental signals seems to have established the phase transition. One is the spinodal decomposition [2] which consider volume instabilities (domain of negative incompressibility). One expect that the system which enter in such a forbidden region will favorably breakup into nearly equal-sized “primitive” fragments in relation to the wavelengths of the most unstable modes [3]. How this simple picture is modified by the asymmetry charge ? Can we expect new signals related to the collision of very asymmetric nuclei ?

# 1 Stability analysis

Let us consider asymmetric nuclear matter (ANM) characterized by a proton and a neutron densities  $\rho_i = \rho_p, \rho_n$ . These densities can be transformed in a set of 2 mutually commuting charges  $\rho_i = \rho_1, \rho_3$  where  $\rho_1$  is the density of baryons,  $\rho_1 = \rho_n + \rho_p$ , and  $\rho_3$  the asymmetry density  $\rho_3 = \rho_n - \rho_p$ . In infinite matter, the extensivity of the free energy implies that it can be reduced to a free energy density :  $F(T, V, N_i) = V\mathcal{F}(T, \rho_i)$ . The system is stable against separation into two phases if the free energy of a single phase is lower than the free energy in all two-phases configurations. This stability criterium implies that the free energy density is a convex function of the densities  $\rho_i$ . A local necessary condition is the positivity of the curvature matrix :

$$[\mathcal{F}_{ij}] = \left[ \frac{\partial^2 \mathcal{F}}{\partial \rho_i \partial \rho_j} \Big|_T \right] \equiv \left[ \frac{\partial \mu_i}{\partial \rho_j} \Big|_T \right] \quad (1)$$

where we have introduced the chemical potentials  $\mu_j \equiv \frac{\partial F}{\partial N_j} \Big|_{T, V, N_i} = \frac{\partial \mathcal{F}}{\partial \rho_j} \Big|_{T, \rho_{i \neq j}}$ .

We represent in Fig. 1 the energy surface as a function of  $\rho_n$  and  $\rho_p$ , deduced from SLy230a Skyrme interaction [10]. In the symmetric case ( $\rho_n = \rho_p$ ), one can see the negative curvature of the energy which defines the spinodal area, whereas in pure neutron matter ( $\rho_p = 0$ ), no negative curvature and so no spinodal instability are predicted. We can also notice that the isovector density dependence is almost parabolic illustrating the positivity of  $\mathcal{F}_{33}$ .

We show in Fig. 2 several aspects of the spinodal contour defined as the region where the matrix  $[\mathcal{F}_{ij}]$  is negative. In the left part is plotted the spinodal contour in ANM for several forces. It exhibits important differences. In the case of SLy230a force (as well as SGII, D1P), the total density at which spinodal instability appears decreases when the asymmetry increases whereas for SIII (as well as D1, D1S) it increases up to large asymmetry and finally decreases. We observe that all forces which fulfill the global requirement that they reproduce symmetric nuclear matter (SNM) equation of state as well as the pure neutron matter calculations, leads to the same curvature of the spinodal region. We can appreciate the reduction of the instability when we go away from SNM. However, large asymmetries are needed to induce a sizable effect. The temperature dependence of the spinodal contour can be appreciated in the right part of Fig. 2. As the temperature increases the spinodal region shrinks up to the critical temperature for which it is reduced

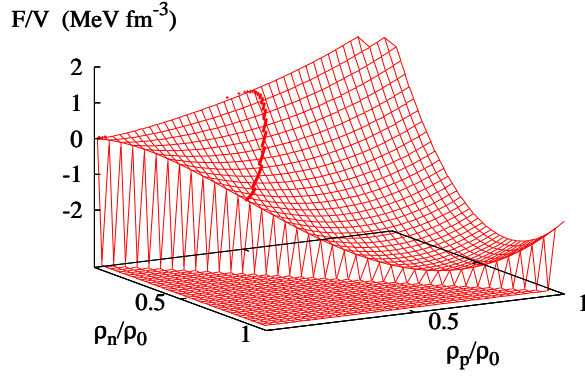


Figure 1: This figure represents the energy surface as a function of the densities  $\rho_n$  and  $\rho_p$  for the SLy230a interaction. The contour delimitate the spinodal area.

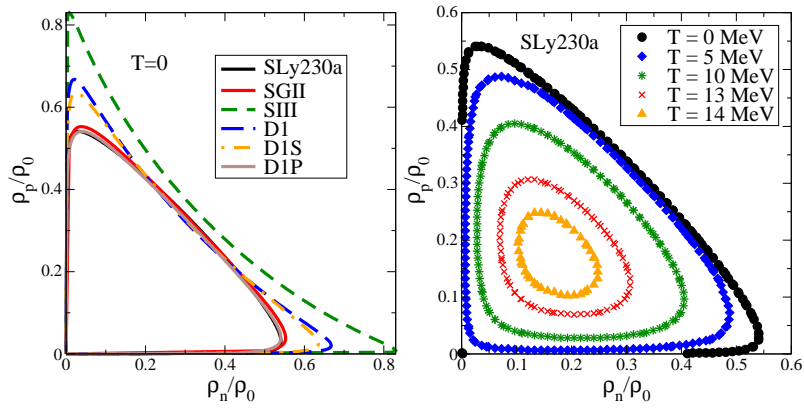


Figure 2: This two figures are a projection of the spinodal contour in the density plane : left, for Skyrme (SLy230a [10], SGII [11], SIII [12]) and Gogny models (D1 [9], D1S [13], D1P [14]) ; right, temperature dependence of the spinodal zone computed for the SLy230a case.

to SNM critical point. However, up to a rather high temperature ( 5 MeV) the spinodal zone remains almost identical to the zero temperature one.

Almost all theoretical predictions has been made with simplified Skyrme interactions : in medium nucleonic masses are taken as the free masses and spin exchange terms proportionnal to  $x_i$  are not explicitly treated. In our case, we have included the standard terms of the interactions that we refer to. Fig. 3 shows a comparison between our calculation and one of those simplified interactions (used by Baran et al [8]). In SNM ( $y=0.5$ ), exploring high temperatures means exploring the  $k$  dependance of the single particle potential, hence the  $k$ -effective mass of the nucleons in the medium. On the counter part, increasing the asymmetry means being sensitive to the isospin dependance of the effective mass. As the  $k$ -effective mass used by Baran et al is independant of the asymmetry parameter but is only a function of the total density, we can conclude that the effective mass (according to SLy230b) reduces the critical temperature in SNM by about 1 MeV while it increases the critical temperature by 1 MeV in very asymmetric matter. This comparison shows that for qualitative predictions, a simple interaction is enough but quantitative predictions requires the standard interaction.

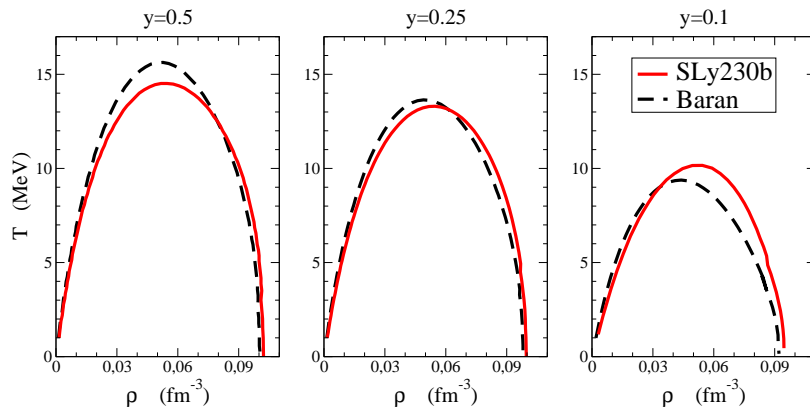


Figure 3: Comparison between the results obtained by Skyrme simplified Baran et al interaction [8] and the one obtained with SLy230b Skyrme interaction.

## 2 Analysis of the curvature matrix $[\mathcal{F}_{ij}]$

In the considered two-fluids system, the  $[\mathcal{F}_{ij}]$  is a  $2 \times 2$  symmetric matrix, so it has 2 real eigenvalues  $\lambda^\pm$  [8] :

$$\lambda^\pm = \frac{1}{2} \left( \text{Tr} [\mathcal{F}_{ij}] \pm \sqrt{\text{Tr} [\mathcal{F}_{ij}]^2 - 4 \text{Det} [\mathcal{F}_{ij}]} \right) \quad (2)$$

associated to eigenvectors  $\delta\rho^\pm$  defined by ( $i \neq j$ )

$$\frac{\delta\rho_j^\pm}{\delta\rho_i^\pm} = \frac{\mathcal{F}_{ij}}{\lambda^\pm - \mathcal{F}_{jj}} = \frac{\lambda^\pm - \mathcal{F}_{ii}}{\mathcal{F}_{ij}} \quad (3)$$

Eigenvectors associated with negative eigenvalue indicate the direction of the instability. It defines a local order parameter since it is the direction along which the phase separation occurs. The eigen values  $\lambda$  define sound velocities,  $c$ , by  $c^2 = \frac{1}{18m} \rho_1 \lambda$ . In the spinodal area, the eigen value  $\lambda$  is negative, so the sound velocity,  $c$ , is purely imaginary and the instability time  $\tau$  is given by  $\tau = d/|c|$  where  $d$  is a typical size of the density fluctuation.

The requirement that the local curvature is positive is equivalent to the requirement that both the trace ( $\text{Tr}[\mathcal{F}_{ij}] = \lambda^+ + \lambda^-$ ) and the determinant ( $\text{Det}[\mathcal{F}_{ij}] = \lambda^+ \lambda^-$ ) are positive

$$\text{Tr}[\mathcal{F}_{ij}] \geq 0, \text{ and } \text{Det}[\mathcal{F}_{ij}] \geq 0 \quad (4)$$

The use of the trace and the determinant which are two basis-independent characteristics of the curvature matrix clearly stresses the fact that the stability analysis should be independent of the arbitrary choice of the thermodynamical quantities used to label the state e.g.  $(\rho_p, \rho_n)$  or  $(\rho_1, \rho_3)$ . If Eq. 4 is violated the system is in the unstable region of a phase transition. Two cases are then possible : i) only one eigenvalue is negative and one order parameter is sufficient to describe the transition or ii) both eigenvalues are negative and two independent order parameters should be considered meaning that more than two phases can coexist. For ANM below saturation density, the case ii) never occurs since the asymmetry energy has always positive curvature ( $\mathcal{F}_{33}$ ). A complete discussion is presented in [4].

Let us now focus on the direction of the instability. If  $\delta\rho^-$  is along  $y=\text{cst}$  then the instability does not change the proton fraction. For symmetry reasons pure isoscalar ( $\delta\rho_3 = 0$ ) and isovector ( $\delta\rho_1 = 0$ ) modes appears only for SNM so it is interesting to introduce a generalization of isoscalar-like and

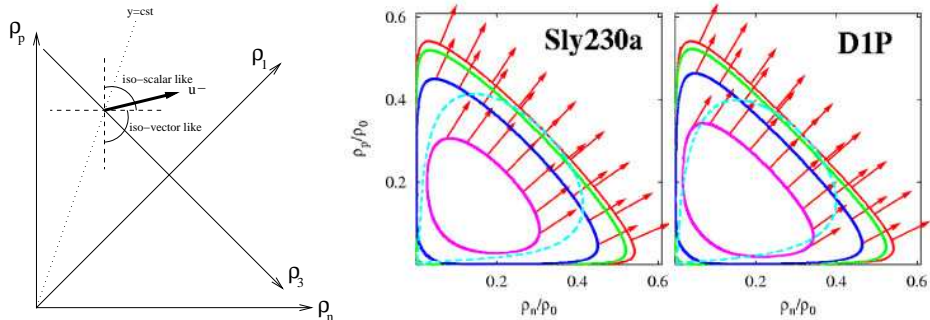


Figure 4: On the left, we illustrate the generalization of the definition of isoscalar and isovector modes. Various contour of equal imaginary sound velocity are also represented for SLy230b and D1P interactions. The more internal curve correspond to the sound velocity  $i0.09c$ , after comes  $i0.06c$ ,  $i0.03c$  and finally 0, the spinodal boarder. We observe that in almost all the spinodal region the sound velocity is larger than  $0.06c$ . The arrows indicate the direction of instability. The mechanical instability is also indicated (dotted line).

isovector-like modes by considering if the protons and neutrons move in phase ( $\delta\rho_n^- \delta\rho_p^- > 0$ ) or out of phase ( $\delta\rho_n^- \delta\rho_p^- < 0$ ). We propose a generalization of the definition of isoscalar and isovector modes in ANM. According to the left graph of Fig. 4, the instability is of isoscalar type if its direction points in the direction of the first bissectix with an absolute value angle less that 45 degrees, while it is of isovector kind if its directions points in the direction of the second bissectrix with an absolute value angle less that 45 degrees. The two figures on the rigth part of Fig. 4 shows the direction of instabilities along the spinodal boarder and some iso-instability lines. We observed that instability is always almost along the  $\rho_1$  axis meaning that it is dominated by total density fluctuations even for large asymmetries. This shows that the unstable direction is of isoscalar nature as expected from the attractive interaction between proton-neutron [8, 4]. The total density is therefore the dominant contribution to the order parameter showing that the transition is between two phases having different densities (i.e. liquid-gas phase transition). The angle with the  $\rho_1$  axis is almost constant along a constant  $y$  line. This means that as the matter enters in the spinodal zone and then dives into it, there are no dramatic change in the instability direction which remains essentially a density fluctuation. Moreover, the unstable eigenvec-

tor drives the dense phase (i.e. the liquid) towards a more symmetric point in the density plane. By particle conservation, the gas phase will be more asymmetric leading to the fractionation phenomenon. Those results are in agreement with recent calculation for ANM [8] and nuclei [15].

### 3 Are they new signals in asymmetric matter ?

A frequent discussion can be found in the literature [5, 7]. It is argued that asymmetric nuclear matter do not only present a mechanical instability for which the density is the order parameter but also a broader chemical instability associated with fluctuations of the matter isospin content [5]. Indeed, it is usually argued that it exists a region in which the compressibility at constant isospin asymmetry is negative (see Fig.1 of [6]) leading to the interpretation that the system is mechanically unstable. Above a maximum asymmetry the isotherms at constant asymmetry does not presents any back bending leading to the idea that the system is mechanically stable. However, looking at the equilibrium of the chemical potentials one can see that above this maximum asymmetry for mechanical instabilities the system may amplify fluctuations in the proton neutron concentration leading to a second instability region usually called chemical instabilities.

However, we have recently shown that this splitting of the spinodal region into two types of instabilities, a mechanical and a chemical one, is not correct and that ANM present only one type of instability [4] (hereafter called the first argument). This result is robust because it can be related to the density dependance of the asymmetry energy reproduced by several models of the nuclear interaction (Skyrme, Gogny, Brueckner). In a recent proceeding [6], we have discussed that figures like Fig.1 of [6] may lead to the unphysical separation of two area in the spinodal. Indeed, these two regions are artefact in the sense that it comes from a 2-dimensionnal projection of the 3-dimensionnal representation of the free energy (see for instance Fig.1). At the entrance of the spinodal region, only one direction is unstable (the direction pointed out by the eigen vector, see Fig.4). The further the system sink inside the spinodal, the wider become the unstable directions and each point in the spinodal is a saddle point. Deep inside the spinodal, the particular direction  $y = \rho_p/\rho = \text{const}$  becomes also unstable but the system do not enter



inside a new phase as it is often claimed.

Here, we will give a third argument against the artificial separation of chemical and mechanical instabilities. This argument is based on the relations first demonstrated by Baran et al [8]. It rely the eigen modes to the chemical and mechanical definitions. It reads:

$$\frac{\partial P}{\partial \rho_1}|_{T,y} = \frac{\lambda^-}{\sqrt{t}}(t \cos \beta + \sin \beta)^2 + \frac{\lambda^+}{\sqrt{t}}(t \sin \beta - \cos \beta)^2 \quad (5)$$

$$\frac{\partial \mu_p}{\partial y}|_{T,P} = \rho_n \lambda^+ \lambda^- \left( \frac{\partial P}{\partial \rho_1}|_{T,y} \right)^{-1} \quad (6)$$

where  $\beta = 1/2 \arctan \mathcal{F}_{np}/(\mathcal{F}_{pp} - \mathcal{F}_{nm})$  and  $t = \rho_n N_0^n / \rho_p N_0^p$ . In SNM,  $t = 1$  and  $\beta = \pi/4$ . Then, the relation between the eigen values  $\lambda^+$ ,  $\lambda^-$  and the definition of mechanical and chemical instabilities is trivial: below saturation density, SNM is unstable toward density fluctuations which means that  $\lambda^-$  or  $\frac{\partial P}{\partial \rho_1}|_{T,y}$  become negative. Beyond saturation density, some interactions manifest an instability in the isospin channel, and once again, it can be associated to the negativity of  $\lambda^+$  or  $\frac{\partial \mu_p}{\partial y}|_{T,P}$ . Hence, it is totally equivalent to discuss the negativity of the eigen values  $\lambda^+$ ,  $\lambda^-$  or the onset of a chemical and mechanical instability. But, in ANM, the simplicity of this equivalence is not preserved as it is shown by Eq. 5 and Eq. 6. Hence, one should trust only the eigen analysis of the curvature matrix  $[\mathcal{F}_{ij}]$  and this analysis have shown that only one of the two eigen values ( $\lambda^-$ ) changes its sign. Then, there is one unstable mode in ANM below saturation density.

## 4 Conclusion

Finally, we have presented three arguments in favor of the fact that ANM does not present two types of spinodal instabilities, a mechanical and chemical, but only one which is dominantly of isoscalar nature. This means that the instability is always dominated by density fluctuations and so can be interpreted as a liquid-gas separation. The instabilities tend to restore the isospin symmetry for the dense phase (liquid) leading to the fractionation of ANM. We have shown that changing the asymmetry up to  $\rho_p < 3\rho_n$  does not change quantitatively the density at which instability appears, neither the imaginary sound velocity compared to those obtained in SNM. The quantitative predictions concerning the shape of the spinodal zone as well as the instability times depends upon the chosen interaction but converge for the

various forces already constrained to reproduce the pure neutron matter calculation.

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